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Loan Portfolio Swaps Under Capital Regulation and Deposit Insurance: A Bilateral Approach

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Abstract: Using a bilateral approach, we document a loan portfolio swap for lending management. This swap provides insurance against credit-related losses through diversification. We find that the bank's optimal non-swap-performing (swap-performing) loan rate is negatively (positively) related to its credit improvement, to its counterparty's credit deterioration, to the capital-to-deposits ratio, and to the deposit insurance premium under strategic substitutes if the bank is sufficiently powerful in the two loan markets. The most obvious application of our result is to the theory of how a bank should select a lending portfolio to compete. The strategic effect on one lending market in another market must be considered. Our findings provide alternative explanations for loan portfolio swap transactions concerning bank loan-rate-setting behavior and regulation.

Keywords: *Loan Portfolio Swap; Loan-Rate Setting; Capital Regulation; Deposit Insurance*

JEL Classification: *G13, G21*

I. Introduction

Berger (2003) presented statistics that illustrate some of the changes in performance and structure of the banking industry in the United States. The annual average growth rate of gross total assets in the industry was only 3.0% during the period 1984-2001. However, the annual average growth in the notional interest-rate swap value was 27.9% in real terms during that period. Longstaff, Santa-Clara, and Schwartz (2001) pointed out that the International Swaps and Derivatives Association (ISDA) reported that the total notional amount for swap-related derivatives outstanding at the end of 1997 was over $4.9 trillion. This was more than 300 times the $15 billion notional for all Chicago Board of Treasury notes and bond futures options combined. These statistical data motivate this analysis, contributing to a view of evolution in development of swap-related derivatives and bank lending under the authority regulation.

Banks and regulatory authorities must always deal with asset quality problems. Many banks have experienced borrower defaults due to their exposure to specialized debts, such as agricultural and industrial loans. One risk of bank lending is credit risk, the risk of borrower default. Neal (1996) pointed out that the credit risk faced by a bank is sufficiently high for two reasons. First, banks with loan portfolios are usually concentrated in particular industries or geographic areas, have limited ability to diversify the credit risk across borrowers. Second, credit risk is pre-dominated because the credit risk premium is fixed when the loan is made to a business. Under these circumstances, a bank tends to conduct a credit derivative to manage and control the credit risk from potential asset quality problems.[*] As Neal suggests, a loan portfolio swap is a credit derivative that provides insurance against credit-related losses through diversification. Thus, banks have defaults due not only to an exposure to common risk factors, but also to firm-specific risks that are termed counterparty risks. The counterparty risk is important to loan portfolio swaps. The

[*] Neal (1996) suggested three types of credit derivatives: credit swaps, credit options, and credit-linked notes, and show how they manage and control credit risk. A loan portfolio swap discussed in this paper is a type of credit swap.

issues of how credit states and regulation conditions jointly determine the optimal bank interest margin, which is the difference between the interest rates the bank charges borrowers and the rate the bank pays to depositors, deserve closer scrutiny.

Academic interest in analyzing swap default risk is not new.[†] Depending on the specifics of the credit derivative instrument, Jarrow and Yu (2001) demonstrated that default risk can enter in the following three ways. First, in the case of an over-the-counter options contract, credit sensitivity can be associated with the underlying asset on which the contract is written, or it can be associated with the writer of the contract. The options contract subject to default is said to the be "vulnerable." Jarrow and Turnbull (1995), for example, provided a model to price vulnerable options. Second, in the case of interest rate swap, credit risk is viewed as a two-sided state because both parties in the swap can default. Duffie and Huang (1996), and Jarrow and Turnbull (1997), established reduced-form models on two-sided default risk applied to swaps. Third, in the case of a default swap, the default risk of the two counterparties and the reference asset must be considered simultaneously. Jarrow and Yu, for example, focused on the counterparty risk and the pricing of defaultable securities.

