

行政院國家科學委員會專題研究計畫 成果報告

雙變量跳躍模型之應用 以輕原油與熱燃油為例

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## Abstracts

This paper investigates the price volatility of WTI crude oil and New York harbor no.2 heating oil over past 20 years using the CBP-GARCH model. Both features of jumps and bivariate are considering in this paper. The empirical results show that the jump variance in crude oil is higher than heating oil and stronger seasonal effects exist in heating oil price volatility. Moreover, the variance and covariance under GARCH model resemble to CBP-GARCH model in peacetime, but higher in the specific periods of high volatility. The variances are overestimation in traditional GARCH model around high volatility periods causing by the continuous assumption. Relatively, the CBP-GARCH model assumes that the specific shocks are independent with normal volatility and lowering the persistence of the abnormal volatility. For the reason that the overestimation in variance and covariance will bias the further application in finance, this paper is useful to any participators in markets for lowering the transaction costs and maximizing the profits.

Keywords: Jumps; Overestimation; Volatility; CBP-GARCH model

## 摘要

本文利用 CBP-GARCH 模型探討過去 20 年西德州原油與紐約港二號熱燃油之價格波動性。本文同時考量跳躍與雙變量之特性。實證研究發現原油的跳躍變異數較熱燃油為高，且熱燃油的價格波動存在高度的季節效果。再者，在一般時候，GARCH 模型下的變異數與共變異數與 CBP-GARCH 模型下之結果相似，但在高波動期間，GARCH 模型所估計的結果明顯較高。顯示 GARCH 模型中的連續性假設促使其變異數在高波動期間有過度估計的現象；相對的，CBP-GARCH 模型假設跳躍與一般波動相互獨立，可以降低異常波動的持續性。過度估計變異數與共變異數將使許多後續應用產生偏誤，對市場參與者而言，這篇文章能降低交易成本，使獲利極大化。

關鍵詞：跳躍，過度估計，波動性，CBP-GARCH 模型

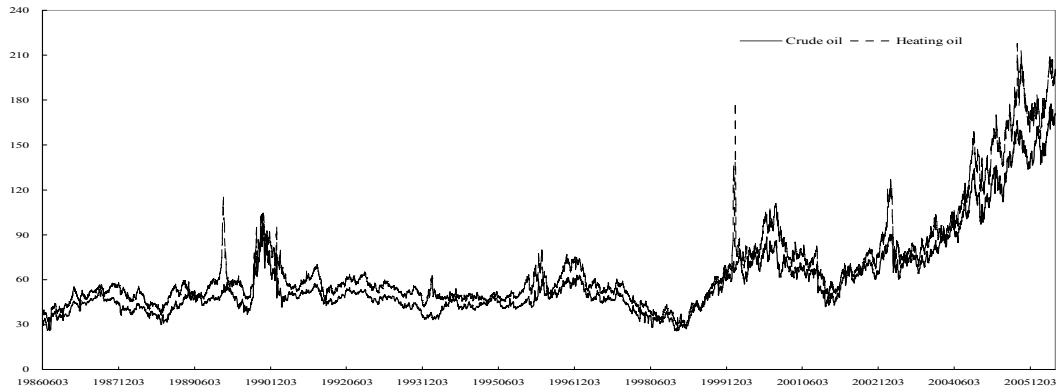
## INTRODUCTION

Volatility estimation and forecasting is the main task in the financial market for the past two decades, and it is the roots of most financial issues, such as asset pricing, portfolio selection, volatility relationship, hedging, risk, and so on. Most of researches assume that time series data follows a smooth and continuous volatility process, and generalized autoregressive conditional heteroscedasticity model (GARCH) is now widely accepted in this field (see the survey in Poon and Granger, 2003; Bauwens et al., 2006). However, the existence of jumps implies that diffusion models are misspecified statistically. Jorion (1988) ever argues that time-varying volatility and occasional jumps are perhaps two most distinctive features of daily financial time series. Park (2002) mentions that the standardized residuals of GARCH model still have excess kurtosis, albeit less in the raw financial returns (also see Bollerslev, 1987; Baillie and Bollerslev, 1989; Hsieh, 1989). Chan (2003) points out that although multivariate GARCH models are adequate terms of accounting for heteroskedasticity, these models do not fully capture the stylized fact of leptokurtosis in the unconditional distributions, often observed in financial data. Therefore, financial econometrics further investigate the volatility with jumps (for examples, Chang and Kim, 2001; Pan, 2002; Eraker, Johannes and Polson, 2003; Chan and Maheu, 2002; Johannes, 2004; Maheu and McCurdy, 2003). Most jump models have been successfully applied to the analysis of foreign exchange and stock market returns, may improve on the performance of capturing price behavior in physical commodities. In metal markets, Chan and Young (2006) find that the jump model fit the cooper spot and futures data well. Further, this paper introduces the jump GARCH model into the energy market, and investigates the price behavior on crude oil and heating oil, most liquid trading assets in New York Mercantile Exchange (NYMEX). The traditional GARCH model is obviously misspecified in energy assets with high volatility, especially while the enormous jumps occurring.

