行政院國家科學委員會專題研究計畫 成果報告

研發與空間聚集
研究成果報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 96-2415-H-032-009-
執 行 期 間 ：96 年 08 月 01 日至 97 年 07 月 31 日
執 行 單 位 ：淡江大學產業經濟學系

計畫主持人：梁文榮

計畫參與人員：博士班研究生-兼任助理人員：林晏如
博士班研究生-兼任助理人員：曾靜枝
博士班研究生-兼任助理人員：王羿傑

報 告 附 件 ：出席國際會議研究心得報告及發表論文

處 理 方 式 ：本計畫涉及專利或其他智慧財產權，1 年後可公開查詢

中 華 民 國 97 年 08 月 22 日

行政院國家科學委員會補助專題研究計畫 ■ 成 果 報 告
□期中進度報告

研發與空間聚集

計畫類別：■ 個別型計畫　　□ 整合型計畫
計畫編號：NSC 96－2415－H－032－009－
執行期間： 96 年 08 月 01 日 至 97 年 07 月 31 日

計畫主持人：梁文榮
共同主持人：
計畫參與人員： 林晏如、曾靜枝、王羿傑

成果報告類型(依經費核定清單規定繳交)：■精簡報告　□完整報告

本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
■出席國際學術會議心得報告及發表之論文各二份
□國際合作研究計畫國外研究報告書一份

處理方式：除產學合作研究計畫、提升產業技術及人才培育研究計畫、列管計畫及下列情形者外，得立即公開查詢
□涉及專利或其他智慧財產權，■一年□二年後可公開查詢

執行單位：淡江大學產業經濟系

中　華　民　國　97　年　08　月　18　日

1.1 中文摘要

本研究引進產品同時具水平與垂直異質特性，建立一個二維空間模型，本文的重點在探討垂直異質對廠商區位選擇的影響。本研究採用一個二階段賽局，廠商在第一階段同時決定最適區位；在第二階段則分別探討三種訂價政策下，廠商在商品市場進行 Bertrand 價格競爭。我們証明在空間差別訂價下，最小差異法則有可能存在，但最大差異法則卻不可能發生。我們也証明在單一遞送價格與單一出廠價格的訂價政策下，空間聚集是唯一的區位均衡解。

1.2. Abstract

This paper constructs a two-dimensional framework to take into consideration both horizontal and vertical differentiation. The focus of the paper is on the impact of vertical (quality) differentiation to firms' location configuration. It employs a two-stage game, in which firms first simultaneously decide optimal locations and then play Bertrand price competition with three pricing policies. This paper shows that the Principle of Minimum Differentiation may occur while the Principle of Maximum Differentiation can never emerge, if firms engage in spatially discriminatory pricing. It also shows that spatial agglomeration is the unique location equilibrium in both cases where firms charge uniform delivered and mill pricings.

1.3 Key Words: Spatial Agglomeration; Two-dimensional Framework; Vertical Differentiation; Competition Effect; Cost-Saving Effect

2. Introduction

Hotelling (1929) first proposed that two firms of a homogeneous product agglomerate at the center of the line market under linear transportation costs, which has been termed the *Principle of Minimum Differentiation*. However, D'Aspremont *et al*. (1979) challenge this principle by indicating that there exists no price equilibrium in this case and shows that the two firms will locate at the opposite endpoints of the line market under quadratic transportation cost instead. This has been termed the *Principle of Maximum Differentiation*. From then on, many regional economists have tried to deduce the conditions under which the Principle of Minimum Differentiation can be restored. They include: Stahl (1982), De Palma *et al*. (1985), Rhee *et al*. (1992), Anderson and Neven (1991), Jehiel (1992), Friedman and Thisse (1993), Tabuchi (1994), Zhang (1995), Mai and Peng (1999), Liang and Mai (2006), and Matsushima and Matsumura (2006).

3. The Purposes

The purpose of this paper is to determine the conditions under which the Principle of Minimum Differentiation can be restored under three pricing regimes, i.e., the spatially discriminatory, the uniform delivered and the mill pricings, where the firms' quality levels (vertical differentiation) are exogenously given.

4. Literature Survey

Many regional economists have tried to deduce the conditions under which the Principle of Minimum Differentiation can be restored. They include: Stahl (1982) who considers some harmonious conjectural variations; De Palma *et al*. (1985) and Rhee *et al*. (1992), who introduce heterogeneity in both consumers and firms; Anderson and Neven (1991) who assume that firms play Cournot quantity competition instead of Bertrand price competition in the commodity market; Jehiel (1992) and Friedman and Thisse (1993) who adopt price collusion; and Tabuchi (1994) who constructs a model with two dimensions of horizontal differentiation. Tabuchi in particular shows that two firms maximize their distance in one dimension, but minimize their distance in the other dimension. In addition to these researchers, Zhang (1995) imposes a price-matching policy; Mai and Peng (1999) emphasize the importance of the externality-like benefits generated from the exchange of information between firms; Liang and Mai (2006) focus on the crucial influence caused from the vertical subcontracting of the intermediate product; and Matsushima and Matsumura (2006) analyze the mixed-oligopoly economy.

Ferreira and Thisse (1996) employ Launhardt's (1885) spatial oligopolistic model to examine the decisions of the firms' optimal quality levels (vertical differentiation) taking location (horizontal differentiation) as exogenously given. They use transport rate as a measure of quality, a high (low) transport rate representing a low (high) quality level, and the game employed is a two-stage game, in which firms select the optimal quality levels in the first stage and then engage in Bertrand price competition in the commodity market in the second stage. They find an interesting result that firms select to maximize the vertical differentiation when the horizontal differentiation is minimized, while to minimize the vertical differentiation when the horizontal differentiation is maximized. This result is termed the Max-Min and Min-Max result hereafter.

5. Methodology

5.1. Spatially Discriminatory Pricing

Consider a two-dimensional framework, in which the horizontal axis measures the traditional Hotelling line referred to as horizontal characteristic, while the vertical axis measures the tastes of consumers to qualities referred to as vertical characteristic. Two firms, denoted firm 1 and firm 2, are located at x_1 and x_2, with $x_1 \leq x_2$ along a line segment with length $L = 1$ on the horizontal axis. The firms, whose production cost is for simplicity assumed to be nil, sell products with vertically differentiated qualities, α_1 and α_2 with $\alpha_1 \leq \alpha_2$ respectively, to consumers. In a model with vertically differentiated qualities, there must be heterogeneity in consumers' willingness to pay for quality, which is captured by assuming that a continuum of consumers is uniformly distributed over the interval $[\underline{\theta}, \overline{\theta}]$ along the vertical axis with unit density at each point of the Hotelling line. Following Choi and Shin (1987), we assume $\underline{\theta} = \bar{\theta} - 1$, where $\bar{\theta}$>1. Thus, these two characteristics lead to a rectangular distribution of consumers over [0, 1] × $[\bar{\theta}-1, \bar{\theta}]$. A firm faces a continuum of consumers with taste $\theta \in [\bar{\theta}-1, \bar{\theta}]$ at each point of the Hotelling line or a continuum of consumers with different locations for a given taste of the vertical axis. Assume further that the transport cost function of the product is linear and takes the following form: $T(x - x_i) = t|x_i - x|$, where T is the transport cost, and t is the transport rate per unit output per unit distance.

Suppose that firms engage in discriminatory pricing to charge different prices for consumers residing at different locations. The indirect utility of a consumer residing at the location with combination (x, θ) and purchasing from firm i can be expressed as:

$$u(x,\theta) = k + \theta\alpha_i - p_i(x), i = 1,2, \tag{1}$$

where $u(x, \theta)$ is the utility function of the consumer with combination (x, θ); and k is the reservation utility of consuming one unit of commodity; and θ denotes the taste of consumers' preference for quality ranging along the interval $[\bar{\theta}-1, \bar{\theta}]$ with $\bar{\theta}$ is the upper bound of the consumers' tastes; and α_i (i = 1, 2) represents the quality level of the product produced by firm i; and p_i (x) is the delivered price charged by firm i at site x.

The taste of the marginal consumer, who is indifferent between buying one unit of the product from either firm, for a continuum of consumers residing at x can be obtained by equaling the utility levels of buying from the two firms as follows:

$$\hat{\theta}(x) = [p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1), \tag{2}$$

where $\hat{\theta}(x)$ denotes the taste of the marginal consumer for a continuum of consumers residing at x.

Each firm's demand function at site x can be derivable as:

$$q_1(x) = \hat{\theta} - \underline{\theta} = \{[p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1) - (\bar{\theta} - 1)\}, \tag{3.1}$$

$$q_2(x) = \bar{\theta} - \hat{\theta} = \{\bar{\theta} - [p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1)\}. \tag{3.2}$$

Assuming that production costs are zero and the quality cost is fixed, firm i's operating profit function at site x can be expressed as:

$$\pi_i(x) = [p_i(x) - t|x - x_i|]q_i(x), i = 1,2, \tag{4}$$

where $\pi_i(x)$ denotes firm i's operating profit at site x.

The game employed in this paper is a two-stage game as discussed previously. The sub-game perfect Nash equilibrium can be solved by backward induction, beginning with the final stage. Differentiating (4) with respect to $p_i(x)$ respectively, we can derive the profit-maximizing conditions for prices in stage 2. Solving these equations, we have:

$$p_1(x) = (1/3)[(\alpha_2 - \alpha_1)(2 - \bar{\theta}) + t(2|x - x_1| + |x - x_2|)], \tag{5.1}$$

$$p_2(x) = (1/3)[(\alpha_2 - \alpha_1)(\overline{\theta} + 1) + t(|x - x_1| + 2|x - x_2|)]. \tag{5.2}$$

Substituting (5) into (3), we obtain:

$$q_1(x) = [1/3(\alpha_2 - \alpha_1)][(\alpha_2 - \alpha_1)(2 - \overline{\theta}) + t(|x - x_2| - |x - x_1|)], \tag{6.1}$$

$$q_2(x) = [1/3(\alpha_2 - \alpha_1)][(\alpha_2 - \alpha_1)(\overline{\theta} + 1) - t(|x - x_2| - |x - x_1|)]. \tag{6.2}$$

It is worth noting that the upper bound of the quality taste ($\overline{\theta}$) must be smaller than 2 to ensure firm 1's demand being positive, as firms locate at the same site, i.e. $x_1 = x_2$. Consequently, the upper bound of the quality taste lies within the interval [1, 2).

Substituting (5) into (2), we can derive the taste of the marginal consumer residing at site x as follows:

$$\hat{\theta}(x) = [1/3(\alpha_2 - \alpha_1)][(\alpha_2 - \alpha_1)(2\overline{\theta} - 1) + t(|x - x_2| - |x - x_1|)], x \in [0,1]. \tag{7}$$

Differentiating (7) with respect to x, yields:

$$\partial\hat{\theta}(x)/\partial x = \begin{cases} 0 & \text{if } \ x \in [0, x_1], \\ -2t/3(\alpha_2 - \alpha_1) < 0 & \text{if } \ x \in [x_1, x_2], \\ 0 & \text{if } \ x \in [x_2, 1]. \end{cases} \tag{8}$$

We see from (8) that given firms' locations x_1 and x_2, the taste of the marginal consumer remains unchanged for $x \in [0, x_1]$ and $x \in [x_2, 1]$, while taste decreases with respect to x within the interval $[x_1, x_2]$.

Next, we turn to the first stage. Substituting (5) and (6) into (4), we can derive firm i's reduced aggregate operating profit function as follows:

$$\begin{aligned}\Pi_1 = [1/9(\alpha_2 - \alpha_1)]\{&\int_0^{x_1} [(\alpha_2 - \alpha_1)(2 - \overline{\theta}) + t(x_2 - x_1)]^2 dx \\ &+ \int_{x_1}^{x_2} [(\alpha_2 - \alpha_1)(2 - \overline{\theta}) + t(x_2 + x_1 - 2x)]^2 dx \\ &+ \int_{x_2}^{1} [(\alpha_2 - \alpha_1)(2 - \overline{\theta}) - t(x_2 - x_1)]^2 dx\},\end{aligned} \tag{9.1}$$

$$\begin{aligned}\Pi_2 = [1/9(\alpha_2 - \alpha_1)]\{&\int_0^{x_1} [(\alpha_2 - \alpha_1)(\overline{\theta} + 1) + t(x_1 - x_2)]^2 dx \\ &+ \int_{x_1}^{x_2} [(\alpha_2 - \alpha_1)(\overline{\theta} + 1) + t(-x_1 - x_2 + 2x)]^2 dx \\ &+ \int_{x_2}^{1} [(\alpha_2 - \alpha_1)(\overline{\theta} + 1) - t(x_1 - x_2)]^2 dx\}.\end{aligned} \tag{9.2}$$

Differentiating (9) with respect to x_i, respectively, yields the profit-maximizing conditions for locations as follows:

$$\partial\Pi_1 / \partial x_1 = (t)\{-[2t/9(\alpha_2 - \alpha_1)][(x_2 - x_1)(1 - x_2 + x_1)] + (4/9)(1/2 - x_1)(2 - \overline{\theta})\} = 0, \tag{10.1}$$

$$\partial\Pi_2 / \partial x_2 = (t)\{[2t/9(\alpha_2 - \alpha_1)][(x_2 - x_1)(1 - x_2 + x_1)] + (4/9)(1/2 - x_2)(\overline{\theta} + 1)\} = 0, \tag{10.2}$$

where $1 < \overline{\theta} < 2$ and $0 \le x_1 \le x_2 \le 1$.

Recalling that $0 \le x_1 \le x_2 \le 1$ and $\alpha_1 \le \alpha_2$, we find that the first term in the brace of the right-hand side of (10.1) is non-positive. This term can be named the competition effect, which shows that as the two firms move apart, the horizontal differentiation between the two products is increased, implying that price competition between firms is mitigated. Consequently, the competition effect attracts firm 1 to move leftward. Moreover, the competition effect is weakened, as the two products become more vertically differentiated (i.e., α_2 - α_1, is larger) or the transport rate is lower. On the other hand, the second term in the brace is denoted as the transportation cost saving effect (for simplicity, the cost-saving effect, hereafter), whose value is non-negative. This arises because the first term is non-positive. In order to ensure an interior solution, the second term has to be non-negative to make the profit-maximizing condition equal zero. The cost-saving effect reflects firm *i*'s desire to move toward the center in order to save on the transportation cost. Consequently, firm 1's location equilibrium is determined by the balance of the competition and the cost-saving effects. We find from (10.2) that this result applies to firm 2's location equilibrium, in which the competition effect attracts firm 1 to move rightward while the cost-saving effect forces firm 2 to move toward the center of the market.

