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**Social-Optimal Pricing in a Spatial Market: A General Analysis**

Economic benefits comparisons of alternative spatial pricing policies have received much attention in the literature. There are three oft-studied spatial price policies, including (1) ***uniform mill pricing*** **(**hereafter, UMP) under which the seller chooses a constant f.o.b. price and transportation costs are paid by consumers; (2) ***spatial price discrimination*** (SPD) under which different and location-specific prices are selected over space; and (3) ***uniform delivered pricing*** (UDP) under which consumers at different market sites pay the same delivered price. The oft-used measures of economic benefits are total output sold, profit, consumers' surplus, social welfare, and so on. The market the firm faces maybe a spatial monopoly, or Beckmann's *spatial monopolistic competition*, namely that each firm is the sole seller in its own market since it charges its buyers a lower delivered price than neighbor firms can offer (Beckamnn, 1970). Moreover, while firm faces competition, three oft-studied types of spatial competition are the Loschian one under which the market radius is assumed fixed and within the boundary the firm acts like a monopolist (Losch, 1954), and the Hotelling-Smithies competition under which firms assume the price charged by neighboring firm is fixed (Hotelling, 1929, and Smithies, 1941), while the price at the boundary is fixed in the Greenhut and Ohta's market (Greenhut and Ohta, 1975). One of main findings is that some conclusions previously obtained in traditional economic models cannot remain valid when distance is costly (see, for example, Greenhut and Ohta, 1972; Holahan, 1978; Capozza and Van Order, 1978; Norman, 1981; Gronberg and Meyer, 1981; Hwang and Mai, 1990). Another is that in a spatial market, price discrimination maybe socially desirable, of which implication is it cannot be treated as illegal per se.

Almost all efforts in the literature have been devoted to the comparison of resultant economic benefits under alternative spatial pricing policies charged by a profit-maximizing firm, that is, the comparison is made only among those profit-maximization prices. Only few attempts are made to derive the social optimal pricing under alternative spatial pricing policies, and then compare the resultant economic benefits of such pricing with those from its profit-maximizing counterpart. In an interested paper, Holahan (1978) has made such attempt. The analysis is conducted with a set assumptions, including (H1) individual demand is linear, (H2) consumers are uniformly distributed over the market, (H3) the marginal cost of production is constant, and the total fixed cost is positive, that is, the case of short-run, (H4) the social welfare is measured by profits plus consumers surplus on a single market point rather than on the entire market, (H5) both price and market area—the space the firm actually serves-- are chosen in order to maximize social welfare, and (H6) the spatial pricing policy is UMP. Norman (1981) later come to revisit the issue with the model same as Holahan (1978) except that UMP is replaced by SPD.

Both Holahan and Norman show that in order to maximize social welfare, the price under UMP and SPD shall equal the marginal cost of production, a spatial equivalent of marginal cost pricing. This result together with the assumption (H3), in turn, yield that welfare maximization requires subsidy to cover fixed production cost. Moreover, both papers show that either under SPD or UMP, the market area—the space the firm actually serves--is smaller under welfare-maximizing pricing than it would be in a monopoly. In other words, more firms are needed to fill the space in order to maximize social welfare than they would be in the case of spatial monopoly. The cautious reader may cast some doubt about this result. In the situation stipulated by Holahan and Norman, marginal cost pricing require subsidy to cover those fixed costs, and thus, more firms mean more subsidy is needed. Actually, Norman (1981) recognizes this problem, and states: “We should then ask, perhaps, whether welfare should be maximized subject to a non-negative profit constraint, since production subsidy will eventually imply additional consumers taxes and may also imply output reduction in other sectors” (p. 110)

The purpose of this paper is to revisit the spatial equivalent of marginal cost pricing. Our attempt will be made without assumption from (H1) to (H3), that is, demand can be linear or nonlinear, the consumer density can be of any form, the fixed costs maybe positive or zero. We will replace (H4) by the aggregate social welfare over space since the aggregation is the natural way to measure the social welfare as a whole. In addition to SPD and UMP, we will explore the welfare maximizing pricing under UDP. Our model is same as Hsu (1983) with only one exception that the market area is a control variable to maximize social welfare.

The outline of this paper is: In Section 1, we present a basic model. Three sets of optimal prices and market area under alternative spatial pricing policies are derived in Section 2. Section 3 is devoted to the comparison of the optimal market area when the market size varies while Section 4, to the social superiority of three spatial pricing policies.