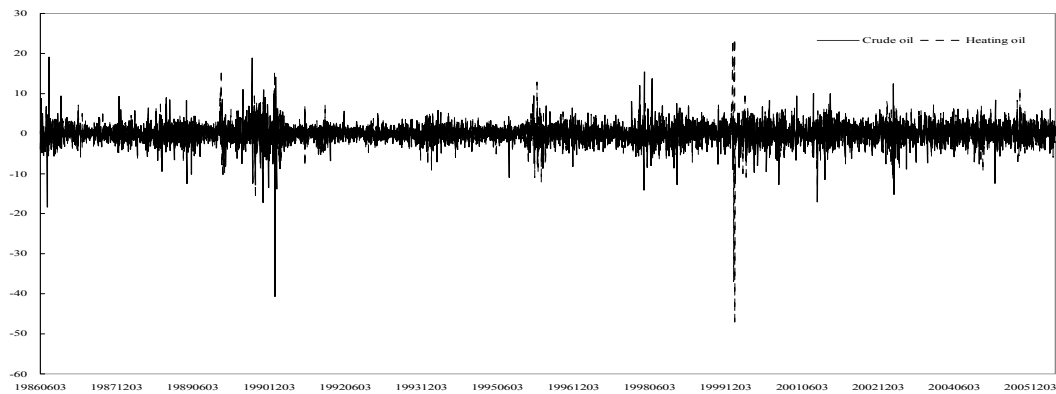
Jumps, specified to model unusual news events as part of the latent news process, have the potential to capture both smooth and sudden movements in price volatility (Chan and Young, 2006). The Poisson jump model used in financial market is first proposed by Press (1967), which introduces an independent jump process with arrival of jumps governed by a Poisson distribution. Although jumps are unobservable, an ex-post filter can always be constructed to infer the probability of jumps. Tucker and Pond (1988), Akgiray and Booth (1988) and Hsieh (1989) all finds that the Poisson jump model provide a well statistic characterization of daily exchange rates. The basic jump models are further extended in a number of directions. Combing with ARCH/GARCH model is an essential application (Jorion, 1988; Vlaar and Palm, 1993), and time-varying jump are emphasized in the following to fit in with

reality (Betas, 1991; Eraker et al., 2002; Das, 2003; Chan and Maheu, 2003; Maheu and McCurdy, 2004). However, the limitation of above models is the univariate setting for capturing the price volatility of specific asset. It is now widely accept that financial volatilities move together over time across assets and markets. Recognizing this feature through a multivariate modelling framework leads to more relevant empirical models than working with separate univariate models. Therefore, Chan (2003) develops a bivariate jump model which combing Correlated Bivariate Poisson (CBP) function and GARCH model to analyze the jump dynamics. The CBP-GARCH model is applied in energy market to investigate the price volatility of crude oil and heating oil.

Crude oil is not only the world's most actively traded commodity, but also the world's largest-volume futures contract trading on a physical commodity in NYMEX. Because of its excellent liquidity and price transparency, the contract is used as a principal international pricing benchmark. No.2 heating oil is a product refined from crude oil. According to EIA's (Energy Information Agency) Petroleum Marketing Monthly (2001), the price to consumers of home heating oil is generally comprised of 42% for crude oil, 12% for refining costs, and 46% for distribution and marketing. That is, while centering on the spot price of New York harbor No. 2 heating oil, the majority price components is the cost of crude oil, hence, the heating oil prices should closely tied to the cost of crude oil prices. Observing the spot prices over past 20 years (Figure 1), we find that the trend of crude oil and heating oil spot prices are familiar in most of time, but the surge in heating oil spot prices appears occasionally obviously in the end of 1989 and the early in 2000 and 2003. The background reasons are subject to swift supply and demand shifts due to weather, refinery shutdowns or the tension regarding oncoming war. As to the Panel B in Figure 1, the phenomena of jumps in returns are significantly and easily found. Traditional continuous and smooth models may fail in capture the dramatically violate in returns. Accordingly, we imply the CBP-GARCH model in crude oil and heating oil for proving the necessary of considering jumps. Besides, almost none of researches investigates jump in crude oil and heating oil in energy markets. GARCH model still the widely used to research the volatility behavior in energy assets (Lin and Tamvakis, 2001; Ewing et al., 2002; Sadorsky, 2002; Hammoudeh et al., 2003). Appropriate estimating the variance and covariance of the assets would improve the performance in forecasting, hedging, risk managing and so on.