The location equilibria are subject to the second-order and the stability conditions as follows:

$$\partial^2\Pi_1 / \partial x_1{}^2 = [2t/9(\alpha_2 - \alpha_1)]\{-2(\alpha_2 - \alpha_1)(2 - \overline{\theta}) - t[2(x_2 - x_1) - 1)]\} \le 0, \tag{11.1}$$

$$\partial^2\Pi_2 / \partial x_2{}^2 = [2t/9(\alpha_2 - \alpha_1)]\{-2(\alpha_2 - \alpha_1)(\overline{\theta} + 1) - t[2(x_2 - x_1) - 1]\} \le 0, \tag{11.2}$$

$$\begin{aligned} J &= (\partial^2\Pi_1 / \partial x_1^2)(\partial^2\Pi_2 / \partial x_2^2) - (\partial^2\Pi_1 / \partial x_1\partial x_2)(\partial^2\Pi_2 / \partial x_2\partial x_1) \\ &= [8t^2 / 81(\alpha_2 - \alpha_1)]\{2(\alpha_2 - \alpha_1)(2 - \overline{\theta})(\overline{\theta} + 1) + 3t[2(x_2 - x_1) - 1]\} \ge 0. \end{aligned} \tag{11.3}$$

In addition, the location equilibria should fulfill the market-serving condition, which requires the output of each firm at its remote endpoint shipped from its equilibrium location be positive. This can be described as follows:

$$q_1(1; x_1, x_2) = [(\alpha_2 - \alpha_1)(2 - \overline{\theta}) - t(x_2 - x_1)] / 3(\alpha_2 - \alpha_1) > 0, \tag{12.1}$$

$$q_2(0; x_1, x_2) = [(\alpha_2 - \alpha_1)(1 + \overline{\theta}) - t(x_2 - x_1)] / 3(\alpha_2 - \alpha_1) > 0. \tag{12.2}$$

Solving (10), we can obtain location equilibria as follows:

$$x_1{}^A = x_2{}^A = 1/2, \tag{13.1}$$

$$x_1^D = [2(\alpha_2 - \alpha_1)(2 + 3\overline{\theta} - \overline{\theta}^3)/9t] + [(1 - 2\overline{\theta})/6],$$
$$x_2^D = [2(\alpha_2 - \alpha_1)(-4 + 3\overline{\theta}^2 - \overline{\theta}^3)/9t] + [(7 - 2\overline{\theta})/6], \tag{13.2}$$

where the superscript "*A*" ("*D*") denotes the variables associated with the case of the agglomeration (disperse) equilibrium, respectively.

There are two possible location equilibria, central agglomeration and spatial dispersion. Substituting (13.1) into (11.1) - (11.3), (12.1) and (12.2), we have:

$$\partial^2\Pi_1/\partial x_1^2\Big|_{x_1^A, x_2^A} \le 0, \text{if } (\alpha_2 - \alpha_1) \ge t/2(2 - \overline{\theta}) \equiv \Delta_1^A, \tag{14.1}$$

$$\partial^2\Pi_2/\partial x_2^2\Big|_{x_1^A, x_2^A} \le 0, \text{if } (\alpha_2 - \alpha_1) \ge t/2(1 + \overline{\theta}) \equiv \Delta_2^A, \tag{14.2}$$

$$J\Big|_{x_1^A, x_2^A} \ge 0, \text{if } (\alpha_2 - \alpha_1) \ge 3t/2(2 - \overline{\theta})(1 + \overline{\theta}) \equiv \Delta_3^A, \tag{14.3}$$

$$q_1(1; x_2 = x_1 = 1/2) = (2 - \overline{\theta})/3 > 0, \tag{15.1}$$

$$q_2(0; x_2 = x_1 = 1/2) = (1 + \overline{\theta})/3 > 0. \tag{15.2}$$

Recall that $1 < \overline{\theta} < 2$. We figure out from (14) that $\Delta_3^A > \Delta_1^A > \Delta_2^A$. Thus, central agglomeration arises only if the degree of vertical differentiation is sufficiently large, say $(\alpha_2 - \alpha_1) \ge \Delta_3^A$. The intuition behind this result can be stated as follows. We have argued that the location equilibrium is determined by the competition and the cost-saving effects. We have also shown that the higher the degree of vertical differentiation, the weaker the competition effect will be. Therefore, as the degree of vertical differentiation is no less than the critical value Δ_3^A, the competition effect is dominated by the cost-saving effect such that the two firms agglomerate at the center of the Hotelling line.

Next, substituting (13.2) into (11.1) - (11.3), (12.1) and (12.2), we have:

$$\partial^2\Pi_1/\partial x_1^2\Big|_{x_1^D, x_2^D} \le 0, \text{if } (\alpha_2 - \alpha_1) \le 3t/2(2 - \overline{\theta})(2\overline{\theta} - 1) \equiv \Delta_1^D, \tag{16.1}$$

$$\partial^2\Pi_2/\partial x_2^2\Big|_{x_1^D, x_2^D} = 2t[-2(\alpha_2 - \alpha_1)(1 + \overline{\theta})(2\overline{\theta} - 1) - 5t]/27(\alpha_2 - \alpha_1) \le 0, \tag{16.2}$$

$$J\Big|_{x_1^D, x_2^D} \ge 0, \text{if } (\alpha_2 - \alpha_1) \le 3t/2(2 - \overline{\theta})(1 + \overline{\theta}) \equiv \Delta_2^D \equiv \Delta_3^A, \tag{16.3}$$

$$q_1(1; x_1^D, x_2^D) > 0, \text{if } (\alpha_2 - \alpha_1) > 4t/(2 - \overline{\theta})(5 + 2\overline{\theta}) \equiv \Delta_3^D, \tag{17.1}$$

$$q_2(0; x_1^D, x_2^D) > 0, \text{if } (\alpha_2 - \alpha_1) > 4t/(1 + \overline{\theta})(7 - 2\overline{\theta}) \equiv \Delta_4^D. \tag{17.2}$$

We find from (16) that $\Delta_2^D < \Delta_1^D$, and from (17) that $\Delta_3^D > \Delta_4^D$. We also find from (14.3) and (16.3) that $\Delta_3^A = \Delta_2^D$. Accordingly, we can derive that spatial dispersion arises as the degree of vertical differentiation lies in between [Δ_3^D, Δ_2^D], i.e., $\Delta_3^D \le (\alpha_2 - \alpha_1) \le \Delta_3^A$. The same intuition applies to this result. The competition effect is strong enough to push the two firms apart, as the degree of vertical differentiation is no greater than the critical value, Δ_3^A.

Next, we explore the invalidity of the Principle of Maximum Differentiation. First of all, we examine this principle as the interior location equilibrium arises, which can be done via (13.2). Recall that $1 < \bar{\theta} < 2$ and $\alpha_1 \leq \alpha_2$. We can find from (13.2) that $x_2^D = [2(\alpha_2 - \alpha_1)(-4 + 3\bar{\theta}^2 - \bar{\theta}^3)/9t] + [(7 - 2\bar{\theta})/6] < 5/6$. This shows that firm 2 would never locate at the right end of the Hotelling line. Thus, the Principle of Maximum Differentiation will never emerge in this case. Secondly, we examine this Principle when a corner solution for location equilibrium occurs. This arises as the conditions $\partial\Pi_1/\partial x_1 < 0$ and $\partial\Pi_2/\partial x_2 > 0$ hold. We can calculate from (10.1) that the former condition holds if $(\alpha_2 - \alpha_1) < t$ $(x_2 - x_1)(1 - x_2 + x_1) / (1 - 2x_1)(2 - \bar{\theta}) \equiv \Delta_1^c$ and from (10.2) that the latter condition holds if $(\alpha_2 - \alpha_1) < t\ (x_2 - x_1)(1 - x_2 + x_1) / (2x_2 - 1)(1 + \bar{\theta}) \equiv \Delta_2^c$. We find that $\Delta_1^c = \Delta_2^c = 0$ as $x_1 = 0$ and $x_2 = 1$. Thus, the conditions hold only if $\alpha_2 - \alpha_1 < 0$, which contradicts the assumption $\alpha_1 \leq \alpha_2$ and excludes the possibility of the Principle of Maximum Differentiation as the corner solution for location equilibrium emerges.

5.2. Uniform Delivered and Mill Pricings

In this section, we will examine firms' location equilibria as firms undertake uniform delivered and mill pricings in the commodity market. We first discuss the case of uniform delivered pricing. Firms charge the same delivered price at each point of the Hotelling line, respectively, in this case. Thus, the indirect utility of a consumer, who purchases from firm *i*, can be expressed as:

$$u = k + \theta\alpha_i - p_i^u, i = 1,2, \tag{21}$$

where the superscript "*u*" denotes the variables associated with the case of uniform delivered pricing.

The marginal consumer, who is indifferent between buying one unit of product from either firm, following (21), acts so as to satisfy:

$$\hat{\theta}^u = (p_2^u - p_1^u)/(\alpha_2 - \alpha_1). \tag{22}$$

Firms' demand functions under the case of uniform delivered pricing are therefore equal to:

$$q_1^u = [(p_2^u - p_1^u)/(\alpha_2 - \alpha_1)] - (\bar{\theta} - 1), \tag{23.1}$$

$$q_2^u = \bar{\theta} - [(p_2^u - p_1^u)/(\alpha_2 - \alpha_1)]. \tag{23.2}$$

Firm's aggregate operating profits can be obtained by integrating its operating profit of each point along the Hotelling line, and can be expressed as:

$$\Pi_i^u = \int_0^1 (p_i^u - t|x - x_i|) q_i(x) dx = q_i^u [p_i^u - t(x_i^2 - x_i + \frac{1}{2})], i = 1,2. \tag{24}$$

In order to save space, we skip the procedure of stage 2 and jump directly to the profit-maximizing conditions for location in stage 1. These conditions are as follows:

$$\partial \Pi_i^u / \partial x_i = 4t(\alpha_2 - \alpha_1) q_1^u [(1/2) - x_i] = 0, i = 1,2. \tag{25}$$

It is shown from (25) that the term on the right-hand side is denoted as a cost-saving effect, while the competition effect vanishes. This arises because firms charge the same price for every point along the Hotelling line, which leads to the result that firms are unable to increase price and profits by locating further away from each other. Thus, the competition effect disappears. The only effect left is the cost-saving effect, where firms will locate at the center of the Hotelling line to minimize transport costs. This result can be supported by solving (25) while considering all three constraints; viz. the second-order, the stability and the market serving conditions. We may write the firms' optimal locations as follows:

$$x_i^u = \frac{1}{2}, i = 1,2. \tag{26}$$

Next, we turn to examine the case of mill pricing. The indirect utility of a consumer residing at site x can be rewritten as:

$$u(x) = k + \theta \alpha_i - p_i^f - t|x - x_i|, \tag{27}$$

where the superscript "*f*" denotes the variables associated with the case of mill pricing.

The marginal consumer's choice satisfies:

$$\hat{\theta}^f(x) = [p_2^f - p_1^f + t(|x - x_2| - |x - x_1|)] / (\alpha_2 - \alpha_1). \tag{28}$$

Firms' demand functions under the case of mill pricing can be expressed as:

$$q_1^f(x) = [(p_2^f - p_1^f) + t(|x - x_2| - |x - x_1|) / (\alpha_2 - \alpha_1)] - (\bar{\theta} - 1), \tag{29.1}$$

$$q_2^f(x) = \bar{\theta} - [(p_2^f - p_1^f) + t(|x - x_2| - |x - x_1|) / (\alpha_2 - \alpha_1)]. \tag{29.2}$$

Firms' aggregate operating profits functions under the case of mill pricing can similarly be written as:

$$\begin{aligned} \Pi_1^f &= \int_0^1 q_1^f(x) p_1^f dx \\ &= p_1^f [(p_2^f - p_1^f) + t(x_2 - x_1)(x_2 + x_1 - 1) - (\bar{\theta} - 1)(\alpha_2 - \alpha_1) / (\alpha_2 - \alpha_1)], \end{aligned} \tag{30.1}$$

$$\begin{aligned} \Pi_2^f &= \int_0^1 q_2^f(x) p_2^f dx \\ &= p_2^f [\bar{\theta}(\alpha_2 - \alpha_1) - (p_2^f - p_1^f) - t(x_2 - x_1)(x_2 + x_1 - 1) / (\alpha_2 - \alpha_1)]. \end{aligned} \tag{30.2}$$

Similarly, the profit-maximizing conditions of location can be derived as follows:

$$\partial\Pi_i^{\,f} / \partial x_i = 4t(\alpha_2 - \alpha_1)q_1^{\,u}[(1/2) - x_i] = 0, i = 1,2. \tag{31}$$

Solving (31) with due consideration of all three constraints yields the firms' optimal locations as follows:

$$x_i^{\,f} = \frac{1}{2}, i = 1,2. \tag{32}$$

We see from (30) that central agglomeration is the unique location equilibrium. This occurs because the competition effect is no longer present due to charging the same mill price at each point of the Hotelling line. The same intuition stated in the case of uniform delivered pricing applies to this case.

6. Conclusion and Suggestions

First of all, assuming that firms engage in discriminatory pricing in the commodity market, firms agglomerate at the market center when the degree of vertical differentiation is sufficiently high, while they move apart when it lies in between $\Delta_3^{\,D} \le (\alpha_2 - \alpha_1) \le \Delta_3^{\,A}$. Moreover, firms locate further apart in the disperse equilibrium when the degree of vertical differentiation between products is lower or the transport rate is higher. However, the Principle of Maximum Differentiation can never emerge.

Secondly, firms agglomerate at the market center as long as the degree of vertical differentiation is greater than zero in both cases, where firms charge uniform delivered and mill pricings. The reason why the disperse equilibrium can not be existent in these two cases is that firms charge the same price for every point along the Hotelling line leading to the result that the competition effect vanishes and only cost-saving effect remains.