## 1. Basic Model

Consider a linear market over which consumers are continuously dispersed, and in which a single firm produces and sells a delivered product, subject to a constant freight rate, $t$. In order to highlight the pure effect of economic space, we assume in accordance with Hoover (1937) and Smithies (1941) that all individual demand are identical of the following form

(1) $$q(x) = f(p), \quad f' < 0$$

where $x$ denotes distance from the seller's mill, $q(x)$ = the quantity demanded at the market site $x$, $p$ = the *delivered* price, the amount a consumer shall pay for a unit of commodity, and $f'$ = the slope of $f(p)$. Equation (1) includes liner demand postulated by Holahan (1978) and Norman (1981) as a special case.

The total output actually sold, namely, the *spatial demand*, can be generally defined as

(2) $$Q = \int_0^B f(p)\gamma(x)dx$$

where $B$ denotes the extent of the market area over which the seller actually serves, and $\gamma(x)$, the density of consumers ate the market site $x$. The profit is used to measure the firm's benefit, which can be generally expressed as:

(3) $$\pi = \int_0^B [mq(x)]\gamma(x)dx - C(Q)$$

where, $m$ denotes the *mill* price, that is, the amount the seller can receive by selling one unit of product, and $C(Q)$ denotes total costs of production, a function of total output $Q$. Total production cost function here is of a very general form. Its marginal cost can be constant or not. Also, it contains fixed production costs when one focused the short run as done by Holahan (1978) and Norman (1981).

To measure a single consumer's benefit, we adopt a simple concept of consumer's surplus, as defined by the excess amount of money a consumer would be willing to pay for the commodity, over the amount he or she actually pays for. That is, $cs(x) = \int_p^{p_{\max}} f(v)dv$, where $p_{\max}$ = the reservation price, the maximum price a consumer is willing to pay for a unit of commodity. Accordingly, the aggregate benefits of all consumers (hereafter the *consumer surplus*) is:

(4) $$CS = \int_0^B cs(x)\gamma(x)dx$$

Therefore, the level of social welfare, $W$, is:

(5) $$W = \pi + CS\ .$$

Finally, there are three possible spatial pricing policies to be adopted in a spatial market. Moreover, we assume in accordance with Holahan (1978) and Gorman (1981), that in addition to price, the extent of the market area is endogenously determined. In other words, three sets of social optimal prices and market area under alternative pricing policies will be derived and examined respectively in the subsequent discussion.

## 2. Optimal prices and market areas under alternative pricing policies

Consider first spatial price discrimination under which different mill prices can be charged at different sites. Since under mill pricing, consumers have to pay for freight costs incurred, (4) can be written as

(6) $$W(m_d) = \pi(m_d) + CS(m_d)$$

where $m_d$ denotes a set of discriminatory mill prices. The term $W(m_d)$ is a functional since $m_d$ is a function of distance $x$, and thus, we employ the technique of calculus of variations to derive the optimal shape of $m_d$, denoted by $m_d^w$.

We shall first examine the property of the social optimal market area. The first order condition for welfare-maximization with respected to the market area is that given the mill price schedule is optimal, it is

(7) $$\partial W(m_d)/\partial B_d = [m_d^w - mc(Q_d^w)]q(B_d^w)\gamma(B_d^w) + cs(B_d^w)\gamma(B_d^w) = 0$$

where $mc = dC/dQ$ denotes the marginal costs of production, a function of total production, $Q_d^w = Q(m_d^w)$ is the total output actually sold with optimal mill prices $m_d^w$, and $B_d^w$ denotes the extent of the welfare-maximizing market area under SPD. Now let $m_d = m_d^w + zh(x)$, where $h(x)$ is an arbitrary function of distance $x$, and $z$, an arbitrary scalar. Substituting $m_d = m_d^w + zh(x)$ in (5) yields