Despite these complexities, the pricing of credit derivatives is easily managed by the martingale pricing technique. In the following literature review, we focus on default swaps. Whittaker (1987) used an option approach to exogenously value the credit exposure of interest rate swaps. Cooper and Mello (1991) analyzed the exchange of financial claims from risk swap under the option-pricing framework, and

[†] For example, see Cooper and Mello (1991), and Sorensen and Bollier (1994).

showed that the equilibrium swaps in a perfect market transfer wealth from shareholders to debt holders. Sorensen and Bollier (1994) used a bilateral model to price swap default risk. The primary determinants in Sorensen and Bollier's model included the two parties' credit conditions and the shape and volatility of the yield curve. Neal presented three new financial instruments (credit swaps, credit options, and credit-linked notes) for controlling credit risk, and showed that there is a strong relation between the credit rating and the credit risk premium: the higher the credit rating, the lower the credit premium. Duffie and Huang developed a model for valuing swap claims subject to default by both contracting parties. Duffie and Huang's model demonstrated that the anticipated variation in the credit quality of the contracting parties over the life of the swap is an important factor to determine an optimal swap rate between contracting parties. Using a string market model, Longstaff, Santa-Clara, and Schwarz solved the correlation matrix implied by swaptions and examined the relative valuation of caps and swaptions.

In the above literature, the authors used the Markowitz-Tobin portfolio-theoretic approach as their analytical apparatus concerning the pricing of default risk. The principal advantage of this approach is the explicit treatment of uncertainty, which has long played a crucial role in swaptions discussions. However, this approach omits important aspects of behavioral swap participation modes, specifically banks. It is assumed that under the portfolio-theoretic approach of loan portfolio swaps, loan markets are perfectly competitive so that quantity setting and rate taking are the relevant behavioral modes in the markets. This assumption is not applicable to loan markets because such markets are always concentrated in particular industries or geographic areas where the banks set rates.[‡]

In addition, those authors, except Sorensen and Bollier, and Jarrow and Yu, used a unilateral perspective in pricing default risk. This unilateral default exposure may exist because one party has no probability for default under any future economic scenario. Such an exposure may also exist if most of the swap's expected future values or replacement costs are negative to one party and positive to the counterparty under any future interest rate scenario. However, if both parties have some degree of default risk, there should be a bilateral perspective in pricing default risk. The pricing of the risk will depend on a combination of the two parties credit conditions.

A loan-rate-setting bank needs to value claims subject to default risk when it conducts loan portfolio swaps. Bank regulators require a consistent method for measuring the potential default risk so that they are able to set appropriate capital requirements to regulate the bank's asset quality. Concerns about bank asset quality and bank failures have promoted bank regulators to adopt a risk-based system of capital standards. The regulators designed a system to force bank capital positions to reflect their asset portfolio risks. It is of interest to study this capital regulation affecting bank loan rates and thus bank profits and risks under loan portfolio swaps.

The primary purpose of this paper is to develop a model that integrates the bilateral loan portfolio swap considerations in the portfolio-theoretic approach with the market conditions, and loan-rate-setting behavioral modes of the firm-theoretic

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[‡] Klein (1970) is among the first to question the applicability of the portfolio approach to intermediary behavior and shows that some basic theorems of portfolio theory are not applicable under imperfectly competitive market structures. Hancock (1986) demonstrated bank loan-rate-setting modes of behavior.

approach under capital regulation and deposit insurance. Our model applies the Black-Scholes (1973) option pricing to value the bank's capital equity. The comparative statical results from this model present the influences of the bank's (and its counterparty's) credit deterioration/improvement, capital regulation, and deposit insurance on the bank's optimal loan rate for the loan portfolio swap. We show that under strategic substitutes in the bank's risky-asset portfolio, the bank's optimal loan rate for the loan portfolio swap is a decreasing function of its credit improvement, its counterparty's credit deterioration, the capital-to-deposits ratio, and the deposit insurance premium when the bank has a greater market power. Therefore, this model provides an alternative explanation for a credit derivative loan portfolio swap for managing credit risk concerning bank loan-rate-setting behavior and regulation.

This paper is organized as follows. The next section presents the basic framework for the model. Section III derives and discusses the equilibrium of the model. Section IV analyzes the comparative static results. Section V concludes.