7. Self-evaluation

First of all, this study is fully in accordance with the original proposal. Moreover, I have completed the whole targets raised in the proposal. Secondly, this study derives several striking results leading to the possibility that the study could be published in academic journals.

淡江大學專任教（職）員出國報告書提要

Summary report : For Tamkang Faculty Attending a Conference Abroad/In Service Training

<table>
<tr><td>姓　　名：(Reporter's Name) :
梁文榮</td><td>所屬系所 (Department/Institution) :
產業經濟系</td></tr>
<tr><td>開會(訓練)名稱 (Title of Conference/Training) :
2007 North American Meetings of Regional Science Association.</td><td>職　　稱 (Position) : 教授</td></tr>
<tr><td>會議(訓練)地點 (Place of Conference/Training) :
Savannah, Georgia, USA.</td><td>會期或訓練期間 (Date of the Meeting/Period of Training) :
96 年 11 月 07 日~96 年 11 月 15 日</td></tr>
<tr><td>論文名稱 (Title of Paper Presented) :
A General Analysis of Spatial Cournot Competition in a Linear City Model</td><td>校內外補助單位 (Subsidizing Agencies) :
國科會</td></tr>
<tr><td colspan="2">提要 (Summary Report) :
一、　參加會議經過
本人搭乘長榮航空班機經美國洛杉磯、Charlotte 至 Savannah。本人全程參與會議，共聆聽六個 sessions 之論文宣讀，均為區域經濟相關的論文。由於與會論文均經過嚴格篩選，品質極高，因此獲益匪淺。本人也於 10 日上午發表一篇論文，由 Professor Fu-Chuan Lai 提出建設性的問題，使論文因而增色許多。此外，大會指派本人擔任該 session 之主持人。在會議期間，本人也與法國之 Thisse 教授、日本之 Fujita 與 Tubuchi 教授、香港之孔繁欽、美國之 Ping Wang 教授及台灣之彭信坤與賴孚權教授討論許多區域經濟相關的議題及概念，獲益良多。並將與 Ping Wang 與 Thisse 教授合作一篇論文。再者，本人於會後再搭機至 St. Louis 華盛頓大學經濟系訪問，與 Ping Wang 教授就雙方合作議題繼續討論，直到 11 月 13 日才回國，期間並與在該系訪問的陳明郎與徐美教授有密切互動及討論，收獲極為豐碩。

二、　與會心得
此北美區域科學學會年會為區域科學之學術水準最高且參與學者最多的學術會議，參加此一研討會對個人觀念的啟發與最新研究趨勢的了解，極有幫助。再者，由於與會學者互動密切，對日後推動國際交流也有極大助益。</td></tr>
<tr><td colspan="2">備　　註 (Remark) :</td></tr>
<tr><td>國際交流委員會 (International Exchange Committee) :</td><td>單位主管簽名(Chairman/Chief)</td></tr>
</table>

＊受訓人員請另行檢附詳細報告書連同本提要送國際交流委員會

＊In-Service Trainees : Please give a detailed report to International Exchange Committee

淡江大學專任教（職）員出國報告書提要

Summary report : For Tamkang Faculty Attending a Conference Abroad/In Service Training

<table>
<tr><td>姓　　名：(Reporter’s Name) :
梁文榮</td><td>所屬系所 (Department/Institution) :
產業經濟系</td></tr>
<tr><td>開會(訓練)名稱 (Title of Conference/Training) :
PET 08 (Association for Public Economic Theory)</td><td>職　　稱 (Position) : 教授</td></tr>
<tr><td>會議(訓練)地點 (Place of Conference/Training) :
Seoul, South Korea.</td><td>會期或訓練期間 (Date of the Meeting/Period of Training) :
97 年 06 月 26 日~97 年 06 月 30 日</td></tr>
<tr><td>論文名稱 (Title of Paper Presented) :
Spatial Agglomeration with Vertical Differentiation</td><td>校內外補助單位 (Subsidizing Agencies) :
國科會、淡江大學</td></tr>
</table>

提要 (Summary Report) :

三、　參加會議經過

本人於 6 月 26 日搭乘大韓航空班機至韓國首爾 (Seoul)。本人全程參與會議，共聆聽六個 sessions 之論文宣讀，均為區域與產業經濟相關的論文；另聆聽諾貝爾經濟學獎得主 Mirrlees 的專題演講，講題為 Optimal Taxation of Saving and Inheritance。由於與會論文均經過嚴格篩選，品質極高，因此獲益匪淺。本人也於 28 日上午發表一篇論文，美國 Washington University at St. Louis 的 Marcus Berliant 及 Ping Wang、 Boston University 的 Hideo Konishi 教授與中研院經濟所的彭信坤教授均提出建設性的意見，使論文因而增色許多。在會議期間，本人也與美國之 Marcus Berliant 教授、Qian Wen、Ping Wang 教授及台灣之彭信坤、楊建成、陳虹如與張俊仁教授討論許多區域與公共經濟相關的議題及概念，獲益良多。

四、　與會心得

此 PET 08 為 Association for Public Economic Theory 舉辦之年會，為學術水準極高且發表之論文多達二、三百篇的高水準學術會議，參加此一研討會對個人觀念的啟發與最新研究趨勢的了解，極有幫助。再者，由於與會學者互動密切，對日後推動國際交流也有極大助益。

<table>
<tr><td colspan="2">備　　註 (Remark) :</td></tr>
<tr><td>國際交流委員會 (International Exchange Committee) :</td><td>單位主管簽名(Chairman/Chief)</td></tr>
</table>

＊受訓人員請另行檢附詳細報告書連同本提要送國際交流委員會

＊In-Service Trainees : Please give a detailed report to International Exchange Committee

Spatial Agglomeration with Vertical Differentiation

Ching-Chih Tseng
Department of Industrial Economics
Tamkang University

and

Wen-Jung Liang*
Department of Industrial Economics
Tamkang University
and
Public Economics Research Center
National Taiwan University

and

Kuang-Cheng Andy Wang
Social Science Division
Center for General Education
Chang Gung University

Current Version: August 15, 2008

JEL Classification: R3, L15
Key Words: Spatial Agglomeration; Two-dimensional Framework; Vertical Differentiation; Competition Effect; Cost-Saving Effect

* Corresponding Author: Wen-Jung Liang, Department of Industrial Economics, Tamkang University, Tamsui, Taipei County, Taiwan 25137, ROC. Tel: 886-2-2625-1863, Fax: 886-2-2620-8731, E-mail: wjliang@mail.tku.edu.tw

Abstract

This paper constructs a two-dimensional framework to take into consideration both horizontal and vertical differentiation. The focus of the paper is on the impact of vertical (quality) differentiation to firms' location configuration. It employs a two-stage game, in which firms first simultaneously decide optimal locations and then play Bertrand price competition with three pricing policies. This paper shows that the Principle of Minimum Differentiation may occur while the Principle of Maximum Differentiation can never emerge, if firms engage in spatially discriminatory pricing. It also shows that spatial agglomeration is the unique location equilibrium in both cases where firms charge uniform delivered and mill pricings.

1. Introduction

Hotelling (1929) first proposed that two firms of a homogeneous product agglomerate at the center of the line market under linear transportation costs, which has been termed the *Principle of Minimum Differentiation*. However, D'Aspremont *et al*. (1979) challenge this principle by indicating that there exists no price equilibrium in this case and shows that the two firms will locate at the opposite endpoints of the line market under quadratic transportation cost instead. This has been termed the *Principle of Maximum Differentiation*. From then on, many regional economists have tried to deduce the conditions under which the Principle of Minimum Differentiation can be restored. They include: Stahl (1982) who considers some harmonious conjectural variations; De Palma *et al*. (1985) and Rhee *et al*. (1992), who introduce heterogeneity in both consumers and firms; Anderson and Neven (1991) who assume that firms play Cournot quantity competition instead of Bertrand price competition in the commodity market; Jehiel (1992) and Friedman and Thisse (1993) who adopt price collusion; and Tabuchi (1994) who constructs a model with two dimensions of horizontal differentiation. Tabuchi in particular shows that two firms maximize their distance in one dimension, but minimize their distance in the other dimension. In addition to these researchers, Zhang (1995) imposes a price-matching policy; Mai and Peng (1999) emphasize the importance of the externality-like benefits generated from the exchange of information between firms; Liang and Mai (2006) focus on the crucial influence caused from the vertical subcontracting of the intermediate product; and Matsushima and Matsumura (2006) analyze the mixed-oligopoly economy.

Ferreira and Thisse (1996) employ Launhardt's (1885) spatial oligopolistic model to examine the decisions of the firms' optimal quality levels (vertical differentiation) taking location (horizontal differentiation) as exogenously given.[1] They use transport rate as a measure of quality, a high (low) transport rate representing a low (high) quality level, and the game employed is a two-stage game, in which firms select the optimal quality levels in the first stage and then engage in Bertrand price competition in the commodity market in the second stage. They find an interesting result that firms select to maximize the vertical differentiation when the horizontal differentiation is minimized, while to minimize the vertical differentiation when the horizontal differentiation is maximized. This result is termed the Max-Min and Min-Max result hereafter.

It can be observed in the real world that there exist many industries whose location choice can be regarded as a short-term decision, while quality is a long-term decision. This kind of setting can be found in Mai and Peng (1999) where location is endogenously determined under which R&D (or quality level) is treated as given.[2]

---

[1] According the definition of Ferreira and Thisse (1996, p. 486), two products are said to be horizontally differentiated when both products have a positive demand whenever they are offered at the same price. Neither product dominates the other in terms of characteristics, and heterogeneity in preferences over characteristics explains why both products are present in the market. We can also find a similar definition in Lancaster (1979).

[2] Bonanno and Hawoth (1998) also treat quality as a long-term decision, while process R&D as a

Generally, these cases arise in the industries where set-up costs are lower such that firms are capable of changing their locations easily with fixed qualities. Examples include: various quality levels of restaurants, electronic appliance stores, apparel shops, motels, and so on.

Theoretically, taking into account both horizontal differentiation (i.e., location) and vertical differentiation (i.e., qualities) allows us to explore the substitutability of quality for location and the strategic interactions between location-quality combinations that firms provide.[3] To the best of our knowledge, the decision of firms' optimal location, in which the level of quality is exogenously determined, has yet to be touched upon. This paper aims to fill this gap.

Based on the above analysis, the purpose of this paper is to determine the conditions under which the Principle of Minimum Differentiation can be restored where the firms' quality levels (vertical differentiation) are exogenously given. In order to take into account both horizontal and vertical differentiation, we follow Economides (1993) by introducing a two-dimensional model, in which each differentiated product is defined by one feature of location and one feature of quality. This facilitates the study of the effect of quality differentiation on firms' location decisions.

---

short-term decision.

[3] See Economides (1993, p. 236).

The game in question is a two-stage game, in which firms simultaneously select their optimal locations to maximize profits in the first stage, and then play Bertrand competition in the commodity market in the second stage. Three pricing regimes -- discriminatory, uniform delivered and mill pricings -- are taken into consideration.

We show, in the paper, that the equilibrium location is determined by the balance of two countervailing effects, the centrifugal competition effect and the centripetal cost-saving effect.[4] The competition effect indicates that as the two firms are more distant from each other, they become more dissimilar and therefore competition lessens.[5] Accordingly, the two firms tend to separate more distantly to reduce the competition for earning higher profits via charging higher prices. It is shown that the introduction of vertical differentiation mitigates the competition effect due to enlarging the differentiation of products. On the other hand, the cost-saving effect reflects firm *i'*s desire to move toward the center in order to save on the transportation cost. Accordingly, we will show that firms agglomerate if the degree of the vertical differentiation between products is high enough while separate if the products are less differentiated, as firms engage in discriminatory pricing. However, this competition effect vanishes leading to the result that spatial agglomeration is the unique location equilibrium, as firms conduct uniform delivered and mill pricings. This result arises

[4] The idea of the competition effect can also be found in Liang *et al*. (2006).

[5] Contrarily, if the two firms locate at the center of the market, they are symmetric in terms of production cost plus transport cost at any site of the market and the competition is the highest.

because firms charge the same price for every point over the Hotelling line leading to the outcome that firms are unable to increase price and profits via taking farther apart.

As to be shown in this paper, the agglomeration result, under which the degree of vertical differentiation is higher, can be supported by the industries of restaurant, motel, department store, apparel shop, *etc*. We can observe frequently that both higher quality restaurant and lower quality food stands in food court are located closely in many large hotels and department stores; various quality levels of motels locate within a narrow district in many attractions, for example Best Western Grantree Inn rated 3 star and Comfort Inn Bozeman rated 2 star locate next to each other at the same street, North Seventh Avenue, Bozeman, Montana;[6] and some department stores such as Dillard (higher quality store) and JC Penny (lower quality store) and apparel stores such as Banana Republic (higher quality store) and The Limited (lower quality store) agglomerate at the same mall in many towns of the U.S. In contrast, the disperse result, under which products are less differentiated, can be supported by the supermarket and electronic appliance businesses. There is significant evidence that Wal-mart and K-mart, as well as Circuit City and local electronic appliance stores, never locate at the same site due to narrow quality differentiation.[7]

The remainder of the paper is organized as follows. Section 2 sets up a spatial

[6] Bozeman is a city near Yellow Stone National Park. See the following website for details: http://www.ihsadvantage.com/h/hotels/bozeman/mt/us/?trafficID=531450408&serverID=L8%.

[7] The pricing policy of Wal-mart is every day low price, and that of Circuit City is lowest price

model with products exhibiting exogenously vertical differentiation and analyzes the optimal location in the case of discriminatory pricing. Section 3 examines the optimal location in the cases of uniform delivered and mill pricings. The final section concludes the paper.

2. Spatially Discriminatory Pricing

Consider a two-dimensional framework, in which the horizontal axis measures the traditional Hotelling line referred to as horizontal characteristic, while the vertical axis measures the tastes of consumers to qualities referred to as vertical characteristic, as shown in Figure 1.[8] Two firms, denoted firm 1 and firm 2, are located at x_1 and x_2, with $x_1 \leq x_2$ along a line segment with length $L = 1$ on the horizontal axis. The firms, whose production cost is for simplicity assumed to be nil, sell products with vertically differentiated qualities, α_1 and α_2 with $\alpha_1 \leq \alpha_2$ respectively, to consumers. In a model with vertically differentiated qualities, there must be heterogeneity in consumers' willingness to pay for quality, which is captured by assuming that a continuum of consumers is uniformly distributed over the interval $[\underline{\theta}, \overline{\theta}]$ along the vertical axis with unit density at each point of the Hotelling line.[9] Following Choi and Shin (1987),

---

guaranteed. Both pricing policies demonstrate the feature of Bertrand price competition.