(8) $$W[m_d^w + zh(x)] = \pi[m_d^w + zh(x)] + CS[m_d^w + zh(x)]$$

which is a function of the scalar $z$. Accordingly, we have

(9) $$\partial W(m_d)/\partial z = \partial \pi(m_d)/\partial z + \partial CS(m_d)/\partial z$$

$$= \int_0^B (m_d^w f' + f)h(x)\gamma(x)dx - mc\int_0^B f' h(x)\gamma(x)dx + \int_0^B [\partial cs(x)/\partial z]\gamma(x)dx$$

$$+[(m_d - mc)q(B) + cs(B)]\gamma(B)(\partial B / \partial z)$$

$$= \int_0^B (m_d^w - mc) f' h(x)\gamma(x)dx + [(m_d - mc)q(B) + cs(B)]\gamma(B)(\partial B / \partial z)$$

$$= \int_0^B (m_d^w - mc) f' h(x)\gamma(x)dx = 0$$

where the third equality is based on rearranging and on the fact that $\partial cs(x) / \partial z = d[\int_p^{p_{\max}} f(v)dv] / dz \; = [\int_p^{p_{\max}} f(v)dv]h(x)$, and the last equality is based on (7). Since $h(x)$ is arbitrary, and $f' < 0$, it follows from (9) that the necessary condition for welfare maximization requires, in turn, for any market site:

(10) $$m_d^w = mc(Q_d^w)\,.$$

Thus, substituting (10) in (7) yields that the social optimal market area under SPD must satisfy the following condition

(11) $$cs(B_d^w) = 0$$

Equation (10) states that from society's viewpoint, the set of discriminatory mill prices should be equal to constant marginal costs of production. In other words, although varying prices can be charged under SPD, welfare-maximizing mill prices should be *nondiscriminatory* over the entire market space. Moreover, (11) indicates that for society as a whole, the firm shall serve the market space as far as possible until there is no individual consumer surplus at the boundary of the market area.

Consider next the uniform delivered pricing. Let $p_u$ denote the uniform delivered price. The social welfare now becomes

(12) $$W(p_u) = \pi(p_u) + CS(p_u)$$

The firm's profit is:

(13) $$\pi(p_u) = \int_0^{B_u} [(p_u - tx)q(x)]\gamma(x)dx - C(Q)\,,$$

and the consumer surplus is:

(14) $$CS(p_u) = \int_0^{B_u} [\int_{p_u}^{p_{max}} f(v)dv]\gamma(x)dx = N_u \int_{p_u}^{p_{max}} f(v)dv$$

where $B_u$ denotes the extent of the market area under UDP, and $N_u = \int_0^{B_u} \gamma(x)dx$, the total population served by the firm under UDP. The first order condition of welfare maximization with respect to the market area is:

(15) $$\partial W(p_u)/\partial B_u = \{f(p_u)[p_u - mc(Q_u^w) - tB_u] + cs(B_u)\}\gamma(B_u) = 0$$

where $Q_u^w = Q(p_u^w)$. To derive the welfare maximization price under UDP, one may differentiate (12) to obtain

(16) $$\begin{aligned}\partial W(p_u)/\partial p_u =\ & f'(p_u)\int_0^{B_u}(p_u - tx)\gamma(x)dx + f(p_u)\int_0^{B_u}\gamma(x)dx \\ & - mc(.)\int_o^{B_u} f'(p_u)\gamma(x)dx + N_u\int_{p_u}^{p_{max}} f'(v)dv \\ & + \{f(p_u)[p_u - mc(.) - tB_u] + cs(B_u)\}\gamma(B_u)(\partial B_u/\partial p_u) \\ =\ & f'(p_u)\int_0^{B_u}[p_u - tx - mC(.)]\gamma(x)dx + f(p_u)N_u + N_u\int_{p_u}^{p_{max}} f'(v)dv\end{aligned}$$

in which the second equality is based on (15). Noting that $\int_{p_u}^{p_{max}} f'(v)dv = -f(p)$, (16) becomes:

(17) $$\partial W(p_u)/\partial p_u = f'(p_u)\int_0^{B_u}[p_u - mc(Q_u^w) - tx]\gamma(x)dx$$

Thus, the welfare-maximizing price under UDP, $p_u^w$, is

(18) $$p_u^w = mc(Q_u^w) + t\bar{x}$$

where $\bar{x} = \int_0^{B_u} x\gamma(x)dx / N_u$. The term $\bar{x}$ is the simple spatial mean of consumer distribution, or the *average distance* since $N_u = \int_0^{B_u}\gamma(x)dx$, the total population served by the firm under UDP, and $\gamma(x)/N_u$ is the probability density of consumer locating over space. Equation (18) indicates that although the seller is initially supposed to absorb all freight costs incurred, the welfare-maximizing price under UDP should contain "transferred" transportation costs in addition to marginal cots of production. Moreover, by substituting (18) in (15) and upon rearranging, the extent of the welfare-maximizing market area, $B_u^w$, is[1]:

(19) $$B_u^w = \bar{x} + cs(B_u) / f(p_u)t \,.$$

Consider finally the uniform mill pricing. The level of social welfare is:

(20) $$W(m_f) = \pi(m_f) + CS(m_f)$$

where $m_f$ = the uniform mill price under UMP. The firm's profit is:

(21) $$\pi(m_f) = \int_0^{B_f} m_f f(m_f + tx) r(x) dx - C(Q_f)$$

and the consumer surplus is:

(22) $$CS(m_f) = \int_0^{B_f} [\int_{m_f+tx}^{p_{\max}} f(v)dv] \gamma(x) dx$$

where $Q_f = Q(m_f)$, and $B_f$ denotes the extent o the market area under UMP. The first order conditions for welfare-maximization with respected to the market area is:

(23) $$\partial W(m_f) / \partial B_f = \{[m_f - mc(.)] f(m_f + tB_f^w) + cs(B_f^w)\} \gamma(B_f^w) = 0,$$

where $B_f^w$ denotes the extent of the welfare-maximization market area under UMP. To derive the welfare maximization uniform mill price, one may differentiate (20) to obtain

(24) $$\partial W(m_f) / \partial m_f = \int_0^{B_f} \{ f + (m_f - mc(.)] f' \} \gamma(x) dx + \int_0^{B_f} [\int_{m_f+tx}^{p_{\max}} f'(v)dv] \gamma(x) dx$$
$$+ \{[(m_f - mc(.)] f(m_f + tB_f) + cs(B)\} \gamma(B)(dB_f / dm_f)$$
$$= \int_0^{B_f} [m_f - mc(.)] f' \gamma(x) dx$$

where the second equality is based on (23), and the last equality , on $\int_{p_u}^{p_{\max}} f'(v)dv$ $= -f(p)$. Accordingly, the welfare-maximizing mill price under UMP, $m_f^w$, is

(27) $$m_f^w = mc(Q_f^w)$$

where $Q_f^w = Q(m_f^w)$. Equation (27) shows that in order to maximize social welfare,

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[1] Hsu (1983) has examined the case that the market is bounded, that is, the market area is exogenously determined by the market size, say $R$. It is clear that if $R > B_u^w$, then there is no consumer locating at the market site $B_u^w$, that is, $\gamma(B_u^w) = 0$, and thus, (8) holds. In other words, the model postulated in this paper can include Hsu (1983) as a special case.

the uniform mill price should equal marginal costs of production. Moreover, by substituting (27) in (23), the following relation must hold at the boundary of the market area under UMP:

(28) $cs(B_f^w) = 0$

Here we observe from (10) and (27) that the welfare maximization mill prices are same under SPD and UMP. It follows from (11) and (28) that the firm serves the same market area under SPD as it would be under UMP. We may then refer hereafter both SPD and UMP simply by *mill pricing,* and let $m_m^w = m_d^w = m_f^w$, and $B_f^w = B_d^w = B_m^w$, where $m_m^w$ and $B_m^w$ denotes the welfare-maximization mill price and market area under mill pricing, respectively. Moreover, the welfare maximizing delivered price schedule under mill pricing is $p_m^w = m_m^w + tx$.

Two welfare maximization delivered prices of UDP and mill pricing are equal at the market site $x = \bar{x}$ (see figure 1). Thus, $p_d \geq p_f$ accordingly as $x \leq \bar{x}$, and vise versa. That is, relative to mill pricing, the welfare-maximizing price under UDP imposes a welfare loss on each nearby buyer in the region (0, $\bar{x}$), and results in a welfare gain on the buyer in region ($\bar{x}$, $B_u^w$ or $B_m^w$).

## 3. Welfare-maximizing prices and market areas when the market size varies

In the real world, the length of the spatial market maybe infinite or finite. It is worthy pointing out that when the market is small, (11) or (19) might over-estimate the extent of the market area. What (11) actually states is that once we know specific form of demand, costs of transportation and production, and the optimal price, the value of $B_m^w$ can then be determined, regardless of whether there are any consumers locating at that market site. So does (19). It is clear that if there is no buyer at those market sites, the extent of the market area should equal the length of the market size, say $S$. Accordingly, the following rule should be used to determine the extent of the market area

(29) $B_i^w = \min(S, S_i^w)$

where $S_i^w$ stands for the distance satisfying (11) and (19) respectively under alternative pricing policies.

