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Producers' Hedging Decisions under Incomplete Forward Markets

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Abstract: The concept of "full-hedging on average" is defined for the case of hedgeable price risk with undiversifiable production risk. In such case, prudence is necessary whereas prudence and temperance are sufficient for a risk-averse producer to underhedge on average.

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1. Introduction

In his seminal paper, Sandmo (1971) showed that a risk-averse, competitive producer reduces her output when output price becomes uncertain in the absence of a forward market.[1] Recently, Viaene and Zilcha (1998) generalized and extended Kawai and Zilcha's (1986) analysis by adding statistically independent undiversifiable cost and production risks into the model with diversifiable price and/or exchange rate risks. They pointed out correctly that risk aversion is necessary and sufficient for the full-double hedging of output price and exchange rate risks under an undiversifiable cost risk but is too weak to yield any unambiguous result under an undiversifiable production risk. Unfortunately, they did not derive the conditions under which a risk-averse producer under- or overhedges against the exchange rate and output price risks in the latter case.

This paper focuses on a closed-economy, risk-adverse producer. It attempts to derive the necessary and the sufficient conditions on the producer's preference under which the forward/futures hedging decision against the output price risk is unambiguous in the presence of an undiversifiable production risk.[2] Before carrying out any formal analysis, however, this paper suggests that the concept of "full-hedging" needs to be modified for the case of an undiversifiable production risk. A new concept called "full-hedging on average"

[1]Subsequently, the output and hedging decisions of an open-economy, risk-averse producer under exchange rate risk (see, e.g., Ethier, 1973; Baron, 1976; Kawai, 1981) and of a closed-economy, risk-averse producer under price risk (see, e.g., Stein, 1979; Newbery and Stiglitz, 1981) were analyzed. Similarly, although to a lesser extent, the effects of undiversifiable production and cost uncertainty on output were also analyzed (see, e.g., Losq, 1982; Moschini and Lapan, 1995). For a more detailed summary of the relevant literature, see Viaene and Zilcha (1998).

[2]Notice that Viaene and Zilcha (1998) studied the more complicated case with diversifiable joint risks of price and exchanged rate in the presence of undiversifiable cost or production uncertainty. Therefore, they analyzed the concept of "full double-hedging" instead of simply "full-hedging." This paper makes the simplifying assumption that the diversifiable exchange rate risk is absent so that the analysis can focus on the "full-hedging" of output price risk only.

is introduced. It turns out that with undiversifiable production risk, a necessary condition for underhedging (overhedging) on average is prudence (imprudence) and a sufficient set of conditions is prudence and temperance (imprudence and intemperance). This result is surprising since temperance tends to reduce an individual's exposure to a risk in the presence of another statistically independent risk as suggested by Kimball (1992). The result will also be contrasted with that of the case of undiversifiable cost risk under which risk aversion alone guarantees the full-hedging against output price risk.

2. The Model

The model is a closed-economy version of that of Viaene and Zilcha (1998). A competitive producer produces a single output using a single factor of production l. She faces output price risk, production risk, and cost risk denoted by $\tilde{p}$, $\tilde{\epsilon}$, and $\tilde{\eta}$ with means $\bar{p}$, $\bar{\epsilon}$, and $\bar{\eta}$, respectively. Again, following Viaene and Zilcha, assume that $\tilde{p}$, $\tilde{\epsilon}$, and $\tilde{\eta}$ are statistically independent. When a forward market exists only for the output price risk, the producer's profit equals

$$\tilde{\pi} = \tilde{p}q(l, \tilde{\epsilon}) - c(l, \tilde{\eta}) + (p_f - \tilde{p})x, \tag{1}$$

where p_f is the forward price of output; x is the level of hedging; q is the production function satisfying $q_1 > 0$, $q_2 > 0$, $q_{12} = q_{21} > 0$, $q_{11} < 0$, and $q_{22} < 0$.