Part A. Time series plots of spot prices (cents / per gallon)



Part B. Returns

Figure 1. Time series plots and returns of crude oil and heating oil

The original motivation of this paper is to fit the volatility well while facing the high volatility assets in energy market. We provide a complete analysis to price volatility to crude oil and heating oil over past 20 years, and investigate whether the performance is better in jump model. Another emphasis in this paper is the bivariate jump models are applied for estimating the volatilities of two highly related assets accurately. Not only for the reason that volatility spillovers exist between markets and assets, but also for the covariance between series are important, as well as the variances of the individual series themselves. Therefore, followed Chan (2003), we discuss the volatility features of crude oil and heating oil using correlated bivariate jump model. Further, we investigate that whether the overestimation exists in the traditional GARCH model while considering jump events. The overestimation in variance and covariance will bias the further application, for example, hedging, value of risk, portfolio constructing and so on. In point of hedging, the hedge is important particular during the high volatility period. The overestimation will lead to over hedging along with raising cost and reducing hedge effectiveness. Overestimation in volatility will increases the value at risk and then loss the potential profit. Hence, we should attach important to the problem of overestimation. The rest of the paper is organized as follows. Section 2 describes the methodology of GARCH and

CBP-GARCH models. Section 3 illustrates the data and descriptive statistics. Section 4 presents the empirical results. The conclusions are drawn in the final section.

### METHODOLOGY: CBP-GARCH Model

The CBP-GARCH model is a combination of the GARCH (Bollerslev, 1986) and the Poisson Correlated function (M'Kendrick, 1926; Campbell, 1934). The model is defined as follows:

$$R_t = \hat{R} + \varepsilon_t + J_t, \quad (1)$$

where  $R_t$  is a  $2 \times 1$  vector of returns consisting of a mean equation  $\hat{R}$ , a random disturbance  $\varepsilon_t$ , and a jump component  $J_t$ . The random disturbance follows a bivariate normal distribution with zero mean and variance covariance matrix  $\tilde{H}_t$ .

In a bivariate framework, jump component  $J_t$  has a bivariate normal distribution with zero mean and variance covariance matrix  $\Delta_t$ . The normal disturbance and the jump components are assumed to be independent, defined as:

$$J_t = \begin{bmatrix} \sum_{i=1}^{n_{1t}} Y_{1t,i} - E_{t-1}(\sum_{i=1}^{n_{1t}} Y_{1t,i}) \\ \sum_{j=1}^{n_{2t}} Y_{2t,j} - E_{t-1}(\sum_{j=1}^{n_{2t}} Y_{2t,j}) \end{bmatrix} \quad (2)$$

Here,  $Y_i$  is a random variable, called jump size. The sum of  $Y_i$  means that the return may experience “n” number of jumps, depending on the news content entering the market within any single time period  $t$ . Each of these jump sizes is governed by a normal distribution with constant mean  $\theta$  and variance  $\delta^2$ . In other words, the jump size for the two spots (crude oil and gasoline) can be characterized as:

$$Y_{1t,i} \sim N(\theta_1, \delta_1^2) \text{ and } Y_{2t,j} \sim N(\theta_2, \delta_2^2). \quad (3)$$

In equation (2), two discrete counting variables  $n_{1t}$  and  $n_{2t}$  control the arrival of jumps and they are constructed by three independent Poisson variables, namely,  $n_{1t}^*$ ,  $n_{2t}^*$ , and  $n_{3t}^*$ . Each one of these variables has a probability density function given by

$$P(n_{it}^* = j | \Phi_{t-1}) = \frac{e^{-\lambda_i} \lambda_i^j}{j!}. \quad (4)$$

The expected value and variance of  $n_{it}^*$  are both equal to  $\lambda_i$ , which is also referred

to as the jump intensity. The correlated jump intensity counters (M'Kendrick, 1926; Campbell, 1934) are defined as

$$n_{1t} = n_{1t}^* + n_{3t}^* \quad \text{and} \quad n_{2t} = n_{2t}^* + n_{3t}^*. \quad (5)$$

By construction, each of these counting variables ( $n_{1t}$  and  $n_{2t}$ ) is capable of generating independent jumps ( $n_{1t}^*$  and  $n_{2t}^*$ ) and correlated jumps ( $n_{3t}^*$ ), which contribute jumps to both series.

Using the change of variables method and integrating out  $n_{3t}^*$  yield the joint probability density for  $n_{1t}$  and  $n_{2t}$  as:

$$P(n_{1t} = i, n_{2t} = j | \Phi_{t-1}) = \sum_{k=0}^{\min(i,j)} e^{-(\lambda_1 + \lambda_2 + \lambda_3)} \frac{\lambda_1^{i-k} \lambda_2^{j-k} \lambda_3^k}{(i-k)!(j-k)!k!}. \quad (6)$$

The expected number of jumps is equal to

$$E(n_{it}) = \lambda_i + \lambda_3. \quad (7)$$

According to Chan (2003), the time varying jump intensities are defined as

$$\begin{aligned} \lambda_{1t} &= \lambda_1 + \eta_1^2 r_{1t-1}^2 \\ \lambda_{2t} &= \lambda_2 + \eta_2^2 r_{2t-1}^2 \\ \lambda_{3t} &= \lambda_3 + \eta_3^2 r_{1t-1}^2 + \eta_4^2 r_{2t-1}^2, \end{aligned} \quad (8)$$

where  $r_{it-1}$  is the rate of return for asset  $i$  at time  $(t-1)$  and  $r_{it-1}^2$  is an approximation of the last period's volatility. The jump intensities are assumed to be related to market conditions which are related in volatility. Similarly, the covariance is governed by the variations in the last period's volatilities from both series. The parametric structure not only introduces additional jump dynamics to the model, but also allows a time varying correlation between the counting variables  $n_{1t}$  and  $n_{2t}$ . The correlation is calculated as follows:

$$\text{Corr}(n_{1t}, n_{2t}) = \frac{\lambda_{3t}}{\sqrt{(\lambda_{1t} + \lambda_{3t})(\lambda_{2t} + \lambda_{3t})}}. \quad (9)$$