II. The Model

Consider a bank that makes decisions during a single-period horizon. The loan portfolio swap presented in this paper highlights the economic substance of the model. At the beginning of the period, when the capital constraint is binding, the bank has the following balance sheet:

$$L + L^{*} + B = D + K = K(\frac{1}{q} + 1) \tag{1}$$

where L and L^* are the amounts of two heterogeneous loans, B is a composite variable denoting the bank's net position in the interbank market, D is the quantity of deposit, and K is the stock of equity capital. The bank is a lender in the interbank market when $B > 0$, and a borrower when $B < 0$. Further, the bank can lend and borrow in the interbank market at a known rate R. The bank provides depositors with a rate of return equal to the risk-free rate, R_D. The bank is fully insured by the Federal Deposit Insurance Corporation (FDIC), and pays an insurance premium of P per dollar of deposits. Our model assumes that K is fixed over the period, and this equity capital held by the bank is tied by regulation to be a fixed proportion (q) of the bank's deposits, $K \geq qD$. We assumed that the required capital-to-deposits ratio (q) is an increasing function of the amount of the loans (L and L^*) held by the bank at the beginning of the period, $q' > 0$. Zarruk and Madura (1992) demonstrated that this required minimum capital-to-deposits ratio is risk-based.[§]

Without loss of generality, we assumed that loans $(L$ and $L^*)$ granted by the bank belong to two classes of fixed-rate claims with one-period maturity. L and L^* are the total lending amount, where $L > 0$ is treated as the amount of non-swap-performing (NSP) loans and $L^* > 0$ is the amount of swap-performing (SP) loans in the bank's loan portfolio. This portfolio is composed of some combination of the NSP and SP loans since the bank tends to reduce risk through diversification. We assume that the bank has some market power in lending (see Cosimano and

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[§] Risk-based capital is based on assets, specifically the composition of assets. In this paper, the bank maximizes the market value of equity under its balance sheet constraint. Using the capital-to-deposits ratio or capital-to-loans formula ratio affects none of the qualitative results in the model. However, an advantage of capital-to-deposits ratio used in this paper is for purposes of simplicity of calculation. Further, Our model assumes that q is an increasing function of L and L^*. Adding this complexity affects none of the qualitative results as well.

McDonald, 1998). Both loan markets are imperfect in the sense that the bank is a loan-rate setter. Both loan demands are specified by a downward-sloping function, $L = L(R_L, R_L^*)$ and $L^* = L^*(R_L, R_L^*)$, where R_L is the loan rate of L, with $\partial L/\partial R_L < 0$ and $\partial^2 L/\partial R_L^2 < 0$, and R_L^* is the loan rate of L^*, with $\partial L^*/\partial R_L^* < 0$ and $\partial^2 L^*/\partial R_L^{*2} < 0$. In addition, L and L^* are substitutes when $\partial L/\partial R_L^* > 0$ $(\partial L_L^*/\partial R_L > 0)$ and complements when $\partial L/\partial R_L^* < 0$ $(\partial L_L^*/\partial R_L < 0)$.

Neal pointed out that credit swaps are appealing to commercial banks whose loan portfolios are concentrated in particular industries or geographic areas. Neal's observation allows a loan-setting bank (e.g., an industrial bank) to swap the payments from some of its loans for payments from a different bank (called its counterparty, e.g., an agricultural bank) rather than lending outside its local area or selling some loans and purchasing others to diversify the credit risk. Our model assumes that a part of the loan portfolio granted by the bank's counterparty belongs to a single class of fixed-rate (R_A) claims with one-period maturity. The bank's counterparty operates its lending business in an imperfectly competitive loan market, and the demand for loans faced by the counterparty is a downward-sloping function, $L_A = L_A(R_A)$, where $\partial L_A/\partial R_A < 0$ and $\partial^2 L_A/\partial R_A{}^2 < 0$. L_A can be treated as a multiple-loan function; however, adding this complexity affects none of the qualitative results.