The von Neumann-Morgenstern risk-averse producer's problem is

$$\max_{\{l,x\}} Eu[\tilde{p}q(l, \tilde{\epsilon}) - c(l, \tilde{\eta}) + (p_f - \tilde{p})x], \tag{2}$$

where E is the (joint) expectation operator; u satisfies $u' > 0$ and $u'' < 0$. Assume an interior solution exists for l such that the following first-order conditions are satisfied:

$$E\{[\tilde{p}q_1(l^*, \tilde{\epsilon}) - c_1(l^*, \tilde{\eta})] \cdot u'[\tilde{p}q(l^*, \tilde{\epsilon}) - c(l^*, \tilde{\eta}) + (p_f - \tilde{p})x^*]\} = 0; \tag{3}$$

$$E\{(p_f - \tilde{p}) \cdot u'[\tilde{p}q(l^*, \tilde{\epsilon}) - c(l^*, \tilde{\eta}) + (p_f - \tilde{p})x^*]\} = 0, \tag{4}$$

where l^* and x^* are the optimal factor demand and level of hedging. Denote the optimal level of profit at $\{l^*, x^*\}$ by $\tilde{\pi}^*$. The second-order condition is clearly satisfied given $u'' < 0$

(see Viaene and Zilcha, 1998).

The following unbiasedness assumption on the diversifiable risk, used by Viaene and Zilcha (1998), is employed in this paper:

(A1) *The output price forward market is unbiased with* $p_f = \bar{p}$.

The factor (output) decision under the cases of undiversifiable cost and production risks have been fully analyzed by Viaene and Zilcha (1998) and are thus omitted. Instead, the main purpose of this paper is to compare the hedging decisions under these two cases. A simplified version of Viaene and Zilcha's result under undiversifiable cost risk will be discussed in the next section for comparison purposes, whereas the necessary and sufficient conditions for unambiguous hedging decision of a risk-averse producer will be given in the last section.

3. Output Price Hedging under Undiversifiable Cost Risk

Fixing $\tilde{\epsilon}$ at ϵ (or any other constant value), when the producer faces only a hedgeable output price risk and an undiversifiable cost risk, her profit is given by

$$\tilde{\pi} = \tilde{p}q(l, \overline{\epsilon}) - c(l, \tilde{\eta}) + (p_f - \tilde{p})x = \tilde{p}[q(l, \overline{\epsilon}) - x] - c(l, \tilde{\eta}) + p_f x. \tag{5}$$

She is said to be fully hedging (overhedging/underhedging) against her output price risk if $x^* = (> / <)\ q(l^*, \overline{\epsilon})$. Clearly, $x^* = q(l^*, \overline{\epsilon})$ eliminates the output price risk.

Denote $E_{\tilde{z}}$ as the marginal expectation operator of any random variable $\tilde{z}$. By (A1) and the independence of $\tilde{p}$ and $\tilde{\eta}$, one can rewrite first-order condition (4) as

$$0 = E_{\tilde{p}}[(\overline{p} - \tilde{p})E_{\tilde{\eta}}u'(\tilde{\pi}^*)] = \text{cov}_{\tilde{p}}[-\tilde{p}, E_{\tilde{\eta}}u'(\tilde{\pi}^*)]. \tag{6}$$

It can be checked from (5) that the covariance term in (6) equals zero if and only if $q(l^*, \overline{\epsilon}) = x^*$ such that $\tilde{p}$ is eliminated from the profit function. This suggests that any risk-averse producer fully hedges against the output price risk in the presence of an unbiased forward market and an undiversifiable cost risk.