Combining the GARCH model with the CBP function, the probability density function for  $R_t$  given  $i$  and  $j$  jumps in spot 1 and spot 2 is defined by

$$f(R_t | n_{1t} = i, n_{2t} = j, \Phi_{t-1}) = \frac{1}{(2\pi)^{N/2}} |H_{ij,t}|^{-1/2} \exp[-u'_{ij,t} H_{ij,t}^{-1} u_{ij,t}], \quad (10)$$

where  $u_{ij,t}$  is the usual error term with the jump component  $J_{ij,t}$  representing the

effect of  $i$  and  $j$  jumps:

$$\mathbf{u}_{ij,t} = \mathbf{R}_t - \hat{\mathbf{R}} - \mathbf{J}_{ij,t} = \begin{bmatrix} r_{1t} - \hat{r}_1 - i\theta_1 + (\lambda_1 + \lambda_3)\theta_1 \\ r_{2t} - \hat{r}_2 - j\theta_2 + (\lambda_2 + \lambda_3)\theta_2 \end{bmatrix}. \quad (11)$$

The variance covariance matrix  $\mathbf{H}_{ij,t}$  can be separated into two parts: the variance covariance matrix for the normal random disturbance  $\tilde{\mathbf{H}}_t$  and for the jump components  $\Delta_{ij,t}$ .

First, the variance covariance matrix for the normal random distribution can be defined as

$$\tilde{\mathbf{H}}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}'\tilde{\boldsymbol{\varepsilon}}_{t-1}\tilde{\boldsymbol{\varepsilon}}'_{t-1}\mathbf{A} + \mathbf{B}'\tilde{\mathbf{H}}_{t-1}\mathbf{B}, \quad (12)$$

where  $\mathbf{C}$  is an upper triangular matrix, and  $\mathbf{A}$  and  $\mathbf{B}$  are symmetric matrices. Term  $\tilde{\boldsymbol{\varepsilon}}_{t-1}$  refers to the sum of a disturbance and a jump component. Second, the variance covariance matrix for the jump components is

$$\Delta_{ij,t} = \begin{bmatrix} i\delta_1^2 & \rho_{12}\sqrt{ij}\delta_1\delta_2 \\ \rho_{12}\sqrt{ij}\delta_1\delta_2 & j\delta_2^2 \end{bmatrix}, \quad (13)$$

where  $\rho_{12}$  is the correlation coefficient between  $Y_{1t}$  and  $Y_{2t}$ . The variance covariance matrix for the CBP-GARCH model is then a sum of  $\tilde{\mathbf{H}}_t$  and  $\Delta_{ij,t}$ .

Finally, the conditional density of returns is defined by

$$P(\mathbf{R}_t | \Phi_{t-1}) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} f(\mathbf{R}_t | n_{1t} = i, n_{2t} = j, \Phi_{t-1}) P(n_{1t} = i, n_{2t} = j, \Phi_{t-1}). \quad (14)$$

The log likelihood function is simply the sum of the log conditional densities:

$$\ln L = \sum_{t=1}^N \ln P(\mathbf{R}_t | \Phi_{t-1}). \quad (15)$$

## DATA and DESCRIPTIVE STATISTICS

This paper analysis the price volatility of WTI crude oil spot price and New York harbor No. 2 heating oil spot price using CBP-GARCH model. The sample period is from June 2, 1986 to May 31, 2006 with a total of 5,027 observations. All the data are obtained from the U.S. Department of Energy (DOE). The descriptive statistics for spots returns are shown in Table 1. The mean of returns are similar in crude oil and heating oil, while the standard deviation of heating oil is slightly higher than the crude oil. However, observing the time series plots in Figure 1, we find that



the higher variance of heating oil results from the enormous price change at times. Historically, heating oil prices have been higher during the winter months when the demand is higher (Figure 2), more notably over the sample period is the end of 1989, the early of 2000 and 2003. The biggest heating oil crisis appears in February 2000<sup>1</sup> for the shirking supply, the cold weather drives the demand up in the end of 1989, in addition, the low stock, high winter demand and the specter of war looming cause the high price in the early 2003. Further, both returns are negative skewness and leptokurtic. The skewness of crude oil and heating oil are -1.0576 and -1.8437 respectively, and the excess kurtosis are 19.7809 and 41.0871 for crude oil and heating oil, all the values are significant at 1% level. The covariance/correlation matrix is also listed in Table 1. The static correlation coefficient and covariance is 0.6519 and 4.3803 respectively, and it indicates that the high and positive relationship between crude oil and heating oil over past 20 years.