[8] Economides (1993) extends the circular model of variety-differentiated products constructed by Salop's (1979) to a two-dimensional model, in which both horizontal and vertical differentiation are taken into consideration.

[9] Given two products with different qualities, all consumers would prefer the product with higher quality to that with lower quality at the same price. In order to keep the two firms survive in the market,

we assume $\underline{\theta} = \overline{\theta} - 1$, where $\overline{\theta}$>1. Thus, these two characteristics lead to a rectangular distribution of consumers over [0, 1] × $[\overline{\theta}-1, \overline{\theta}]$. A firm faces a continuum of consumers with taste $\theta \in [\overline{\theta}-1, \overline{\theta}]$ at each point of the Hotelling line or a continuum of consumers with different locations for a given taste of the vertical axis. Assume further that the transport cost function of the product is linear and takes the following form: $T(x - x_i) = t|x_i - x|$, where T is the transport cost, and t is the transport rate per unit output per unit distance.

(Insert Figure 1 here)

Suppose that firms engage in discriminatory pricing to charge different prices for consumers residing at different locations. The indirect utility of a consumer residing at the location with combination (x, θ) and purchasing from firm i can be expressed as:

$$u(x,\theta) = k + \theta\alpha_i - p_i(x), i = 1,2, \tag{1}$$

where $u(x, \theta)$ is the utility function of the consumer with combination (x, θ); and k is the reservation utility of consuming one unit of commodity; and θ denotes the taste of consumers' preference for quality ranging along the interval $[\overline{\theta}-1, \overline{\theta}]$ with $\overline{\theta}$ is the upper bound of the consumers' tastes; and α_i (i = 1, 2) represents the quality level of the product produced by firm i; and p_i (x) is the delivered price charged by firm i at site x.

---

the consumers' willingness to pay for quality must be heterogeneous.

The taste of the marginal consumer, who is indifferent between buying one unit of the product from either firm, for a continuum of consumers residing at x can be obtained by equaling the utility levels of buying from the two firms as follows:[10]

$$\hat{\theta}(x) = [p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1), \tag{2}$$

where $\hat{\theta}(x)$ denotes the taste of the marginal consumer for a continuum of consumers residing at x.

Each firm's demand function at site x can be derivable as:

$$q_1(x) = \hat{\theta} - \underline{\theta} = \{[p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1) - (\overline{\theta} - 1)\}, \tag{3.1}$$

$$q_2(x) = \overline{\theta} - \hat{\theta} = \{\overline{\theta} - [p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1)\}. \tag{3.2}$$

Assuming that production costs are zero and the quality cost is fixed, firm i's operating profit function at site x can be expressed as:[11]

$$\pi_i(x) = [p_i(x) - t|x - x_i|]q_i(x), i = 1,2, \tag{4}$$

where $\pi_i(x)$ denotes firm i's operating profit at site x.

The game employed in this paper is a two-stage game as discussed previously. The sub-game perfect Nash equilibrium can be solved by backward induction, beginning with the final stage. Differentiating (4) with respect to $p_i(x)$ respectively, we

---

[10] Notice that eq. (2) is derived by assuming that the reservation utility k is sufficiently high such that all consumers buy one unit of product, i.e., the market is covered. However, the main results of the paper remain unchanged if the market is uncovered, i.e., some low taste consumers refuse to purchase any product. For simplicity, we use the covered market assumption for the exclusion of tedious expositions.

can derive the profit-maximizing conditions for prices in stage 2. Solving these equations, we have:[12]

$$p_1(x) = (1/3)[(\alpha_2 - \alpha_1)(2 - \overline{\theta}) + t(2|x - x_1| + |x - x_2|)], \tag{5.1}$$

$$p_2(x) = (1/3)[(\alpha_2 - \alpha_1)(\overline{\theta} + 1) + t(|x - x_1| + 2|x - x_2|)]. \tag{5.2}$$

Substituting (5) into (3), we obtain:

$$q_1(x) = [1/3(\alpha_2 - \alpha_1)][(\alpha_2 - \alpha_1)(2 - \overline{\theta}) + t(|x - x_2| - |x - x_1|)], \tag{6.1}$$

$$q_2(x) = [1/3(\alpha_2 - \alpha_1)][(\alpha_2 - \alpha_1)(\overline{\theta} + 1) - t(|x - x_2| - |x - x_1|)]. \tag{6.2}$$

It is worth noting that the upper bound of the quality taste ($\overline{\theta}$) must be smaller than 2 to ensure firm 1's demand being positive, as firms locate at the same site, i.e. $x_1 = x_2$. Consequently, the upper bound of the quality taste lies within the interval [1, 2).

Substituting (5) into (2), we can derive the taste of the marginal consumer residing at site x as follows:

$$\hat{\theta}(x) = [1/3(\alpha_2 - \alpha_1)][(\alpha_2 - \alpha_1)(2\overline{\theta} - 1) + t(|x - x_2| - |x - x_1|)], x \in [0,1]. \tag{7}$$

Differentiating (7) with respect to x, yields:[13]

$$\partial\hat{\theta}(x)/\partial x = \begin{cases} 0 & \text{if } \ x \in [0, x_1], \\ -2t/3(\alpha_2 - \alpha_1) < 0 & \text{if } \ x \in [x_1, x_2], \\ 0 & \text{if } \ x \in [x_2, 1]. \end{cases} \tag{8}$$

We see from (8) that given firms' locations x_1 and x_2, the taste of the marginal consumer remains unchanged for $x \in [0, x_1]$ and $x \in [x_2, 1]$, while taste decreases

---

[11] Firm *i*'s profit equals its operating profit minus fixed quality cost.
[12] Suppose that the second-order conditions are satisfied.
[13] In the second stage, firms' locations x_1 and x_2 have been determined in the first stage. We can thus

with respect to x within the interval [x_1, x_2]. According to eqs. (7) and (8), the relationship of the taste of the marginal consumer and location x along the Hotelling line is depicted as the broken line on Figure 1. The area above the broken line represents the total output of the high quality firm, while the area below denotes the total output of the low quality firm.

Next, we turn to the first stage. Substituting (5) and (6) into (4), we can derive firm i's reduced aggregate operating profit function as follows:

$$\begin{aligned}\Pi_1 = {} & [1/9(\alpha_2 - \alpha_1)]\{\int_0^{x_1}[(\alpha_2 - \alpha_1)(2-\bar{\theta}) + t(x_2 - x_1)]^2 dx \\ & + \int_{x_1}^{x_2}[(\alpha_2 - \alpha_1)(2-\bar{\theta}) + t(x_2 + x_1 - 2x)]^2 dx \\ & + \int_{x_2}^{1}[(\alpha_2 - \alpha_1)(2-\bar{\theta}) - t(x_2 - x_1)]^2 dx\},\end{aligned} \tag{9.1}$$

$$\begin{aligned}\Pi_2 = {} & [1/9(\alpha_2 - \alpha_1)]\{\int_0^{x_1}[(\alpha_2 - \alpha_1)(\bar{\theta}+1) + t(x_1 - x_2)]^2 dx \\ & + \int_{x_1}^{x_2}[(\alpha_2 - \alpha_1)(\bar{\theta}+1) + t(-x_1 - x_2 + 2x)]^2 dx \\ & + \int_{x_2}^{1}[(\alpha_2 - \alpha_1)(\bar{\theta}+1) - t(x_1 - x_2)]^2 dx\}.\end{aligned} \tag{9.2}$$

Differentiating (9) with respect to x_i, respectively, yields the profit-maximizing conditions for locations as follows:

$$\begin{aligned}\partial\Pi_1/\partial x_1 = {} & (t)\{-[2t/9(\alpha_2 - \alpha_1)]\big[(x_2 - x_1)(1 - x_2 + x_1)\big] \\ & + (4/9)(1/2 - x_1)(2-\bar{\theta})\} = 0,\end{aligned} \tag{10.1}$$

have $\partial x_1/\partial x = 0$ and $\partial x_2/\partial x = 0$.

$$\partial\Pi_2/\partial x_2 = (t)\{[2t/9(\alpha_2-\alpha_1)]\big[(x_2-x_1)(1-x_2+x_1)\big] \\ +(4/9)(1/2-x_2)(\overline{\theta}+1)\} = 0, \tag{10.2}$$

where $1<\overline{\theta}<2$ and $0\le x_1\le x_2\le 1$.

Recalling that $0\le x_1\le x_2\le 1$ and $\alpha_1 \le \alpha_2$, we find that the first term in the brace of the right-hand side of (10.1) is non-positive. This term can be named the competition effect, which shows that as the two firms move apart, the horizontal differentiation between the two products is increased, implying that price competition between firms is mitigated. Consequently, the competition effect attracts firm 1 to move leftward. Moreover, the competition effect is weakened, as the two products become more vertically differentiated (i.e., α_2 - α_1, is larger) or the transport rate is lower. On the other hand, the second term in the brace is denoted as the transportation cost saving effect (for simplicity, the cost-saving effect, hereafter), whose value is non-negative. This arises because the first term is non-positive. In order to ensure an interior solution, the second term has to be non-negative to make the profit-maximizing condition equal zero. The cost-saving effect reflects firm *i*'s desire to move toward the center in order to save on the transportation cost. Consequently, firm 1's location equilibrium is determined by the balance of the competition and the cost-saving effects. We find from (10.2) that this result applies to firm 2's location equilibrium, in which the competition effect attracts firm 1 to move rightward while the cost-saving effect forces firm 2 to move toward the center of the market.

The location equilibria are subject to the second-order and the stability conditions as follows:

$$\partial^2 \Pi_1 / \partial x_1{}^2 = [2t/9(\alpha_2-\alpha_1)]\{-2(\alpha_2-\alpha_1)(2-\overline{\theta})-t[2(x_2-x_1)-1)]\} \le 0, \quad (11.1)$$

$$\partial^2 \Pi_2 / \partial x_2{}^2 = [2t/9(\alpha_2-\alpha_1)]\{-2(\alpha_2-\alpha_1)(\overline{\theta}+1)-t[2(x_2-x_1)-1]\} \le 0, \quad (11.2)$$

$$\begin{aligned} J &= (\partial^2\Pi_1/\partial x_1^2)(\partial^2\Pi_2/\partial x_2^2)-(\partial^2\Pi_1/\partial x_1\partial x_2)(\partial^2\Pi_2/\partial x_2\partial x_1) \\ &= [8t^2/81(\alpha_2-\alpha_1)]\{2(\alpha_2-\alpha_1)(2-\overline{\theta})(\overline{\theta}+1)+3t[2(x_2-x_1)-1]\} \ge 0. \end{aligned} \quad (11.3)$$

In addition, the location equilibria should fulfill the market-serving condition, which requires the output of each firm at its remote endpoint shipped from its equilibrium location be positive.[14] This can be described as follows:

$$q_1(1;x_1,x_2) = [(\alpha_2-\alpha_1)(2-\overline{\theta})-t(x_2-x_1)]/3(\alpha_2-\alpha_1) > 0, \quad (12.1)$$

$$q_2(0;x_1,x_2) = [(\alpha_2-\alpha_1)(1+\overline{\theta})-t(x_2-x_1)]/3(\alpha_2-\alpha_1) > 0. \quad (12.2)$$

Solving (10), we can obtain location equilibria as follows:

$$x_1{}^A = x_2{}^A = 1/2, \quad (13.1)$$

$$\begin{aligned} x_1{}^D &= [2(\alpha_2-\alpha_1)(2+3\overline{\theta}-\overline{\theta}^3)/9t]+[(1-2\overline{\theta})/6], \\ x_2{}^D &= [2(\alpha_2-\alpha_1)(-4+3\overline{\theta}^2-\overline{\theta}^3)/9t]+[(7-2\overline{\theta})/6], \end{aligned} \quad (13.2)$$

where the superscript "*A*" ("*D*") denotes the variables associated with the case of the agglomeration (disperse) equilibrium, respectively.

There are two possible location equilibria, central agglomeration and spatial dispersion. Substituting (13.1) into (11.1) - (11.3), (12.1) and (12.2), we have:

---

[14] See Yang *et al*. (2007).

$$\partial^2\Pi_1/\partial x_1^{\,2}\Big|_{x_1^A, x_2^A} \le 0, \text{if } (\alpha_2-\alpha_1) \ge t/2(2-\overline{\theta}) \equiv \Delta_1^{\,A}, \tag{14.1}$$

$$\partial^2\Pi_2/\partial x_2^{\,2}\Big|_{x_1^A, x_2^A} \le 0, \text{if } (\alpha_2-\alpha_1) \ge t/2(1+\overline{\theta}) \equiv \Delta_2^{\,A}, \tag{14.2}$$

$$J\Big|_{x_1^A, x_2^A} \ge 0, \text{if } (\alpha_2-\alpha_1) \ge 3t/2(2-\overline{\theta})(1+\overline{\theta}) \equiv \Delta_3^{\,A}, \tag{14.3}$$

$$q_1(1; x_2 = x_1 = 1/2) = (2-\overline{\theta})/3 > 0, \tag{15.1}$$

$$q_2(0; x_2 = x_1 = 1/2) = (1+\overline{\theta})/3 > 0. \tag{15.2}$$

Recall that $1 < \overline{\theta} < 2$. We figure out from (14) that $\Delta_3^A > \Delta_1^A > \Delta_2^A$. Thus, central agglomeration arises only if the degree of vertical differentiation is sufficiently large, say $(\alpha_2-\alpha_1) \ge \Delta_3^{\,A}$. The intuition behind this result can be stated as follows. We have argued that the location equilibrium is determined by the competition and the cost-saving effects. We have also shown that the higher the degree of vertical differentiation, the weaker the competition effect will be. Therefore, as the degree of vertical differentiation is no less than the critical value Δ_3^A, the competition effect is dominated by the cost-saving effect such that the two firms agglomerate at the center of the Hotelling line.