The loan portfolio swap in this paper consists of an agreement between two banks to exchange in the future two streams of loan payments. Even though a loan portfolio swap reduces the credit risk through diversification, loans are still risky because they are subject to non-performance due to, e.g., operational risk, counterparty risk, liquidity risk and legal risk. Default risk is often a two-way proposition in the case of swap transactions. If both parties do not have perfect information about each other's credit, frictions may arise. The replacement cost valuation should be adjusted to accommodate a two-way analysis. We used Sorensen and Bollier's bilateral approach to price the credit risk from the loan losses at the end of the period, and value the bank's credit risk, CR_{L^*}, in the following:

$$CR_{L^*} = \alpha_A (1+R_L^*) L^*(R_L, R_L^*) - \alpha_{L^*}(1+R_A) L_A(R_A) \tag{2}$$

$$\begin{cases} = 0 & \text{credit risk : non - existent} \\ \neq 0 & \text{credit risk : existent} \end{cases}$$

where $(1+R_L^*)L^*(R_L, R_L^*)$ is the value of the option for the bank to replace the swap, $(1+R_A)L_A(R_A)$ is the value of the option for the bank's counterparty to replace the swap, α_A is the probability that the bank's counterparty will default on the single default date, and α_{L^*} is the probability of $(1+R_L^*)L^*(R_L, R_L^*)$ defaulting.

From the bank's viewpoint, we used Equation (2) to analyze the interchanging effect between $(1+R_L^*)L^*(R_L, R_L^*)$ and $(1+R_A)L_A(R_A)$, and to value the credit risk in the model. If the option to receive a fixed loan payment is equal to option to pay a fixed loan payment: $(1+R_L^*)L^*(R_L, R_L^*) = (1+R_A)L_A(R_A)$, the direction of the midmarket credit risk adjustment depends on the ratio between the probability defaulting by the two swap parties. If the credit conditions for the two swap parties are equal: $\alpha_{L^*} = \alpha_A$, the direction of the adjustment depends on the difference between the two parties' options. Under these circumstances, the bank's value earned from its risky assets during the period is:

$$V(R_L, R_L^*)\begin{cases} = (1+R_L)L(R_L, R_L^*) + (1-\alpha_{L^*})(1+R_A)L_A(R_A) + \alpha_A(1+R_L^*)L^*(R_L, R_L^*) \\ \quad \text{credit risk : non - existent} \\ \\ \neq (1+R_L)L(R_L, R_L^*) + (1-\alpha_{L^*})(1+R_A)L_A(R_A) + \alpha_A(1+R_L^*)L^*(R_L, R_L^*) \\ \quad \text{credit risk : existent} \end{cases} \tag{3}$$

Given the balance sheet constraint, the bank's value earned from its earning-asset portfolio under credit swap transaction is:

$$A = V(R_L, R_L^*) + (1+R)[K(\frac{1}{q}+1) - L(R_L, R_L^*) - L^*(R_L, R_L^*)] \tag{4}$$

The residual value of the bank after meeting all of its debt obligations is the value of the bank's equity capital at the end of the period. Thus,

$$S = \begin{cases} A - Z & \text{if solvency } (A > Z) \\ \\ 0 & \text{if solvency } (A \le Z) \end{cases} \tag{5}$$

where $Z = (1+R_D)D + PD + C_L(L(R_L, R_L^*), L^*(R_L, R_L^*))$, $\partial C_L / \partial L > 0$, $\partial^2 C_L / \partial L^2 > 0$, $\partial C_L / \partial L^* > 0$, and $\partial^2 C_L / \partial L^{*2} > 0$. For simplicity, we further assumed that $C_L' = \partial C_L / \partial L = \partial C_L / \partial L^*$. The bank's total costs (Z) are composed of deposit payment cost, deposit insurance cost, and administrative loan cost, respectively.[**]

[**] The swap transaction between the two banks also involves an intermediary. The intermediary receives a small fee for arranging the transaction. The marginal swap transactions costs, the marginal administrative deposit costs, the fixed costs are omitted for simplicity because they will have the same qualitative effect on the optimal loan-rate settings as the marginal administrative loan costs.