Table 1. Descriptive statistics

	M ean	Std. deviation	Mi n.	Max	Skewne ss	Excess kurtosis
Crude oil	0.0327	2.4999	-40.6395	19.1506	-1.0576***	19.7809***
Heating oil	0.0315	2.6879	-47.0116	22.9538	-1.8437***	41.0871***

Covariance/Correlation Matrix		
	Crude oil	Heating oil
Crude oil	6.2483	0.6519
Heating oil	4.3803	7.2237

Notes: \*\*\* represents significance under 1% level. The covariance/correlation matrix has the covariance on and below the diagonal and the correlation above it.

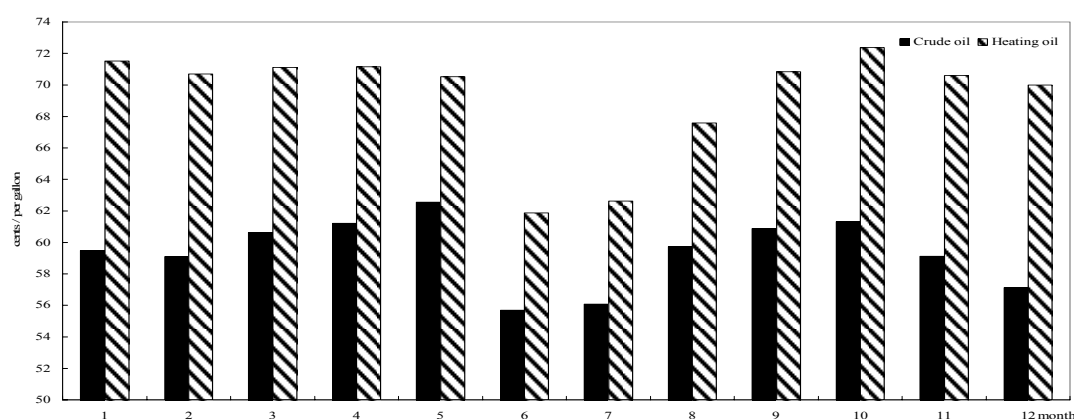


Figure 2. The average monthly spot price from June 1986 to May 2006.

<sup>1</sup> According to DOE, consumers paid an average of \$1.21 per gallon throughout the winter in 1999, however, during late January to early February 2000 the prices quickly rise from \$1.21 to \$1.99 per gallon, up to 64%. DOE are required to establish the Northeast Heating Oil Reserve in July 2000 for against potential shortfalls and price spikes.

## EMPIRICAL RESULTS

### *Estimation results of GARCH and CBP-GARCH model*

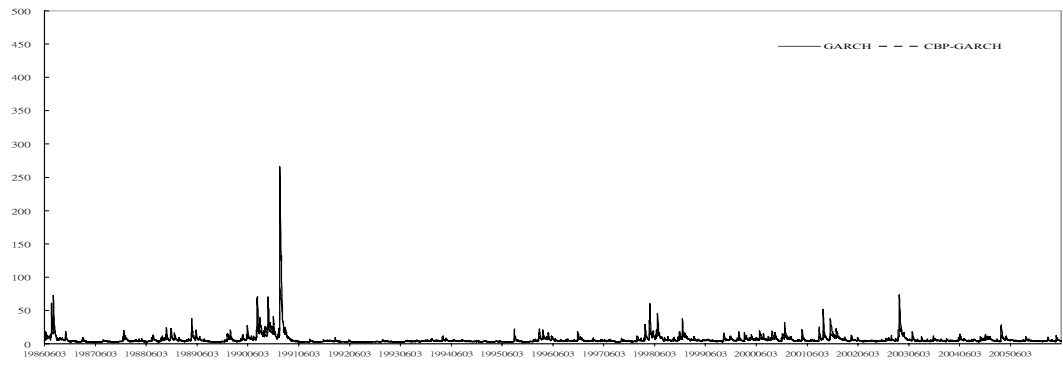
The empirical results of GARCH and CBP-GARCH models list in Table 2 and the volatility and covariance are drawn in Figure 3. In the part of GARCH model, all the parameters in conditional variance equation are significant under 1% level. It indicates that the volatility either of crude oil or heating oil ( $h_{11}$  and  $h_{22}$ ) is directly affected by its own past innovation and volatility. Higher levels of conditional volatility in the past are associated with higher conditional volatility in the current periods. The characteristics of volatility clustering also appear in conditional covariance ( $h_{12}$ ). Moreover, as to the part of CBP-GARCH model, all the parameters in the GARCH volatility terms are significant under 1% level, the same with GARCH model. Further, the jump components of jump size and intensity are discussed in the following. The mean of jump size is significant negative in crude oil ( $\theta_1$ ) and insignificant in heating oil ( $\theta_2$ ). The variance of jump size ( $\delta$ ) is 4.8767 and 4.5012 in crude oil and heating oil separately, both are significant under 1% level, and indicating that higher jump variance occurs in crude oil. The jump correlation is up to 0.9092 between crude oil and heating oil, and it reveals that the bivariate jump setting is highly essential in this study.