Next, substituting (13.2) into (11.1) - (11.3), (12.1) and (12.2), we have:

$$\partial^2\Pi_1/\partial x_1^{\,2}\Big|_{x_1^D, x_2^D} \le 0, \text{if } (\alpha_2-\alpha_1) \le 3t/2(2-\overline{\theta})(2\overline{\theta}-1) \equiv \Delta_1^{\,D}, \tag{16.1}$$

$$\partial^2\Pi_2/\partial x_2^{\,2}\Big|_{x_1^D, x_2^D} = 2t[-2(\alpha_2-\alpha_1)(1+\overline{\theta})(2\overline{\theta}-1)-5t]/27(\alpha_2-\alpha_1) \le 0, \tag{16.2}$$

$$J\Big|_{x_1^D, x_2^D} \ge 0, \text{if } (\alpha_2-\alpha_1) \le 3t/2(2-\overline{\theta})(1+\overline{\theta}) \equiv \Delta_2^{\,D} \equiv \Delta_3^{\,A}, \tag{16.3}$$

$$q_1(1; x_1^{\,D}, x_2^{\,D}) > 0, \text{if } (\alpha_2-\alpha_1) > 4t/(2-\overline{\theta})(5+2\overline{\theta}) \equiv \Delta_3^{\,D}, \tag{17.1}$$

$$q_2(0; x_1^D, x_2^D) > 0, \text{ if } (\alpha_2 - \alpha_1) > 4t/(1+\overline{\theta})(7-2\overline{\theta}) \equiv \Delta_4^D. \qquad (17.2)$$

We find from (16) that $\Delta_2^D < \Delta_1^D$, and from (17) that $\Delta_3^D > \Delta_4^D$. We also find from (14.3) and (16.3) that $\Delta_3^A = \Delta_2^D$. Accordingly, we can derive that spatial dispersion arises as the degree of vertical differentiation lies in between $[\Delta_3^D, \Delta_2^D]$, i.e., $\Delta_3^D \leq (\alpha_2 - \alpha_1) \leq \Delta_3^A$. The same intuition applies to this result. The competition effect is strong enough to push the two firms apart, as the degree of vertical differentiation is no greater than the critical value, Δ_3^A.

Based on the above analysis, we can establish:

Proposition 1. *Assuming that firms engage in discriminatory pricing, we yield*:

(i) *Firms agglomerate at the center of the Hotelling line as the degree of vertical differentiation is high enough, i.e.,* $(\alpha_2 - \alpha_1) \geq \Delta_3^A$.

(ii) *Spatial dispersion emerges as the degree of vertical differentiation lies in between* $\Delta_3^D \leq (\alpha_2 - \alpha_1) \leq \Delta_3^A$.

Note that this result is significant different from that derived in Hotelling (1929), in which price will be undercut to zero as firms agglomerate at the same site. This result emerges because in a model with vertically differentiated qualities, consumers' willingness to pay for quality is heterogeneous. Consequently, firms are able to charge different prices as long as the degree of vertical differentiation between the two products is greater than zero. As mentioned previously, examples supporting the agglomeration result include various quality levels of restaurants, motels, department stores, and apparel shops, while supermarkets and electronic appliance stores against this result.

Next, we explore the invalidity of the Principle of Maximum Differentiation. First of all, we examine this principle as the interior location equilibrium arises, which can be done via (13.2). Recall that $1 < \bar{\theta} < 2$ and $\alpha_1 \leq \alpha_2$. We can find from (13.2) that $x_2^D = [2(\alpha_2 - \alpha_1)(-4 + 3\bar{\theta}^2 - \bar{\theta}^3)/9t] + [(7 - 2\bar{\theta})/6] < 5/6.$[15] This shows that firm 2 would never locate at the right end of the Hotelling line. Thus, the Principle of Maximum Differentiation will never emerge in this case. Secondly, we examine this Principle when a corner solution for location equilibrium occurs. This arises as the conditions $\partial \Pi_1 / \partial x_1 < 0$ and $\partial \Pi_2 / \partial x_2 > 0$ hold. We can calculate from (10.1) that the former condition holds if $(\alpha_2 - \alpha_1) < t\,(x_2 - x_1)(1 - x_2 + x_1) / (1 - 2x_1)(2 - \bar{\theta}) \equiv \Delta_1^c$ and from (10.2) that the latter condition holds if $(\alpha_2 - \alpha_1) < t\,(x_2 - x_1)(1 - x_2 + x_1) / (2x_2 - 1)(1 + \bar{\theta}) \equiv \Delta_2^c$. We find that $\Delta_1^c = \Delta_2^c = 0$ as $x_1 = 0$ and $x_2 = 1$. Thus, the conditions hold only if $\alpha_2 - \alpha_1 < 0$, which contradicts the assumption $\alpha_1 \leq \alpha_2$ and excludes the possibility of the Principle of Maximum Differentiation as the corner solution for

[15] This arises because $\alpha_2 - \alpha_1 \geq 0$, $(-4 + 3\bar{\theta}^2 - \bar{\theta}^3) < 0$, and $[(7 - 2\bar{\theta})/6] < 5/6$.

location equilibrium emerges.

Based on an analysis of eqs. (13) – (17), we can depict the relationship between firms' location equilibria and the degree of vertical differentiation as shown in Figure 2. The locus D_1AE represents firm 1's location equilibrium, while locus D_2AE denotes firm 2's location equilibrium.[16] Figure 2 shows that as the degree of vertical differentiation, α_2 - α_1 is no less than Δ_3^A, two firms agglomerate at the center of Hotelling line, while they take apart as the degree of vertical differentiation lies in between (Δ_3^D, Δ_3^A).

(Insert Figure 2 here)

Accordingly, we have:

Proposition 2. *Assuming that firms engage in discriminatory pricing, the Principle of Maximum Differentiation can never emerge.*

By assuming location is determined prior to quality in a one-dimensional model with mill pricing, Ferreira and Thisse (1996) derive the Max-Min and Min-Max result.

---

[16] Manipulating eq. (13.2), we yield

$\partial x_1^D / \partial(\alpha_2 - \alpha_1) = 2(2 + 3\overline{\theta} - \overline{\theta}^3)/9t > 0,$

$\partial x_2^D / \partial(\alpha_2 - \alpha_1) = 2(-4 + 3\overline{\theta}^2 - \overline{\theta}^3)/9t < 0,$ and

$\partial^2 x_i^D / \partial(\alpha_2 - \alpha_1)^2 = 0, i = 1,2.$

Accordingly, we find that D_1A and D_2A are linear, the slope of D_1A (D_2A) is positive (negative) and D_2A is steeper than D_1A due to the absolute value of the slope of D_2A larger than that of D_1A.

Moreover, letting location be endogenously determined, Economides (1989) finds that firms locate as far apart as possible. However, by reversing the temporal ordering of location and quality decisions in a two-dimensional model, we show that the Principle of Minimum Differentiation can be valid if the degree of vertical differentiation is sufficiently high. By contrast, spatial dispersion emerges as the degree of vertical differentiation becomes sufficiently low. However, the Principle of Maximum Differentiation can never occur.

We now examine the impact of the transport rate on the critical values of the determination of central agglomeration and spatial dispersion. Differentiating Δ_3^A and Δ_3^D with respect to t, we obtain:

$$\partial\Delta_3^{A} / \partial t = 3/2(2-\overline{\theta})(1+\overline{\theta}) > 0, \tag{18.1}$$

$$\partial\Delta_3^{D} / \partial t = 4/(2-\overline{\theta})(5+2\overline{\theta}) > 0. \tag{18.2}$$

Equations (18.1) demonstrate that other things equal, a rise in the transport rate increases the critical value of the degree of vertical differentiation for firms to remain agglomerating at market center. This arises because the competition effect gets to be stronger as the transport rate is higher. In order to balance this stronger separating effect, the critical value of the degree of vertical differentiation has to be higher to keep firms agglomerate at the market center. We see from (18.2) that a rise in the transport rate increase the critical value of the degree of vertical differentiation for serving the whole market. This happens because the delivered prices are increased, which reduces the demand of the remote endpoint, as the transport rate is higher. In order to keep two firms competing at the remote endpoint, the critical value of the degree of vertical differentiation has to be higher to balance the strengthened competition effect. Accordingly, we yield the following Lemma:

Lemma 1. *Other things being equal, the critical values of the degree of vertical differentiation for firms to keep agglomerate at the market center as well as to serve the entire market get to be higher, as the transport rate is higher.*

Manipulating (13.2), we can derive the distance (degree of horizontal differentiation) between the two firms' location equilibria under the case of spatial dispersion as follows:

$$x_2{}^D - x_1{}^D = 1 - (2/3t)(\alpha_2 - \alpha_1)(2 - \overline{\theta})(\overline{\theta} + 1). \tag{19}$$

Differentiating the distance with respect to the degree of vertical differentiation as well as the transport rate, we have:

$$\partial(x_2{}^D - x_1{}^D)/\partial(\alpha_2 - \alpha_1) = -(2 - \overline{\theta})(\overline{\theta} + 1)/3t < 0, \tag{20.1}$$

$$\partial(x_2{}^D - x_1{}^D)/\partial t = 2(\alpha_2 - \alpha_1)(2 - \overline{\theta})(\overline{\theta} + 1)/3t^2 > 0. \tag{20.2}$$

We see from (20) that the distance, between the two firms, decreases as the degree of vertical differentiation rises, while it increases as the transport rate increases. Intuitively, the products become more differentiated leading to a weaker competition effect as the degree of the vertical differentiation is higher. The upshot is that the two firms approach each other spatially. On the other hand, the competition effect strengthens due to higher delivered prices as the transport rate rises. This induces firms to move further apart, while they charge higher prices and earn higher profits.

Accordingly, we can establish:

Proposition 3. *Assuming that firms engage in discriminatory pricing, firms locate further apart in the disperse equilibrium, as the degree of vertical differentiation is lower or the transport rate is higher.*

3. Uniform Delivered and Mill Pricings

In this section, we will examine firms' location equilibria as firms undertake uniform delivered and mill pricings in the commodity market. We first discuss the case of uniform delivered pricing. Firms charge the same delivered price at each point of the Hotelling line, respectively, in this case. Thus, the indirect utility of a consumer, who purchases from firm i, can be expressed as:

$$u = k + \theta\alpha_i - p_i^u, i = 1,2, \tag{21}$$

where the superscript "*u*" denotes the variables associated with the case of uniform delivered pricing.

The marginal consumer, who is indifferent between buying one unit of product from either firm, following (21), acts so as to satisfy:

$$\hat{\theta}^{u} = (p_2{}^{u} - p_1{}^{u})/(\alpha_2 - \alpha_1). \tag{22}$$

Firms' demand functions under the case of uniform delivered pricing are therefore equal to:

$$q_1{}^{u} = [(p_2{}^{u} - p_1{}^{u})/(\alpha_2 - \alpha_1)] - (\overline{\theta} - 1), \tag{23.1}$$

$$q_2{}^{u} = \overline{\theta} - [(p_2{}^{u} - p_1{}^{u})/(\alpha_2 - \alpha_1)]. \tag{23.2}$$

Firm's aggregate operating profits can be obtained by integrating its operating profit of each point along the Hotelling line, and can be expressed as:

$$\Pi_i{}^{u} = \int_0^1 (p_i{}^{u} - t|x - x_i|)q_i(x)dx = q_i{}^{u}[p_i{}^{u} - t(x_i{}^2 - x_i + \frac{1}{2})], i = 1,2. \tag{24}$$

In order to save space, we skip the procedure of stage 2 and jump directly to the profit-maximizing conditions for location in stage 1. These conditions are as follows:[17]

$$\partial \Pi_i{}^{u} / \partial x_i = 4t(\alpha_2 - \alpha_1)q_1{}^{u}[(1/2) - x_i] = 0, i = 1,2. \tag{25}$$

It is shown from (25) that the term on the right-hand side is denoted as a

---

[17] The second-order conditions are:
$\partial^2 \Pi_1{}^{u} / \partial x_1{}^2 \le 0$, if $(\alpha_2 - \alpha_1) \ge [t/2(2 - \overline{\theta})][6x_1(x_1 - 1) - 2x_2(x_2 - 1) + 1] \equiv \Delta_1{}^{u}$.
$\partial^2 \Pi_2{}^{u} / \partial x_2{}^2 \le 0$, if $(\alpha_2 - \alpha_1) \ge [t/2(1 + \overline{\theta})][6x_2(x_2 - 1) - 2x_1(x_1 - 1) + 1] \equiv \Delta_2{}^{u}$.

cost-saving effect, while the competition effect vanishes. This arises because firms charge the same price for every point along the Hotelling line, which leads to the result that firms are unable to increase price and profits by locating further away from each other. Thus, the competition effect disappears. The only effect left is the cost-saving effect, where firms will locate at the center of the Hotelling line to minimize transport costs. This result can be supported by solving (25) while considering all three constraints; viz. the second-order, the stability and the market serving conditions. We may write the firms' optimal locations as follows:[18]

$$x_i^u = \frac{1}{2}, i = 1,2. \tag{26}$$

Accordingly, we have the following proposition:

Proposition 4. *Assuming that firms undertake uniform delivered pricing, central agglomeration is the unique location equilibrium if the degree of vertical differentiation is greater than zero.*

This result is sharply different from that derived in the case of discriminatory pricing, in which spatial dispersion occurs as the degree of vertical differentiation is

[18] Substituting $x_1^u = x_2^u = 1/2$ into the second-order conditions, we can calculate that the critical values $\Delta_1^u = \Delta_2^u = 0$. Moreover, the stability condition is definitely greater than zero and the market-serving is also satisfied.

sufficiently low. This arises because firms charge the same price for every point along the Hotelling line leading to the result that the competition effect vanishes and only cost-saving effect remains.

Next, we turn to examine the case of mill pricing. The indirect utility of a consumer residing at site x can be rewritten as:

$$u(x) = k + \theta\alpha_i - p_i^f - t|x - x_i|, \tag{27}$$

where the superscript "f" denotes the variables associated with the case of mill pricing.