4. Output Price Hedging under Undiversifiable Production Risk

Fixing $\tilde{\eta}$ at $\overline{\eta}$ (or any other constant value), when the producer faces only a hedgeable output price risk and an undiversifiable production risk, her profit is given by

$$\tilde{\pi} = \tilde{p}q(l, \tilde{\epsilon}) - c(l, \overline{\eta}) + (p_f - \tilde{p})x. \tag{7}$$

Comparing (5) and (7) immediately reveals that the undiversifiable cost risk and undiversifiable production risk affect the firm's hedging decision of her output price risk very differently. Particularly, the cost function containing the cost risk is additive whereas the production function containing the production risk is multiplicative to the output price risk. The multiplicative nature makes it impossible to eliminate the output price risk by hedging through the forward output price market unless by coincidence. As a consequence, the concept of "full-hedging" is no longer meaningful. The following definition for "full-hedging" of the output price risk seems reasonable:

Definition *A producer fully hedges (overhedges/underhedges) on average against her output price risk if* $E_{\tilde{\epsilon}}q(l^*, \tilde{\epsilon}) = (< / >)\ x^*$.[3]

According to this definition, a producer fully-hedges against the output price risk if she eliminates the risk when her optimal output is at its average level.

The following proposition states the necessary conditions for underhedging (overhedging) on average. Following Kimball's terminology, a producer is said to be "prudent" if her utility satisfies $u''' > 0$.

Proposition 1. (Necessity) *Suppose assumption (A1) is satisfied. In the presence of an undiversifiable production risk, a risk-averse producer underhedges (overhedges) on average against her output price risk only if she is prudent (imprudent).*

[3] The concept of "full-hedging on average" parallels that of "full-insurance on average" employed in the literature of incomplete insurance market (see Hau, 1999).

Proof: By the independence of $\tilde{p}$ and $\tilde{\epsilon}$, rewrite first-order condition (4) as

$$0 = E_{\tilde{p}}[(\bar{p} - \tilde{p})E_{\tilde{\epsilon}}u'(\tilde{\pi}^*)] = \mathrm{cov}_{\tilde{p}}[-\tilde{p}, E_{\tilde{\epsilon}}u'(\tilde{\pi}^*)]. \tag{8}$$

The first equality in (8) is due to assumption (A1). For any realization p of $\tilde{p}$, consider the following differentiation:

$$\frac{dE_{\tilde{\epsilon}}u'(\tilde{\pi}^*)}{dp} = E_{\tilde{\epsilon}}\{[q(l^*,\tilde{\epsilon}) - x^*]u''(\tilde{\pi}^*)\} = \mathrm{cov}_{\tilde{\epsilon}}[q(l^*,\tilde{\epsilon}), u''(\tilde{\pi}^*)] + E_{\tilde{\epsilon}}[q(l^*,\tilde{\epsilon}) - x^*]E_{\tilde{\epsilon}}u''(\tilde{\pi}^*). \tag{9}$$

Suppose $x^* < E_{\tilde{\epsilon}}q(l^*,\tilde{\epsilon})$ (i.e., the firm underhedges on average). Then the second expression on the right side of the second equality in (9) is negative. Suppose further that $u''' < 0$. Then the covariance term on the right side of (9) is also negative. These together imply that $E_{\tilde{\epsilon}}u'(\tilde{\pi}^*)$ decreases unambiguously when p increases. Therefore, the covariance term in (8) is strictly positive. A contradiction. Hence, for (8) to hold, u''' must be positive such that the right side of the second equality in (9) can possibly be zero. □

The following proposition states the sufficient conditions under which a risk-averse producer underhedges (overhedges) on average against the output price risk. Following the literature of decision under uncertainty, a producer is said to be "temperate" if her utility function satisfies $u'''' < 0$ (see, e.g., Kimball, 1992; Mahul, 2000).

Proposition 2. (Sufficiency) Suppose assumption (A1) is satisfied. In the presence of an undiversifiable production risk, a risk-averse producer underhedges (overhedges) on average against her output price risk if she is prudent and temperate (imprudent and intemperate).