Both jump intensities ( $\lambda$ ) are significantly related to its past volatility, and the persistence is stronger in crude oil. However, the covariance of jump intensity is constant while the parameters  $\eta_3$  and  $\eta_4$  are not significant. The jump intensities are plotted in Figure 4. We also graph the average monthly jump intensity in Figure 5, and the seasonal effect is clear in heating oil. On average, jump intensity of heating oil is highest in February, January and December in sequence. Additionally, jump intensity of crude oil is more stable and higher than heating oil except in January and February. Otherwise, the correlation between numbers of jumps is graphed in Figure 6. The average over last 20 years is 0.7141, and the correlation coefficient is lower when the range between crude oil and heating oil is getting wider, especially the heating oil prices diverge from crude oil price.

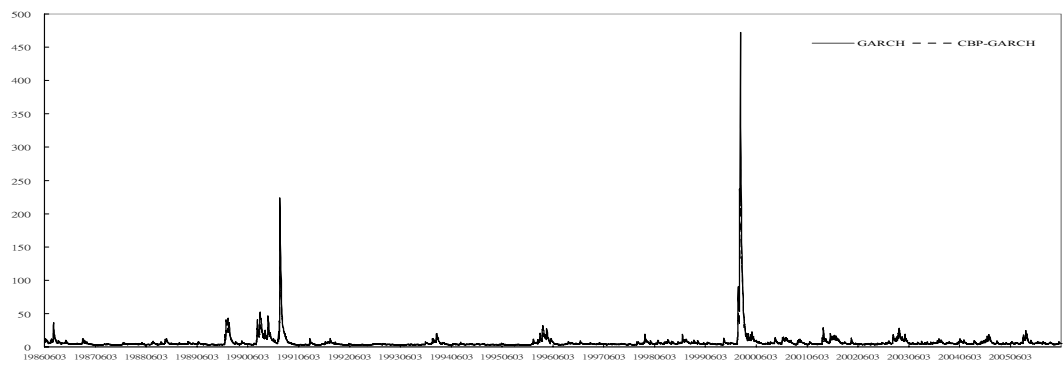
Table 2. Empirical results of GARCH and CBP-GARCH model

	GARCH model	CBP-GARCH model
<i>Mean equation</i>		
$\mu_1$	-0.0077	0.0207
$\mu_2$	0.0363	0.0538 *
<i>Variance equation</i>		
$c_{11}$	0.2019 ***	0.0919 ***
$c_{12}$	0.1734 ***	0.0921 ***
$c_{22}$	0.1817 ***	0.1181 ***
$a_{11}$	0.1496 ***	0.0438 ***
$a_{12}$	0.1202 ***	0.0467 ***
$a_{22}$	0.1153 ***	0.0592 ***
$b_{11}$	0.8314 ***	0.9146 ***
$b_{12}$	0.8451 ***	0.9077 ***
$b_{22}$	0.8586 ***	0.8958 ***
<i>Jump size</i>		
$\theta_1$		-0.5640 ***
$\theta_2$		-0.1685
$\delta_1$		4.8767 ***
$\delta_2$		4.5012 ***
$\rho$		0.9092 ***
<i>Jump intensity</i>		
$\lambda_1$		0.0133 ***
$\lambda_2$		0.0125 **
$\lambda_3$		0.0453 ***
$\eta_1$		0.0463 ***
$\eta_2$		-0.0324 ***
$\eta_3$		-0.0226
$\eta_4$		-0.0052
Log-likelihood value	-20137.9301	-19667.7251

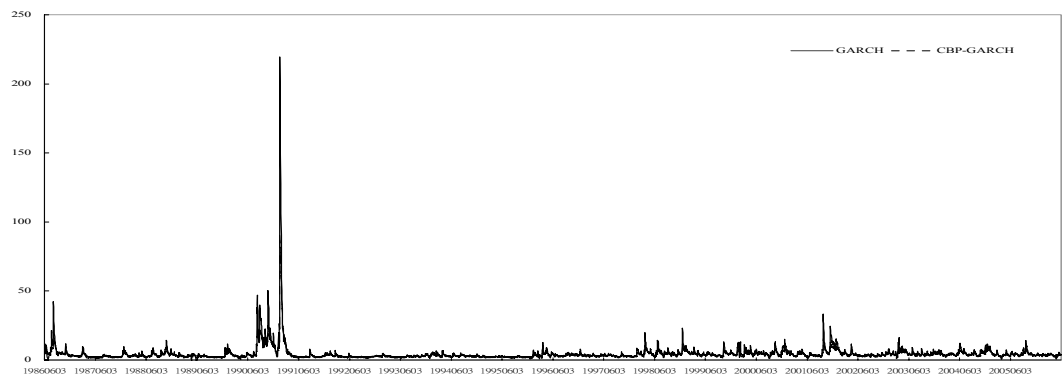
Notes: \*, \*\*, \*\*\* represent significance under 10%, 5% and 1% levels, respectively.