Following a number of previous writers (see, for example, Merton, 1977; Crouhy and Galai, 1991; Mullins and Pyle, 1994; Lin and Teng, 2001), the objective function at the end of the period as described by Equation (5) has the features of a contingent claim, which is written on the current market value of the bank's earning assets. Applying Mullins and Pyle, the bank's market value of equity capital can be a Black-Scholes value of call options effectively purchased by the shareholders of the bank. To illustrate this, we rewrite Equation (5) as

$$S = V(R_L, R_L^*)N(d_1) - \{Z - (1+R)[K(\frac{1}{q}+1) - L(R_L, R_L^*) - L^*(R_{L,} R_L^*)]\}e^{-\mu}N(d_2) \qquad (6)$$

where,

$$d_1 = \frac{1}{\hat{\sigma}}\{[\ln\frac{V}{Z-(1+R)[K(\frac{1}{q}+1)-L-L^*]}]+\mu+\frac{1}{2}\hat{\sigma}^2\}$$

$$d_2 = d_1 - \hat{\sigma}$$

$$\hat{\sigma}^2 = \sigma_v^2 + \sigma_1^2 - 2\rho_{v,1}\sigma_v\sigma_1$$

$$\mu = R - R_D$$

Equation (6) separates the bank's equity value into two components. The first component is the bank's risk adjusted present value earned from its risky assets expressed by the combined standard deviation of the return of the portfolio. The second component is the risk adjusted present value of the bank's net obligations to its initial depositors above and beyond its default-free net lending in the interbank market with its non-interest costs. In this objective function, the cumulative standard normal distributions of $N(d_1)$ and $N(d_2)$ are the risk adjusted factors of the two components. $\hat{\sigma}^2$ is the variance with σ_v and σ_1 being the instantaneous standard derivation of the rates of return on the risky and default-free assets, respectively. $\rho_{v,1}$ is the instantaneous correlation coefficient between the two earning assets of the bank's portfolio. μ is the net deposit spread rate, which is the difference between R and R_D.

III. Equilibrium

The first-order conditions for an optimum of Equation (6) are

$$\frac{\partial V}{\partial R_L} N(d_1) - [C_L' - (1+R)(\frac{Kq'}{q^2}+1)](\frac{\partial L}{\partial R_L} + \frac{\partial L^*}{\partial R_L})e^{-\mu} N(d_2) = 0 \qquad (7\text{-}1)$$

$$\frac{\partial V}{\partial R_L^*} N(d_1) - [C_L' - (1+R)(\frac{Kq'}{q^2}+1)](\frac{\partial L}{\partial R_L^*} + \frac{\partial L^*}{\partial R_L^*})e^{-\mu} N(d_2) = 0 \qquad (7\text{-}2)$$

Equations (7-1) and (7-2) determine both the bank's optimal NSP and SP loan rates, hence its earning-asset portfolio size and composition. We assumed that the equilibrium is Cournot-Nash. There are two reasons for this assumption. First, a conjectural variation demonstrates market conducting when firms recognize their interdependence. In this model, the bank is an imperfectly competitive financial institution that "produces" two loans of NSP and SP. A product nature rather than a competitive nature of conjectural variation demonstrates market conducting when the bank recognizes the independence between the two loans. This recognition allows us to rule out cooperative or collusive behavior in multi-loan setting. Second, for purposes of simplicity, we used a Cournot-Nash nature between two loans (the bank's swap-performing loan and its counterparty's loan) to analyze the swap transactions. It should be apparent in what follows that this abstraction does not affect the basic conclusions of this paper. Intuitively, both the equilibrium conditions demonstrate that the risk-adjusted present value of the bank's marginal loan repayment earned from its risky assets equals the risk-adjusted present value of the marginal net obligations of the loan-rate settings.

Equation (7-1) indicates that the bank's marginal loan repayment value earned from the risky assets of its NSP loan rate is less than -1, $\partial V/\partial R_L < -1$. That is, the bank will operate on the elastic portion of its NSP loan demand curve, just as a monopolistic bank would do. This negative value in turn implies that the bank's NSP and SP loan are complements. This negative value also implies that NSP and SP loans may be substitutes since the NSP loan from a change in its loan rate (denoted by the own effect, $\partial L/\partial R_L$, in the model) is generally assumed to be more significant than the SP loan from a change in R_L (denoted by the cross effect, $\partial L^*/\partial R_L$). Thus, we may argue that the bank tends to conduct the loan portfolio swap to manage credit-related losses through diversification where NSP and SP loans are either complements or substitutes. This is especially true to the extent that the bank is heavily involved in industry-based lending. The bank's NSP and NP loans are complements when its borrowers may be classified as up- and down-stream firms in a particular industry, substitutes when its borrowers may be recognized as competitive firms in a particular industry.