Proof: For any realization p of $\tilde{p}$, consider the following derivative:

$$\begin{aligned} & \frac{d}{dp}[E_{\tilde{\epsilon}}u'(\tilde{\pi}^*) - u'(E_{\tilde{\epsilon}}\tilde{\pi}^*)] \\ = \; & E_{\tilde{\epsilon}}\{[q(l,\tilde{\epsilon}) - x]u''(\tilde{\pi}^*)\} - u''(E_{\tilde{\epsilon}}\tilde{\pi}^*)E_{\tilde{\epsilon}}[q(l,\tilde{\epsilon}) - x] \\ = \; & \mathrm{cov}_{\tilde{\epsilon}}[q(l^*,\tilde{\epsilon}), u''(\tilde{\pi}^*)] + E_{\tilde{\epsilon}}[q(l,\tilde{\epsilon}) - x] \cdot [E_{\tilde{\epsilon}}u''(\tilde{\pi}^*) - u''(E_{\tilde{\epsilon}}\tilde{\pi}^*)]. \end{aligned} \tag{10}$$

Now, suppose the producer is prudent and temperate. Clearly, for any realization p of

$\tilde{p}$, $u''' > 0$ implies that $\text{cov}_{\tilde{\epsilon}}[q(l^*, \tilde{\epsilon}), u''(\tilde{\pi}^*)] > 0$ whereas $u'''' < 0$ implies that $E_{\tilde{\epsilon}}u''(\tilde{\pi}^*) - u''(E_{\tilde{\epsilon}}\tilde{\pi}^*) < 0$ by the Jensen's inequality.

Suppose $x^* \geq E_{\tilde{\epsilon}}q(l^*, \tilde{\epsilon})$. This has two implications. First, the sign of the sum on the third line of (10) is positive. Therefore, $\text{cov}_{\tilde{p}}[-\tilde{p}, E_{\tilde{\epsilon}}u'(\tilde{\pi}^*) - u'(E_{\tilde{\epsilon}}\tilde{\pi}^*)] < 0$. This, together with (8), implies that $\text{cov}_{\tilde{p}}[-p, u'(E_{\tilde{\epsilon}}(\tilde{\pi}^*))] > 0$. Second, for any realization p of $\tilde{p}$, it can be checked that $dE_{\tilde{\epsilon}}(\tilde{\pi}^*)/dp < 0$. This together with $u'' < 0$ implies that $\text{cov}_{\tilde{p}}[-p, u'(E_{\tilde{\epsilon}}(\tilde{\pi}^*))] < 0$ as $u'' < 0$. The two implications contradict each other. Hence, $x^* < E_{\tilde{\epsilon}}q(l^*, \tilde{\epsilon})$. $\square$

Notice that the concept of "prudence" has been used extensively to describe the precautionary saving behavior (see, e.g., Kimball, 1990). In addition, according to Mossin (1968), DARA is an appealing concept, particularly, when referred to the negative effect of wealth on insurance purchase. It can be checked that given risk aversion, prudence is necessary for DARA. More recently, Kimball (1993) showed that decreasing absolute prudence (DAP) and decreasing absolute risk aversion (DARA) are jointly necessary and sufficient for standard risk aversion and jointly sufficient for proper risk aversion, a concept introduced by Pratt and Zeckhauser (1987). It can be checked that for prudent individuals, a necessary condition for DAP is temperance. Very recently, Mahul (2000) showed that the effect of an increase in wealth on the price selection of a multiple peril crop insurance depends crucial on whether a producer is prudent and/or temperate.

Kimball (1993) and Eeckhoudt et al. (1995) suggested that the class of utility functions exhibiting both DARA and DAP (e.g., power utility and logarithm utility) is broad and interesting. If these authors are right, then we will see risk-averse producers facing undiversifiable production risk underheging on average against their output price risk. This is surprising because Kimball (1992) suggested that temperate agents respond to an "additive" idiosyncratic risk by reducing exposure to another statistically independent risk. Clearly, the multiplicative nature of an undiversifiable production risk makes a temperate producer more willing to accept a diversifiable risk "on average."

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