Part A. The conditional variance of crude oil



Part B. The conditional variance of heating oil



Part C. The covariance between crude oil and heating oil

Figure 3. The conditional variance and covariance under GARCH and CBP-GARCH model

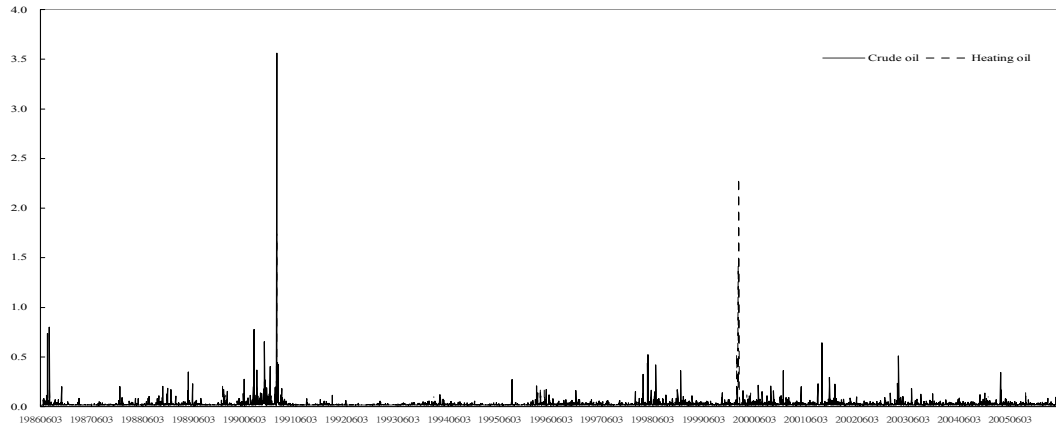


Figure 4. The jump intensity of crude oil and heating oil

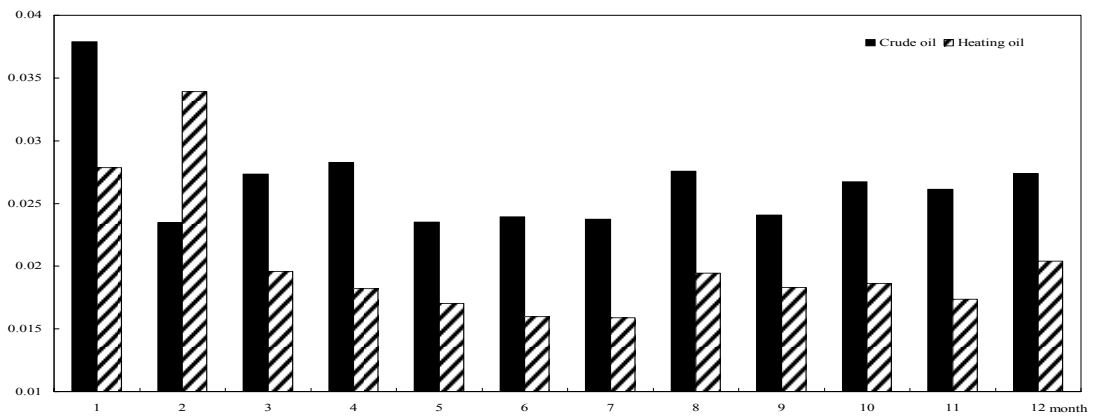


Figure 5. The average monthly jump intensity

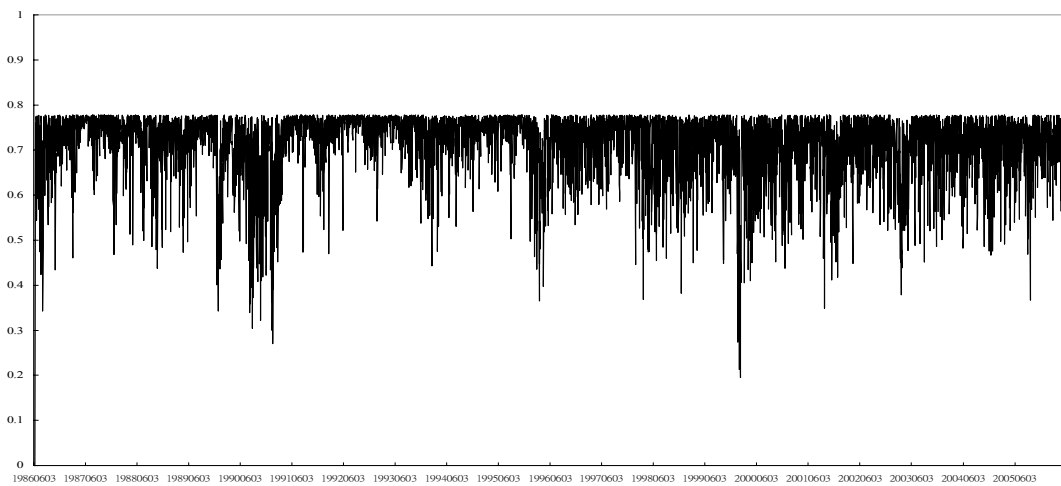


Figure 6. The correlation between the number of jumps of crude oil and heating oil