In addition, the marginal net obligations of NSP loan rate, $C_L' + (1+R)(Kq'/q^2 + 1)$, are positive. This term demonstrates both the marginal administrative cost of serving NSP loans (denoted by the cost effect of the NSP loan-rate setting in the model) and the marginal revenue of borrowing/lending in the interbank market (denoted by the portfolio redistribution effect of the NSP loan-rate setting). The cost effect indicates the behavioral modes of the firm-theoretic approach while the portfolio redistribution effect demonstrates the return-risk trade-off conditions of the portfolio-theoretic approach. Thus, this equilibrium integrates the risk conditions of the portfolio-theoretic approach with the firm-theoretic approach market modes. The basic concept of this integrated approach presented in this paper follows Sealey (1980) concerning bank rate-setting behavior. The interpretation of equation (7-2) follows a similar argument as in the case of equation (7-1).

IV. Comparative Statics

In this section, we consider the effects on optimal loan-rate setting from changes in the model parameters. The second-order conditions and the unique market equilibrium are:

$$\Delta = \frac{\partial^2 S}{\partial R_L^2}\frac{\partial^2 S}{\partial R_L^{*2}} - \frac{\partial^2 S}{\partial R_L \partial R_L^*}\frac{\partial^2 S}{\partial R_L^* \partial R_L} > 0$$

$$\frac{\partial^2 S}{\partial R_L^2} < 0, \quad \frac{\partial^2 S}{\partial R_L^{*2}} < 0$$

$\partial^2 S / \partial R_L \partial R_L^*$ indicates the bank's best-reply operation between its NSP and SP loan-rate settings, and denotes a change in the marginal interest value of NSP loan-rate setting influenced by a change in the SP loan-rate setting. Applying Bulow, Geankopolos, and Klemperer's (1985) product portfolio selection in which to compete, we demonstrate that both the bank's NSP and SP loan-rate settings are strategic substitutes if a given bank's marginal interest value of NSP loan-rate setting falls when the bank increases the SP loan-rate setting: the bank's best-reply function is downward sloping. The strategic substitutes in the model demonstrate the bank's best loan composition management allocation in its earning-asset portfolio. This demonstration analyzes a best-reply effect that the loan contract has on swaptions. It is a case of strategic complements when the bank decreases the SP loan-rate setting: the bank's best-reply function is upward sloping. The interpretation of $\partial^2 S / \partial R_L^* \partial R_L$ follows a similar argument as in the case of $\partial^2 S / \partial R_L \partial R_L^*$.

First, consider the impact on the bank's NSP and SP loan-rate settings from a change in its own probability of $(1+R_L^*)L^*(R_L, R_L^*)$ defaulting, α_L^*. The comparative static results derived from equations (7-1) and (7-2) are presented in the following:

$$\frac{\partial R_L}{\partial \alpha_L^*} = -\frac{I}{\Delta}\frac{(1+R_A)L_A}{\hat{\sigma} V} \tag{8-1}$$

$$\frac{\partial R_L^*}{\partial \alpha_L^*} = -\frac{I^*}{\Delta}\frac{(1+R_A)L_A}{\hat{\sigma} V} \tag{8-2}$$

where,

$$I = G\frac{\partial^2 S}{\partial R_L^{*2}} - G^*\frac{\partial^2 S}{\partial R_L \partial R_L^*}$$

$$I^* = G^*\frac{\partial^2 S}{\partial R_L^2} - G\frac{\partial^2 S}{\partial R_L^* \partial R_L}$$

$$G = -\frac{\partial V}{\partial R_L}\frac{\partial N}{\partial d_1} + [C_L' + (1+R)(\frac{Kq'}{q^2}+1)](\frac{\partial L}{\partial R_L} + \frac{\partial L^*}{\partial R_L})e^{-\mu}\frac{\partial N}{\partial d_2}$$

$$G^* = -\frac{\partial V}{\partial R_L^*}\frac{\partial N}{\partial d_1} + [C_L' + (1+R)(\frac{Kq'}{q^2}+1)](\frac{\partial L^*}{\partial R_L^*} + \frac{\partial L}{\partial R_L^*})e^{-\mu}\frac{\partial N}{\partial d_2}$$