The marginal consumer's choice satisfies:

$$\hat{\theta}^f(x) = [p_2^f - p_1^f + t(|x - x_2| - |x - x_1|)]/(\alpha_2 - \alpha_1). \tag{28}$$

Firms' demand functions under the case of mill pricing can be expressed as:

$$q_1^f(x) = [(p_2^f - p_1^f) + t(|x - x_2| - |x - x_1|)/(\alpha_2 - \alpha_1)] - (\overline{\theta} - 1), \tag{29.1}$$

$$q_2^f(x) = \overline{\theta} - [(p_2^f - p_1^f) + t(|x - x_2| - |x - x_1|)/(\alpha_2 - \alpha_1)]. \tag{29.2}$$

Firms' aggregate operating profits functions under the case of mill pricing can similarly be written as:

$$\begin{aligned}\Pi_1^f &= \int_0^1 q_1^f(x) p_1^f dx \\ &= p_1^f[(p_2^f - p_1^f) + t(x_2 - x_1)(x_2 + x_1 - 1) - (\overline{\theta} - 1)(\alpha_2 - \alpha_1)/(\alpha_2 - \alpha_1)],\end{aligned} \tag{30.1}$$

$$\begin{aligned}\Pi_2^f &= \int_0^1 q_2^f(x) p_2^f dx \\ &= p_2^f[\overline{\theta}(\alpha_2 - \alpha_1) - (p_2^f - p_1^f) - t(x_2 - x_1)(x_2 + x_1 - 1)/(\alpha_2 - \alpha_1)].\end{aligned} \tag{30.2}$$

Similarly, the profit-maximizing conditions of location can be derived as follows:

$$\partial \Pi_i^{\,f} / \partial x_i = 4t(\alpha_2 - \alpha_1) q_1^{\,u} [(1/2) - x_i] = 0, i = 1,2. \tag{31}$$

Solving (31) with due consideration of all three constraints yields the firms' optimal locations as follows:

$$x_i^{\,f} = \frac{1}{2}, i = 1,2. \tag{32}$$

We see from (30) that central agglomeration is the unique location equilibrium. This occurs because the competition effect is no longer present due to charging the same mill price at each point of the Hotelling line. The same intuition stated in the case of uniform delivered pricing applies to this case. Consequently, we can establish:

Proposition 5. *Assuming that firms charge mill pricing, central agglomeration is the unique location equilibrium if the degree of vertical differentiation is greater than zero.*

4. Concluding Remarks

This paper has constructed a two-dimensional framework to take into account both the features of horizontal and vertical differentiation. It has shown that firms' location decisions depend on two countervailing forces: the centrifugal competition effect and the centripetal cost-saving effect. The focus of this paper is on the impact of vertical differentiation to firms' location decision through affecting the competition effect. We have argued that the higher the degree of vertical differentiation, the weaker the competition effect will be. This weakens the centrifugal competition effect leading to the existence of the Principle of Minimum Differentiation. Moreover, firms will not engage in price undercutting, as firms agglomerate at the same site due to the heterogeneity of the consumer's willingness to pay for quality. Several striking results are derived as follows.

First of all, assuming that firms engage in discriminatory pricing in the commodity market, firms agglomerate at the market center when the degree of vertical differentiation is sufficiently high, while they move apart when it lies in between $\Delta_3^{D} \le (\alpha_2 - \alpha_1) \le \Delta_3^{A}$. Moreover, firms locate further apart in the disperse equilibrium when the degree of vertical differentiation between products is lower or the transport rate is higher. However, the Principle of Maximum Differentiation can never emerge.

Secondly, firms agglomerate at the market center as long as the degree of vertical differentiation is greater than zero in both cases, where firms charge uniform delivered and mill pricings. The reason why the disperse equilibrium can not be existent in these two cases is that firms charge the same price for every point along the Hotelling line leading to the result that the competition effect vanishes and only cost-saving effect remains.

References

Anderson, S.P., and D.J. Neven, 1991, "Cournot Competition yield Spatial Agglomeration," *International Economic Review*, 32, 793-808.

Bonanno, G., and B. Haworth, 1998, "Intensity of Competition and the Choice between Product and Process Innovation," *International Journal of Industrial Organization*, 16, 495-510.

Choi, C.J., and H.S. Shin, 1992, "A Comment on a Model of Vertical Product Differentiation," *Journal of Industrial Economics*, 40, 299-231.

D'Aspremont, C., J.J. Gabzewicz, and J.-F. Thisse, 1979, "On Hotelling's Stability in Competition," *Econometrica*, 47, 1145-1150.

De Palma, A., V. Ginsburgh, Y.Y. Papageorgiou, and J.-F. Thisse, 1985, "The Principle of Minimum Differentiation Holds under Sufficient Heterogeneity," *Econometrica*, 53, 767-781.

Economides, N., 1989, "Quality Variations and Maximal Variety Differentiation," *Regional Science*s *and Urban Economics*, 19, 21-29.

Economides, N., 1993, "Quality Variations in the circular model of variety-differentiated products," *Regional Science*s *and Urban Economics*, 23, 235-257.

Friedman, J.M., and J.-F. Thisse, 1993, "Partial Collusion Fosters Minimum Product Differentiation," *RAND Journal of Economics*, 24, 631-645.

Ferreira, R. D. S., and J.-F. Thisse, 1996, "Horizontal and Vertical Differentiation: The Launhardt Model," *International Journal of Industrial Organization*, 14, 485-506.

Hotelling, H., 1929, "Stability in Competition," *Economic Journal*, 39, 41-57.

Jehiel, P., 1992, "Product Differentiation and Price Collusion," *International Journal of Industrial Organization*, 10, 633-643.

Lancaster, K., 1979, *Variety*, *Efficiency and Equity* (Columbia University Press, New York).

Launhardt, W., 1885, Mathematische Begründung der Volkswirtschaftslehre (B.G.. Teubner, Leipaig).

Liang, W.J., and C.C. Mai, 2006, "Validity of the Principle of Minimum Differentiation under Vertical Subcontracting," *Regional Science and Urban Economics*, 36, 373-384.

Liang, W.J., H. Hwang, and C.C. Mai, 2006, "Spatial Discrimination: Bertrand vs. Cournot with Asymmetric Demands," *Regional Science and Urban Economics* 36, 790-802.

Mai, C.C., and S.K. Peng, 1999, "Cooperation vs. Competition in a Spatial Model," *Regional Science and Urban Economics*, 29, 463-472.

Matsushima, N., and T. Matsumura, 2006, "Mixed Oligopoly, Foreign Firms, and Location Choice," *Regional Science and Urban Economics*, 36, 753-772.

Rhee, B.-D., A. De Palma, C. Fornell, and J.-F. Thisse, 1992, "Restoring the Principle of Minimum Differentiation in Product Positioning," *Journal of Economics & Management Strategy*, 1, 475-505.

Stahl, K., 1982, "Location and Spatial Pricing Theory with Nonconvex Transportation Cost Schedules," *The Bell Journal of Economics and Management Science*, 13, 575-582.

Salop, S. C., 1979, "Monopolistic competition with outside goods," *Bell Journal of Economics*, 10, 141-156.

Tabuchi, T., 1994, "Two-stage two-dimensional Spatial Competition Between Two Firms," *Regional Science and Urban Economics*, 24, 207-27.

Yang, B.S., W.J. Liang, H. Hwang, and C.C. Mai, 2007, "A General Analysis of Spatial Cournot Competition in a Linear City Model," Paper Presented in 54*th* *Annual North American Meetings of the Regional Science Association International*, November 7-10, Savannah, Georgia, USA.

Zhang, Z. J., 1995, "Price-Matching Policy and the Principle of Minimum Differentiation," *Journal of Industrial Economics*, 43, 287-299.

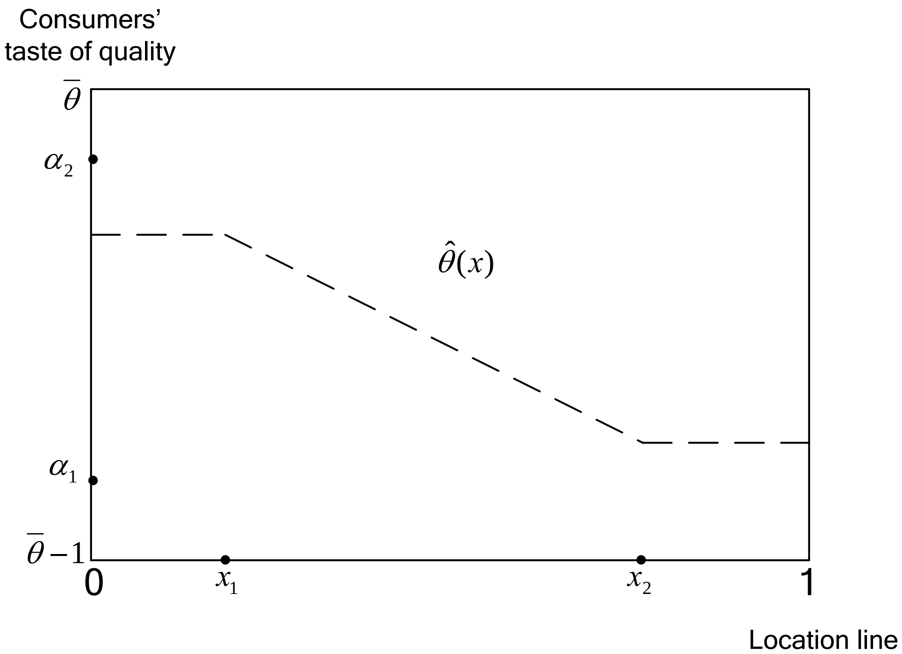

Fig. 1. The two-dimensional framework

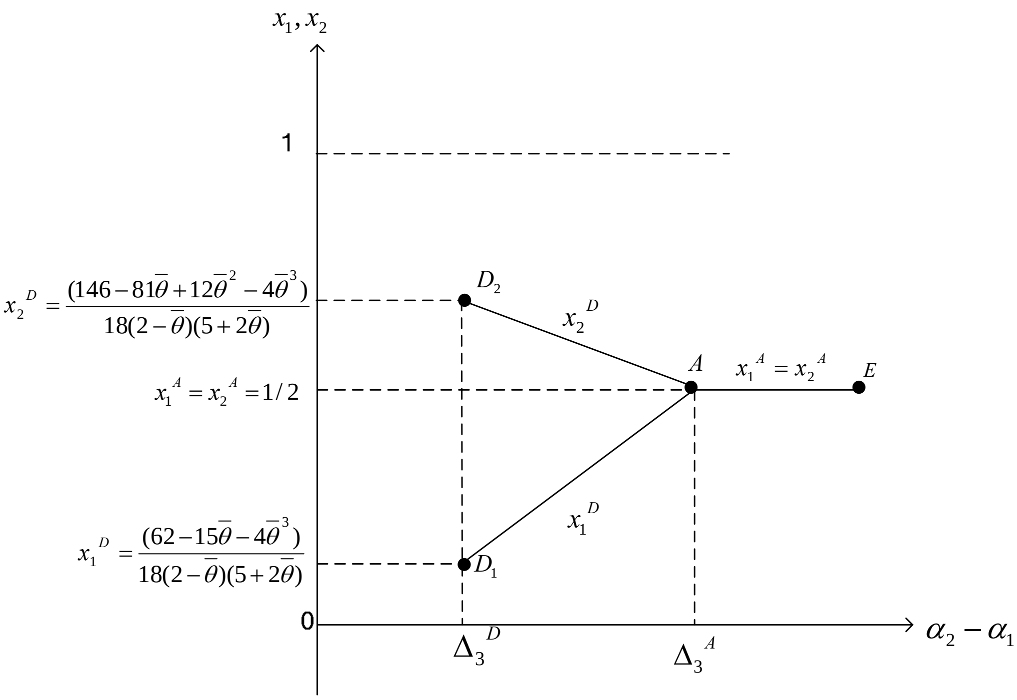

Fig.2. The relationship between firms' location equilibrium and the degree of vertical differentiation

A General Analysis of Spatial Cournot Competition in a Linear City Model

Biing-Shiunn Yang
Department of Economics
Tamkang University

and

Wen-Jung Liang
Department of Industrial Economics
Tamkang University
and
Public Economics Research Center
National Taiwan University

and

Hong Hwang
Department of Economics
National Taiwan University

and

Chao-Cheng Mai*
Department of Industrial Economics

Tamkang University

and

Research Center for Humanities and Social Sciences

Academia Sinica

(Current Version: December 13, 2007)

* Corresponding Author: Chao-Cheng Mai, Research Center for Humanities and Social Science, Academia Sinica, Nankang, Taipei, Taiwan 11529, ROC. Tel: 886-2-2782-1693, Fax: 886-2-2785-4160, E-mail: ccmai@gate.sinica.edu.tw

JEL Classification: R32, L22
Key Words: Cournot Competition; Spatial Agglomeration; Location Configuration

Abstract

Anderson and Neven (1991) show that central agglomeration is the only location equilibrium for n firms if they play Cournot competition and discriminate over space in a linear city model. Using a less restrictive market-serving condition, this paper shows that the location configuration is of spatial dispersion (central agglomeration) if the transport rate is high (low). In the case of spatial dispersion, the location pattern depends crucially on the number of firms being odd or even. For the former, the location configuration is such that the middle firm is located at the center and two equal groups of the rest of the firms are located at the two sides of the middle firm symmetrically. For the latter, it is of two equal groups of firms located symmetrically at the two sides of the linear market.

1. Introduction

In his seminal paper, Hotelling (1929) presented a model of two firms competing to sell a homogeneous product to consumers distributed evenly along a linear market. He showed that in equilibrium the duopolists should be located at the center of the market under linear transportation costs. This kind of location pattern has been termed the *Principle of Minimum Differentiation* (PMD hereafter). Since then, Hotelling's model has been widely cited and extended in various ways. For example, Lerner and Singer (1937) extended the model by increasing the number of firms to three and noted that the continuance of oscillations in location led to location instability; Eaton and Lipsey (1975) examined the cases to which PMD applies and also those other principles which apply for a small group of firms; in particular, d'Aspremont *et al*. (1979) argued that PMD would never hold in a location-price model *a la* Hotelling under linear transportation costs. Moreover, they claimed that under quadratic transportation costs, the two firms necessarily are located at the endpoints of the line market. It should be noted that most of the theoretical models for strategic location decisions, including those we have cited, assume the firms in question engage in Bertrand price competition.