Equation (29) implies that only when the market size is small such that $S < S_u^w$, and $S < S_m^w$, the extent of the welfare-maximization market area is exogenously fixed by the market size. That is, $B_m^w = S$, and $B_u^w = S$. One may refer this case to the *small market*. Note also that this small market is the case examined by Hsu (1983). Nevertheless, the analysis of Section 2 applies to the small market, as shown below.

In a small market, the market area remains unchanged with a change in price, that is,

(30) $dB_d / dz = dB_f / dm_f = dB_u / dp_u = 0$.

Thus, one of necessary conditions for welfare maximization under alternative pricing policies in Section 2 can be omitted, namely, (9), (15) and (23). Moreover, by substituting (30) in the relevant first order welfare maximization condition with respected to the price, that is, (8), (14), and (24), the welfare maximization prices under alternative pricing policies shall have the same form as those presented in Section 2.

We can now compare the extents of the market area under alternative pricing policies. Since the demand function given by (1) is too general to derive any analytical conclusion, we assume in accordance with Greenhut, Hwang, and Ohta (1975) that the individual demand function is of the general form:[2]

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[2] Equation (31) implies that $f'' = \partial^2 f(p)/\partial p^2 = [(1-v)/b^2)][(a-bp)]^{(1/v-2)]}$ since $f' = \partial f(p)/\partial p = (-v/b)(a-bp)^{[(1-v)/v]}$. Thus, $v < 1$, $v = 1$, and $v > 1$ respectively yields that $f''$ is greater than, equal to, or less than zero and thereby, demand curves are convex, linear, or convex from above. Accordingly, (31) "is, in fact, completely general" for the purpose of spatial price theory (see Greenhut, Hwang, and Ohta, 1975, especially, fn. 10, p. 673 for their argument)

(31) $q(x) = f(p) = (a-bp)^{(1/v)}$

Thus, the reservation price is $p_{\max} = a/b$, and $S_m^w = (a-bc)/bt$. Moreover, since $cs(x) = \int_p^{p_{\max}} f(v)dv = (a-bp)^{1/v+1}[v/(1+v)b]$, the market area under UDP is:

(32) $S_u^w = \bar{x} + cs(B_u^w)/f(p_u^w)t = \bar{x} + (a-bp_u^w)[v/(1+v)b]$

$$= \bar{x} + (a-bc-bt\bar{x})[v/(1+v)bt] = (a-bc)[v/(1+v)bt] + \bar{x} - \bar{x}[v/(1+v)]$$

$$= [(a-bc)/bt][v/(1+v)] + \bar{x}/(1+v).$$

Note also that $S_m^w - S_u^w = (a-bc-bt\bar{x})/[bt(1+v)] > 0$ since $a-bc-bt\bar{x} > 0$ is required for $f(p_u) > 0$. Thus, the following results hold

(33) $B_m^w = B_u^w = S \le S_u^w$

(34) $B_m^w > B_u^w$ when $S > S_u^w$

Formally,

<u>Proposition 1</u>: *Except the market is small, the welfare-maximizing market area is greater under mill pricing than it would be under UDP*

Proposition 1 implies that except the market is small, the welfare maximizing pricing under UDP will produce more plants than mill pricing. Moreover, it demonstrates that those results obtained by Hsu (1983) hold only when the market is small.

## 4. Social superiority among welfare-maximizing pricing policies

Social superiority among these pricing policies in terms of social welfare is:

<u>Proposition 2</u>: *The welfare maximization milling pricing should be nondiscriminatory over space, and equal to marginal costs of production, while the uniform delivered pricing will never be socially superior to mill pricing.*

The first part of Proposition 2 is based on the fact that the welfare maximization mill prices under mill pricing are same, and constant over space although varying mill prices is allows to being charged under SPD. To show the second part of Proposition

2, note first that the welfare-maximization problem under SPD is an optimization without any constraint, but that under UDP, a constrained one. Accordingly, the level of social welfare cannot be larger under UDP than it would be under SPD, and so does the mill pricing in general.

The first part of Proposition 2 is a spatial equivalent of marginal cost pricing. Moreover, Proposition 2 holds for any spatial buyer density with different shapes and various sizes, and thus, it includes the result of Hsu (1983) obtained with the small market assumption as a special case.