Note that $\partial V/\partial R_L = L(1+\eta) - \partial L^*/\partial R_L$ and $\partial V/\partial R_L^* = R_L\,\partial L/\partial R_L^* - \partial L^*/\partial R_L^*$, where η is the NSP loan demand interest rate elasticity at the loan rate R_L. Since η is proportional to the Lerner index of the bank, η is a measure of the bank's market power.[††] If the bank's credit deteriorates (hence α_{L^*} increases), then the magnitude of the SP loan default risk increases. If the bank has a greater market power in both the L and L^* markets and both products are substitutes, then an increase in the bank's credit deterioration will decrease the optimal NSP loan rate and increase the optimal SP loan rate under strategic substitutes. Because the bank's required credit risk of the SP loan increases, the bank will pay a lower fixed coupon. Therefore, the optimal SP loan-rate is set increasingly and the SP loan demand decreases. In an imperfect NSP loan market, the bank must reduce its loan rate in order to increase the amount of loans due to the nature of the substitutes in the bank's loan portfolio. Thus, if the bank is sufficiently powerful in both the loan markets and these two loans are substituted, in which the bank's borrowers are recognized as competitors in a particular industry, we present the following proposition.

Proposition1: An increase in the bank's SP loan credit deterioration decreases the NSP loan rate and increases the SP loan rate under strategic substitutes and has an indeterminate effect on the bank's NSP and SP's loan rates under strategic complements.

[††] See Tirole (1988, p.66).

The strategic substitutes between two heterogeneous loans demonstrate that the bank increasing its NSP loan rate is the best response when it decides to decrease its SP loan rate. Rather than emphasizing a bank and its rival's best-reply competitive strategy in Bulow, Geanakoplos and Klemperer's sense, we suggest an interactive strategy of the loan portfolio allocation under swap transactions. Both loan markets faced by the bank are imperfectly competitive so that rate settings are the relevant behavioral modes not only in both markets but also in its loan portfolio management under swap transactions. Thus, Proposition 1 demonstrates a strategy that integrates a swap transaction with the market conditions and rate-setting behavioral modes of the bank.

Second, consider the impact on the bank's NSP and SP loan-rate settings from a charge in its counterparty's probability of $(1+R_A)L(R_A)$ defaulting, α_A. The comparative static results derived from Equations (7-1) and (7-2) are:

$$\frac{\partial R_L}{\partial \alpha_A} = \frac{I}{\Delta}\frac{(1+R_L^*)L^*}{\hat{\sigma}V} \tag{9-1}$$

$$\frac{\partial R_L^*}{\partial \alpha_A} = \frac{I^*}{\Delta}\frac{(1+R_L^*)L^*}{\hat{\sigma}V} \tag{9-2}$$

If the bank has a greater market power in both the L and L^* markets and both products are substitutes, then an increase in the counterparty's credit deterioration will increase the optimal NSP loan rate and decrease the optimal SP loan rate under strategic substitutes. The interpretation of this result follows a similar argument as in the case of a change in α_{L^*}. Basically, just as a higher-rated bond issuer pays a lower coupon, the higher-rated swap party, relatively speaking, due to an increase in its conterparty's credit deterioration, must pay (receive) a lower (higher) fixed coupon. In imperfect loan markets, the bank must decrease the SP loan rate and increase the NSP loan rate in order to increase the SP loan and decrease the NSP loan for the bank's risky earning-asset portfolio. To summarize, if the bank is sufficiently powerful in both the loan markets, and these two loans are substitutes, then a proposition is stated as follows.

Proposition 2: An increase in the bank's counterparty's credit deterioration increases the NSP loan rate and decreases the SP loan rate under strategic substitutes and has an indeterminate effect on the bank's NSP and SP's loan rates under strategic complements.