In contrast, the body of literature on spatial competition that uses Cournot-type models is relatively small. Hamilton, Thisse, and Weskamp (1989) (hereafter HTW) and Anderson and Neven (1991) (hereafter AN) carried out pioneering work on location models with Cournot quantity competition. They assumed that firms behave as Cournot oligopolists and discriminate over space, and demonstrated that for linear demand and linear transport costs, central agglomeration is the only location configuration in equilibrium for an n-firm location-quantity game. Their result replicates Hotelling's PMD.

AN's finding is interesting, but is sensitive to their definition of the so-called market-serving condition, which requires that each firm's sale to any point in the linear market be positive. This definition has been widely used in the literature. For example, Pal (1998) assumes locations being chosen sequentially, and shows that firms agglomerate at the center in a linear city, while locate equidistant from each other in a circular city. Matsushmura and Shimizu (2005) introduce a consumer density function into a linear city model with n firms to investigate the welfare effect of location choice. They show that central agglomeration emerges if the density at the center of the market is sufficiently high. Otherwise, spatial dispersion occurs. There is another strand of literature which employs the same market-serving condition and examines location equilibrium pattern using a circular city model, see for example, Gupta *et al*. (1997), Chamorro-Rivas (2000*a*), Matsushima (2001), Gupta *et al*. (2004) and Matsushima and Matsumura (2006).

Even though the market-serving condition defined by AN has been widely adopted in the literature, its rigorousness has not been challenged. AN's market-serving condition is implemented in the second (output) stage which requires *for any given location* each firm's sale to its remote endpoint be positive[19]. We find the condition unnecessarily restrictive. In contrast, the present paper argues that the market-serving condition should be that, *at the equilibrium location*, each firm's sale to its remote market be positive. By using this definition, we shall prove that the location configuration in the linear market could be either central agglomeration or spatial dispersion, depending critically on the transport rate. Specifically, it is of spatial dispersion (central agglomeration) if the transport rate is high (low). Furthermore, we shall show that in the case of spatial dispersion, the location pattern depends on the number of firms being odd or even. For the former, the location configuration is such that the middle firm is located at the center and two equal groups of the rest of the firms are located at the two sides of the middle firm symmetrically. For the latter, it is of two equal groups of firms located symmetrically at the two sides of the line market. Chamorro-Rivas (2000b) also utilizes the same market-serving condition as ours to analyze firms' location decision where there are only two firms in the market, and obtains that the dispersion equilibrium is *unstable*.

The remainder of the paper is organized as follows: Section 2 develops a basic model with n firms playing Cournot competition in the commodity market *a la* AN. Section 3 analyzes the firms' location choices with linear transport costs for an even number of firms, and Section 4 does so for an odd number of firms. The final section concludes the paper.

2. The Basic Model

Consider a framework *a la* AN (1991). There are n firms in the market, located at $x_1, x_2, \ldots, x_n$, respectively, with $0 \le x_1 \le x_2 \le \ldots \le x_n \le L$, along a linear market with length L. They produce, at zero production cost for simplicity, a homogeneous product and sell their output to consumers residing along the line market. Assume that the demand function at each point $x \in [0, L]$ is linear and symmetric, given by $P = a - bQ$, where P is the delivered price, Q is the quantity demanded by the consumer at x, and a and b are constants. Assume further that the transport cost function of the product is: $T(x - x_i) = t|x - x_i|$, where T is the transport cost per unit of output, t is the transport rate per unit output per unit distance, and x_i is the location of firm i.

Based on the above setting, we can specify firm i's profit from point x as follows:

$$\pi_i(x) = [a - bQ(x) - t|x - x_i|]q_i(x), \ i = 1, \ldots, n, \tag{1}$$

where $Q(x) = \sum_{i=1}^{n} q_i(x)$ and $q_i(x)$ is firm i's sales at x.

The game in question consists of two stages – the n firms choose their locations simultaneously in the first stage followed by Cournot quantity competition in each market in the second stage. As usual, the game is solved by backward induction with the

[19] AN (1991, p.801) wrote "...assume $a > n\tau L$ (where a is the price intercept of the linear demand, n the number of firms, τ the transport rate and L the length of the linear market). This condition guarantees all firms will serve the whole market regardless of their locations." (*Italics* are ours.) Alternatively, HTW (1989, p. 91) specify that the monopoly price is no less than the given value of transport rate under consideration, even if the firms are at the maximum distance, i.e., the endpoints. Given this market-serving condition, both papers have implicitly imposed a ceiling on the transport rate, which excludes the possibility of a non-agglomerated equilibrium.

second stage being worked out first. Standard calculations yield the second stage Cournot equilibrium output and profit for firm i at market site x as follows:

$$q_i(x) = [1/(n+1)b][a + t\sum_{j=1}^{n}\left|x - x_j\right| - (n+1)t\left|x - x_i\right|],\ i = 1,\dots,n, \tag{2.1}$$

$$\pi_i(x) = (1/b)\{[a + t\sum_{j=1}^{n}\left|x - x_j\right| - (n+1)t\left|x - x_i\right|]/(n+1)\}^2,\ i = 1,\dots,n. \tag{2.2}$$

Summing the profits from the entire market, we obtain the total profit function of firm i as follows:

$$\pi_i = \int_0^L \pi_i(x)dx,\ i = 1,\ \dots,n. \tag{3}$$

Differentiating (3) with respect to x_i yields the following first-order condition for profit-maximization:

$$\begin{aligned}
&[b(n+1)^2/2nt](\partial\pi_i/\partial x_i)\\
&= -\int_0^{x_1}\left[a + t\sum_{j=1}^{n}\left|x - x_j\right| - (n+1)t(x_i - x)\right]dx - \dots\\
&- \int_{x_{i-1}}^{x_i}\left[a + t\sum_{j=1}^{n}\left|x - x_j\right| - (n+1)t(x_i - x)\right]dx\\
&+ \int_{x_i}^{x_{i+1}}\left[a + t\sum_{j=1}^{n}\left|x - x_j\right| - (n+1)t(x - x_i)\right]dx + \dots\\
&+ \int_{x_n}^{L}\left[a + t\sum_{j=1}^{n}\left|x - x_j\right| - (n+1)t(x - x_i)\right]dx = 0, \qquad i = 1,\dots,n.
\end{aligned} \tag{4}$$

These n equations can solve for $x_1, x_2, \dots, x_n$, the optimal locations of the n firms. Note that the solutions are subject to the following three conditions: the second-order, the stability and the market-serving conditions. As the second-order condition for the n symmetric firm equilibrium is part of the stability condition, we need only check whether the solutions in (3) satisfy the latter two conditions. In what follows, we shall check whether the solutions of (3) can survive the two conditions.

First of all, let us check the stability condition. Note that the second-order condition is simply the first-order determinant of the first order of the stability conditions due to the symmetric assumption. Hence, we need to consider only the stability conditions. According to Sydsaeter and Hammond (1995), the necessary stability conditions require that the first leading principal minor of the determinant from (4) be non-positive, the second be non-negative and then the following leading principal minors change alternatively in sign accordingly.[20] That is

$$(-1)^i J_i^n(X) \geq 0,\ i = 1,2,\dots,n, \tag{5}$$

where X denotes the vector of equilibrium location combination and $J_i^n(X)$ is the leading principal minor of order i in the n-firm case.[21]

The market-serving condition requires that the output of each firm at any site of the market segment be positive. As the

[20] It can be found that the second-order condition is part of the stability conditions.

[21] Sydsaeter and Hammond (1995, p. 639) show that $(-1)^i J_i^n(X) \geq 0$ (i = 1, 2, …, n) are the necessary conditions of stability for X being local maximum equilibrium, where n is the number of firms.

demands at the market sites are symmetric, if each firm's sale to its farthest market site is positive, the sales to the other markets must also be positive, and the market-serving condition is satisfied. In terms of algebra, the market-serving condition requires:

$$q_1(L;X) = [1/(n+1)b][a + t\sum_{j=1}^{n}\left|L - x_j\right| - (n+1)t\left|L - x_1\right|] > 0. \tag{6}$$

As the location configurations depend on the number of firms being odd or even, in what follows we shall examine their location configurations separately.

3. The Equilibrium Location for an Even Number of firms

In this section, we examine the equilibrium location for an even number of firms. We proceed by studying the case of n = 2 first. Substituting n = 2 into (4), the first-order condition is reduced to $\partial\pi_1/\partial x_1 = 0$ and $\partial\pi_2/\partial x_2 = 0$, which after some manipulation, become respectively:

$$(2a - tL)[(L/2) - x_1] - t[L - (x_2 - x_1)](x_2 - x_1) = 0, \tag{7.1}$$

$$(2a - tL)[(L/2) - x_2] + t[L - (x_2 - x_1)](x_2 - x_1) = 0. \tag{7.2}$$

The first terms in (7.1) and (7.2) are named the transportation cost saving effect (the cost-saving effect hereafter). This effect reflects firm *i*'s desire to move toward the center in order to save on the transportation cost. The second term (including the negative sign in front of the term) is called the competition effect.[22] It indicates that as the two firms are more distant from each other, they become more dissimilar and therefore the competition lessens.[23] In sum, the cost-saving effect tends to move the firms toward the center of the market, whereas the competition effect works in an opposite way. The two effects determine jointly the equilibrium location of the firms.

Substituting n = 2 into (5) and (6), we can derive the corresponding stability and market-serving conditions in (8) and (9) as follows:

$$J_1^2 = \partial^2\pi_1/\partial x_1^2 = (-8t/9b)[(a - tL) + t(x_2 - x_1)] \le 0, \tag{8.1}$$

$$\begin{aligned} J_2^2 &= (\partial^2\pi_1/\partial x_1^2)(\partial^2\pi_2/\partial x_2^2) - (\partial^2\pi_1/\partial x_1\partial x_2)(\partial^2\pi_2/\partial x_2\partial x_1) \\ &= (4t/9b)^2(2a - tL)[(1/4)(2a - 3tL) + t(x_2 - x_1)] \ge 0. \end{aligned} \tag{8.2}$$

$$q_1(L; x_1, x_2) = (1/3b)[a - 2t\left|x_1 - L\right| + t\left|x_2 - L\right|] > 0. \tag{9}$$

Again, the solutions from (7) are meaningful only if they satisfy the stability and market-serving conditions in (8) and (9). Solving (7) and making use of (8) and (9), we come up with the following location configurations:

---

[22] The idea of the competition effect can also be found in Liang *et al*. (2006).
[23] Contrarily, if the two firms locate at the center of the market, they are symmetric in terms of production cost plus transport cost at any site of the market and the competition is the highest.

$$x_1 = L/2, \text{ and } x_2 = L/2, \text{ for } t \le (2/3)(a/L), \tag{10.1}$$

$$x_1 = (2a - tL)/4t, \text{ and } x_2 = L - (2a - tL)/4t, \quad \text{for } (2/3)(a/L) \le t \le (10/11)(a/L). \tag{10.2}$$

Equation (10) shows that there is only one stable equilibrium. It is central agglomeration if the transport rate is no greater than (2/3)(*a*/*L*) (i.e., (10.1)); it is spatial dispersion if the transport rate is between (2/3)(*a*/*L*) and (10/11) (*a*/*L*) (i.e., (10.2)). Our finding generalizes the results of AN and HTW, who claim that central agglomeration is the only location equilibrium. Their result is different from ours mainly because their market-serving condition is unnecessarily stringent, as it requires, for any given location (instead of at the equilibrium location), that each firm's sales at any market site be positive.

Intuitively, the two effects—cost-saving and competition—jointly determine the equilibrium locations of the two firms. The cost-saving effect tends to move firms toward the market center; but the competition effect tends to pull the two firms away from the center. As the transport rate is higher than (2/3)(*a*/*L*) but lower than (10/11)(*a*/*L*), which is the upper limit bounded by the market-serving condition, only the disperse equilibrium survives—the two firms are located symmetrically but separately such that the cost-saving effect equals the competition effect. Note that the two firms' equilibrium locations move closer to the market center as the transport rate falls. When the transport rate falls to (2/3)(*a*/*L*), both the cost-saving effect and the competition effect vanish, which makes the two firms be located at the market center. This location configuration also applies to the cases where the transport rate falls below (2/3)(*a*/*L*).

Proceeding as before, we can derive the location configuration for the case of $n = 4$ as follows:

$$x_1 = \ldots = x_4 = L/2, \text{ for } t \le (2/5)(a/L), \tag{11.1}$$

$$x_1 = x_2 = (2a - tL)/8t, \text{ and } x_3 = x_4 = L - x_1., \quad \text{for } (2/5)(a/L) \le t < (18/29)(a/L). \tag{11.2}$$

Similarly, we can also derive the location configuration for the cases of $n = 6$ and 8, which to save space are not reported in the paper, and we apply mathematical induction to derive the following general location pattern (see Appendix A for the proof):[24]

$$x_1 = \ldots = x_n = L/2, \text{ for } t \le [2/(n+1)]\{a/L), \tag{12.1}$$

$$\begin{aligned} &x_1 = \ldots = x_{n/2} = (2a - tL)/2nt, \text{ and} \\ &x_{(n/2)+1} = \ldots = x_n = L - x_1, \\ &\quad \text{for } [2/(n+1)](a/L) \le t < [(4n+2)/(n^2+3n+1)](a/L). \end{aligned} \tag{12.2}$$

This indicates that central agglomeration emerges if the transport rate is lower than [2/(n+1)], while spatial dispersion occurs if

[24] Liang *et al*. (2007) show that monopoly regions emerge when the transport rate is higher than [(4n+2)/(n^2+3n+1)](*a*/*L*) and if this is the case, firms tend to locate more distantly in a two firm case.

transport rate lies between $[2/(n+1)](a/L)$ and $[(4n+2)/(n^2+3n+1)](a/L)$. Moreover, it is straightforward to show that the two groups of firms are located more distantly in the disperse equilibrium as the number of firms rises.

Accordingly, we can establish:

Proposition 1. *With an even number of firms, we have:*

(1) *Central agglomeration is the location equilibrium when the transport rate is no greater than* $[2/(n+1)](a/L)$, *while dispersion emerges as the equilibrium with two equal groups of firms located symmetrically at the two sides of the line when the transport rate lies between* $[2/(n+1)](a/L) \le t < [(4n+2)/(n^2+3n+1)](a/L)$.

(2) *In the case of disperse equilibrium, half of the firms cluster at* $[(2a-tL)/2nt]$, *while the other half cluster at* $[L-(2a-tL)/2nt]$. *The two groups of firms are located more distantly as the number of firms rises.*

4. The Equilibrium Location for an Odd Number of Firms

In this section, we explore the location configurations for an odd number of firms. First of all, we examine the 3-firm case. Substituting $n = 3$ into (4) yields:

$$\begin{aligned}(8b/3t)(\partial\pi_1/\partial x_1) = (2a-tL)[(L/2)-x_1] - t[L-(x_2-x_1)](x_2-x_1) \\ - t[L-(x_3-x_1)](x_3-x_1) = 0,\end{aligned} \tag{13.1}$$

$$\begin{aligned}(8b/3t)(\partial\pi_2/\partial x_2) = (2a-tL)[(L/2)-x_2] + t[L-(x_2-x_1)](x_2-x_1) \\ - t[L-(x_3-x_2)](x_3-x_2) = 0,\end{aligned} \tag{13.2}$$

$$\begin{aligned}(8b/3t)(\partial\pi_3/\partial x_3) = (2a-tL)[(L/2)-x_3] + t[L-(x_3-x_1)](x_3-x_1) \\ + t[L-(x_3-x_2)](x_3-x_2) = 0.\end{aligned} \tag{13.3}$$

Solving (13), we derive two location equilibria, a central agglomeration equilibrium and a dispersion equilibrium as follows:

$$x_1 = x_2 = x_3 = L/2, \tag{14.1}$$

$$x_1 = (4a-3tL)/10t,\ x_2 = L/2,\ \text{and}\ x_3 = L-(4a-3tL)/10t. \tag{14.2}$$

Once again, after checking the second-order, stability, market-serving and symmetric conditions for (14.1) and (14.2), we find that the central agglomeration solution is stable if $t \le (1/2)(a/L)$; it satisfies the market-serving condition if the transport rate is lower than the upper limit, i.e., $t < 2(a/L)$. Combining the two conditions, we can conclude that the firms agglomerate at the market center if $t \le (1/2)(a/L)$. Likewise, firms would be located separately if $(1/2)(a/L) \le t < (26/37)(a/L)$.

Next, we can proceed further by examining the case for $n = 5$. Substituting $n = 5$ into (4), we obtain five location equilibria: one central agglomeration and four dispersion equilibria. But only the central agglomeration and one of the dispersion equilibria satisfy the second-order, stability, and market-serving conditions. Hence, the location configurations are as follows:

$$x_1 = ... = x_5 = L/2, \text{ for } t \le (1/3)(a/L), \tag{15.1}$$

$$\begin{aligned} &x_1 = x_2 = (4a - 3tL)/18t, \ x_3 = L/2, \text{ and} \\ &x_4 = x_5 = L - x_1, \text{ for } (1/3)(a/L) \le t < (14/27)(a/L). \end{aligned} \tag{15.2}$$

From the location configurations in (14) and (15), we can similarly derive the location configurations for the cases of n = 7 and 9, and apply mathematical induction to derive a general pattern of the location configurations for an odd number of firms (i.e., n = 2m+1) as follows (see Appendix 2 for the proof):

$$x_1 = ... = x_n = L/2, \text{ for } t \le [2/(n+1)](a/L), \tag{16.1}$$

$$\begin{aligned} &x_1 = ... = x_m = \{(4a - 3tL)/[(4n-2)t]\}, \ x_{m+1} = L/2, \text{ and} \\ &x_{m+2} = ... = x_n = L - x_1, \\ &\qquad \text{for } [2/(n+1)](a/L) \le t < [2(4n+1)/(2n^2 + 6n + 1)](a/L). \end{aligned} \tag{16.2}$$

Eq. (16.1) shows that central agglomeration emerges, if transport rate is lower than [2/(n+1)], while (16.2) demonstrates that spatial dispersion occurs, if the transport rate lies between [2/(n+1)](a/L) and [2(4n+1)/(2n^2+6n+1)](a/L). Moreover, the two groups of firms are located more distantly in the dispersion equilibrium as the number of firms rises. This is because competition among firms becomes fiercer as the number of firms rises.

Based on the aforementioned analysis, we can establish:

Proposition 2. *With an odd number of firms*,

(1) *central agglomeration (spatial dispersion) is the location equilibrium if the transport rate is lower (higher) than* [2/(n+1)](a/L).

(2) *in the case of spatial dispersion, the location configuration is such that the middle firm is located at the market center and the rest of the firms are divided evenly and located symmetrically at* {(4a-3tL)/[(4n-2)t]} *and* {L-(4a-3tL)/[(4n-2)t]}, *respectively. Moreover, the two groups of firms are located more distantly as the number of firms rises.*

Note that the general pattern for an odd number of firms is different from the one for an even number of firms. For an odd number of firms, if there exists a dispersion equilibrium, the location configuration is for a middle firm located at the market center and two equal groups of firms located at the two sides of the middle firm symmetrically.[25] But for an even number of firms, the

[25] This location configuration leads to the result that the middle firm faces different situation than other firms. It may be interesting to compare the profit levels in the text among them. For simplicity, let us consider the three-firm case.

Substituting (14) and n = 3 into (2.2) and (3) yields each firm's profit as follows:

disperse equilibrium consists of two even groups of firms located on the two sides of the line segment. In either case (the number of firms being odd or even) of the dispersion equilibrium, the two groups of outside firms are located more distantly as the number of firms rises.

5. Concluding Remarks

AN (1991) set up a linear city model with n firms which behave as Cournot oligopolists and discriminate over space. They conclude that if the demand and transport cost are linear, central agglomeration is the only location configuration. This paper has employed a more reasonable and less restrictive market-serving condition, and shown that the location configurations can be of either central agglomeration or spatial dispersion, depending on the value of the transport rate. The location configuration is of spatial dispersion (central agglomeration) if the transport rate is high (low). Moreover, in the case of spatial dispersion, the location pattern depends crucially on the number of firms being odd or even. If it is odd, the location configuration is such that the middle firm is located at the center and two equal groups of the remaining firms are located at the two sides of the middle firm symmetrically. On the other hand, if it is even, it is of two equal groups of firms located symmetrically at the two sides of the line market. Furthermore, no matter that the number of firms is even or odd, the two groups of firms are located more distantly in the disperse equilibrium as the number of firms rises. These results are in sharp contrast to those found in AN (1991).

It would be interesting to know what the location configuration would be if the transport rate is beyond the upper limit bounded by the market-serving condition. This high transport rate which invalidates the market-serving condition, renders at least one of the firms not be able to profitably serve its remote region and results in a monopoly region at each end of the linear market. The emergence of the monopoly region tends to move the firm to locate farther away from the center of the linear market.[26]

$$\begin{aligned} \pi_2^D &= \{832a^3 - 1572a^2tL + 1554at^2L^2 + 1149t^3L^3\}/24000bt, \text{ and} \\ \pi_1^D &= \pi_3^D = \{704a^3 - 804a^2tL + 18at^2L^2 + 2173t^3L^3\}/24000bt, \end{aligned} \tag{F.1}$$

where the superscripts "D" denote the variables associated with the case of spatial dispersion.

Manipulating (F.1), we obtain:

$$\pi_1^D - \pi_2^D = -2(a-2tL)^3/375bt \geq 0, \text{ for } (1/2)(a/L) \leq t < (26/37)(a/L). \tag{F.2}$$

It follows immediately from (F.1) and (F.2) that the profits of the two outside firms, i.e., firm 1 and firm 3, are identical, but higher than that of the middle firm. Since we have ranked the locations of the firms by the order $x_1 \leq x_2 \leq x_3$, the disperse equilibrium is proved to be a stable equilibrium although the profits of the middle firm are lower than the other firms.

[26] The more detailed discussion can be obtained from the authors upon request.

References

Anderson, S.P., and D.J. Neven, 1991, "Cournot Competition Yields Spatial Agglomeration," *International Economic Review* 32, 793-808.

Chamorro-Rivas, J.M., 2000a, "Plant Proliferation in a Spatial Model of Cournot Competition," *Regional Science and Urban Economics* 30, 507-18.

--------------, 2000b, "Spatial dispersion in Cournot Competition," *Spanish Economic Review* 2,145-52.

d'Aspremont, C., J. Gabzewicz, and J.-F. Thisse, 1979, "On Hotelling's Stability in Competition," *Econometrica* 47, 1145-50.

Eaton, B.C., and R.G. Lipsey, 1975, "The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition," *Review of Economic Studies* 42, 27-49.

Gupta, B., D. Pal and J. Sarkar, 1997, "Spatial Cournot Competition and Agglomeration in a Model of Location Choice," *Regional Science and Urban Economics* 27, 261-82.

Gupta, B., F.C. Lai, D. Pal, J. Sarkar, and C.M. Yu, 2004, "Where to Locate in a Circular City?" *International Journal of Industrial Organization* 22, 759-82.

Hamilton, J.H., J.F Thisse, and A. Weskamp, 1989, "Spatial Discrimination: Bertrand vs. Cournot in a Model of Location Choice," *Regional Science and Urban Economics* 19, 87-102.

Lerner, A., and H. Singer, 1937, "Some Notes on Duopoly and Spatial Competition," *Journal of Political Economy* 45, 145-86.

Liang, W.J., H. Hwang, and C.C. Mai, 2006, "Spatial Discrimination: Bertrand vs. Cournot with Asymmertric Demands," *Regional Science and Urban Economics* 36, 790-802.

Liang, W.J., B.S. Yang, H. Hwang, and C.C. Mai, 2007, **"**Does Cournot Competition Yield Spatial Agglomeration in a Linear City Model?" *Tamkang University*, *Working Paper*.

Matsumura, T., and D. Shimizu, 2005, "Spatial Cournot Competition and Economic Welfare: A Note," *Regional Science and Urban Economics* 35, 658-70.

Matsushima, N., 2001, "Cournot Competition and Spatial Agglomeration Revisited," *Economics Letters* 73, 175-77.

Matsushima, N., and T. Matsumura, 2006, "Mixed Oligopoly, Foreign Firms, and Location Choice," *Regional Science and Urban Economics* 36, 753-72.

Pal, D., 1998, "Does Cournot Competition Yield Spatial Agglomeration?" *Economics Letters* 60, 49-53.

Sydsaeter, K., and P.J. Hammond, 1995, *Mathematics for Economic Analysis*, Prentice-Hall International, London.

Appendix A

In this Appendix, we prove the existence of the general pattern for an even numbers of firms (i.e., (12)) by mathematical induction. This can be accomplished by the following procedures. First, we prove that the general pattern holds for the cases of n = 2 and 4 by taking into account the stability and market-serving conditions. This has been done in Section 3. Secondly, we assume that this general pattern is valid for the case of $n = 2k$, where k is any positive integer. Finally, we are going to prove that the general pattern holds for the case of $n = 2(k+1)$. We know from the general pattern that $x_1 = \ldots = x_{k+1}$ and $x_{k+2} = \ldots = x_n$. Substituting these relationships and $n = 2(k+1)$ into the firm's first-order conditions for profit-maximization (i.e., (4)), we can rewrite (4) as follows:

$$\left[\frac{b(n+1)^2}{2nt}\right]\left(\frac{\partial \pi_i}{\partial x_i}\right) = \left(\frac{L}{2}-x_1\right)\left\{2a+t\left[-4(k+1)\left(\frac{L}{2}-x_1\right)-(2k+3)L\right]\right\}=0, \text{for } i=1,\ldots,k+1, \tag{A.1}$$

$$\left[\frac{b(n+1)^2}{2nt}\right]\left(\frac{\partial \pi_i}{\partial x_i}\right) = \left(\frac{L}{2}-x_n\right)\left\{2a+t\left[-4(k+1)\left(\frac{L}{2}-x_n\right)-(2k+3)L\right]\right\}=0, \text{for } i=k+2,\ldots,n. \tag{A.2}$$

Substituting the spatial agglomeration and spatial dispersion equilibria in (12) into (A.1) and (A.2) respectively, we find that the first-order conditions for profit maximization in (4) still hold. Meanwhile, substituting $n = 2(k+1)$ into the market-serving condition, i.e., eq. (6), and calculating the stability conditions with any integer k, we can show that the general pattern is valid for the case of $n = 2(k+1)$. Accordingly, the general pattern of the location configuration is good for any even number of firms.

Appendix B

Proceeding as before, we can show that the general pattern of location configuration for an odd number of firms is valid for the cases of n = 2 and 4 by taking into account the stability and market-serving conditions. Assume that this holds for the case of $n = 2k+1$. This implies that $x_1 = \ldots = x_{k+1}$, $x_{k+2} = L/2$, and $x_{k+3} = \ldots = x_n$. Substituting this result and $n = 2(k+1)+1$ into (4), we can rewrite (4) as follows:

$$\left[\frac{b(n+1)^2}{2nt}\right]\left(\frac{\partial \pi_i}{\partial x_i}\right) = \left(\frac{L}{2}-x_1\right)\left\{2a+t\left[(4k+5)\left(\frac{L}{2}-x_1\right)-2(k+2)L\right]\right\}=0, \text{for } i=1,\ldots,k+1, \tag{B.1}$$

$$\left[\frac{b(n+1)^2}{2nt}\right]\left(\frac{\partial\pi_i}{\partial x_i}\right)$$
$$=\left(\frac{L}{2}-x_{k+2}\right)\{2a+t[2(k+1)(x_n-x_1)-(2k+3)L]\}=0, \text{for } i=k+2, \tag{B.2}$$

$$\left[\frac{b(n+1)^2}{2nt}\right]\left(\frac{\partial\pi_i}{\partial x_i}\right)$$
$$=\left(\frac{L}{2}-x_n\right)\left\{2a+t\left[-(4k+5)\left(\frac{L}{2}-x_n\right)-2(k+2)L\right]\right\}=0, \text{for } i=k+3,\ldots,n. \tag{B.3}$$

Moreover, substituting the spatial agglomeration and spatial dispersion equilibria in (16) into (B.1), (B.2) and (B.3), we can prove that the first-order conditions in (4) hold. Meanwhile, substituting n = 2k+1 into the market-serving condition and calculating the stability conditions with any integer k, we can show that the general pattern is also valid for the case of n = 2(k+1)+1. Accordingly, the general pattern of the location configuration in (16) is valid for any odd number of firms.