Third, consider the impact on the bank's NSP and SP loan-rate settings from a change in the capital-to-deposits ratio. Implicit differentiations of the first-order conditions with respect to q yield

$$\frac{\partial R_L}{\partial q} = -\frac{I}{\Delta}\frac{(1+R)K}{\hat{\sigma}\{Z-(1+R)[K(\frac{1}{q}+1)-L-L^*]\}q^2} \tag{10-1}$$

$$\frac{\partial R_L^*}{\partial q} = -\frac{I^*}{\Delta}\frac{(1+R)K}{\hat{\sigma}\{Z-(1+R)[K(\frac{1}{q}+1)-L-L^*]\}q^2} \tag{10-2}$$

If the bank has a greater market power and L and L^* are substitutes, then an increase in the capital-to-deposits ratio will increase the optimal NSP loan-rate setting and decrease the optimal SP loan-rate setting under strategic substitutes. Intuitively, as the bank is forced to increase its capital relative to its deposit level, it must provide a return to a larger equity base. One way the bank may attempt to augment its total returns is by shifting its investments to its SP loan (by decreasing the SP loan rate) and away from its investments to its NSP loan (by increasing the NSP loan rate) as well as the interbank market. Thus, capital regulation encourages the bank to conduct a loan portfolio swap; accordingly, credit swaps reduce credit risk through diversification. The results observed from Equations (10-1) and (10-2) are stated in the following proposition.

Proposition 3: An increase in the capital-to-deposits ratio increases the NSP loan rate and decreases the SP loan rate under strategic substitutes and has an indeterminate effect on the bank's NSP and SP's loan rate under strategic complement.

Finally, it is of interest to consider the impact on the bank's NSP and SP loan-rate settings from a change in the deposit insurance premium. Differentiating the first-order conditions with respect to the deposit insurance premium yields

$$\frac{\partial R_L}{\partial P} = -\frac{I}{\Delta}\frac{D}{\hat{\sigma}\{Z-(1+R)[K(\frac{1}{q}+1)-L-L^*]\}} \quad (11\text{-}1)$$

$$\frac{\partial R_L^*}{\partial P} = -\frac{I^*}{\Delta}\frac{D}{\hat{\sigma}\{Z-(1+R)[K(\frac{1}{q}+1)-L-L^*]\}} \quad (11\text{-}2)$$

The results from Equations (11-1) and (11-2) are demonstrated in the following proposition. We note that the results are limited to two assumptions: the bank is sufficiently powerful in both the loan markets and these two loans are substitutes.

Proposition 4: An increase in the deposit insurance premium increases the NSP loan rate and decreases the SP loan rate under strategic substitutes and has an indeterminate effect on the bank's NSP and SP loan rates under strategic complements.

The interpretation of this result follows a similar argument as in the case of a change in capital-to-deposits ratio. Basically, an increase in the cost of deposit insurance encourages the bank to shift investments to its risky assets from default-free assets (see Zarruk and Madura, 1992, p.148). Proposition 4 implies that an increase in the cost of deposit insurance encourage the bank to shift investments to its SP loans from NSP loans. This implication is consistent with Zarruk and Madura's finding if an increase in the deposit insurance does not decrease the bank's total loan portfolio amount. Thus, Proposition 4 provides an alternative observation for the loan portfolio size determination.

V. Conclusions

In this paper, a firm-theoretic model was developed to study a loan-rate-setting bank operating a loan portfolio swap credit derivative for controlling credit risk. This model shows how credit risk, capital regulation and deposit insurance conditions jointly determine the optimal loan-rate decisions. Our main point is that in a sense, the most obvious application of our results is to the theory of how a bank should select a lending loan portfolio in which to compete. The strategic effect on one lending market in another market must be considered. This is especially true to the extent that a bank is heavily involved in industry-based or geography-based lending and attempts to manage and control its credit risk through diversification.

When the bank has greater market power and both loans are substitutes, we find that the optimal NSP loan rate is negatively related to the bank's credit deterioration, positively related to the credit deterioration of the bank's counterparty, to the capital-to-deposits ratio, and to the deposit insurance premium under strategic substitutes. However, the comparative-static results of the optimal SP loan rate follow a contrary argument as in the above optimal NSP loan rate case. Our findings provide alternative explanations for bank behavior that integrates the risk considerations of loan portfolio swap with the market conditions, regulations, and loan-rate-setting behavioral asset management modes.

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