### *The overestimation in GARCH models during jump periods*

The descriptive statistics of variance are listed in Table 3. The average of the crude oil and heating oil is 6.7533 and 7.1848 under the GARCH model, whereas 5.7394 and 6.5128 under CBP-GARCH model. The covariance is 4.5214 under GARCH model and 4.1574 under CBP-GARCH model. All these results reveal that under CBP-GARCH model, the estimation results are smooth and the variation range is relative narrow. We select five events to analysis the phenomena in depth, including three jump events in heating oil which describing previously (Panel A to C in Figure 7) and two jump events in crude oil for the reason of Gulf wars (Panel D and E in Figure 7). We find that the variance and covariance is familiar under GARCH and CBP-GARCH model in peacetime, and diverge in the specific periods. The variances under the GARCH model are higher than CBP-GARCH model, and same as in the covariance. Taking Panel B to be the example, the shirking supply skyrockets the heating oil price in Feb. 2000, and the different estimations between two models are obviously during that period. Moreover, taking a look into the price jumps in crude oil during the periods of Gulf Wars, estimations of variance and covariance in GARCH model are relative higher.

The findings are also supported by the evidence shown in Table 4. In Table 4, we compare the volatility with the realized volatility<sup>2</sup> and test by the mean absolute percentage error (MAPE)<sup>3</sup>. Since the greatly seasonal effects, we list the results of monthly average estimation errors. First of all, inspecting the crude oil, we find that the CBP-GARCH model are better than GARCH model most of time. To contrast with the monthly jump intensity in Figure 5, the percentage error is much lower under CBP-GARCH model than GARCH model while in the month of higher jump intensity. However, the consequence is not consistent in August to October and it may be for the reason of offset while computing the average. Ignoring the seasonal factor, we find that the MAPE is 25.7473 under CBP-GARCH model lower than 26.8962 under GARCH model. Further, we discuss the percentage errors in heating oil. As shown in Table 4, CBP-GARCH model performs much better in January, February and December, which accompany with the highest jump intensity month. With regard to remnant months, the GARCH model can capture the volatility pretty well when the jump intensities are much smaller. The findings are highly consistent with the characteristics of heating oil. The MAPE is 29.6286 under CBP-GARCH model slightly higher than 29.2018 under GARCH model during the whole period, and the difference is much smaller than the crude oil. That is, the CBP-GARCH model

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<sup>2</sup> According to Anderson, Bollerslev, Diebold and Labys (2001), realized volatility is an unbiased and highly efficient estimator of return volatility based on the theory of quadratic variation, and also the logarithmic realized volatility and covariance are approximately Gaussian.

<sup>3</sup> MAPE is the average value of the absolute values of errors expressed in percentage terms.

performs better in crude oil with higher jump intensity, and just performs identically to GARCH model in heating oil during the extremely low jump intensity month.

We argue that the variance may overestimation in traditional GARCH model around high volatility periods causing by the continuous assumption. In other words, the overall shocks do not distinguish into normal or abnormal shocks, and stimulate the volatility to a superior level in next period. Nevertheless, the CBP-GARCH model assumes the specific shock as a jump, independent with normal volatility, and lowering the persistence of the abnormal volatility. Accordingly, the variances in GARCH model would higher than in CBP-GARCH model while facing the specific events or the assets with great highly volatility. Then, the further applications are easily biased based on the overestimation in variance and covariance.

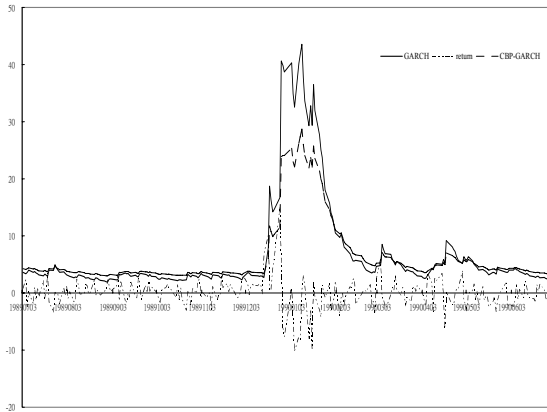
Table 3. Descriptive statistics of variance

	Mean	Std. deviation	Min.	Max.
<i>Crude oil</i>				
GARCH	6.7535	9.9897	1.4587	265.9064
CBP-GARCH	5.7394	4.5593	2.7404	84.6170
<i>Heating oil</i>				
GARCH	7.1848	17.3113	1.5118	472.5200
CBP-GARCH	6.5128	10.8476	2.6005	266.1891
<i>Covariance</i>				
GARCH	4.5214	7.0681	-0.5972	219.5728
CBP-GARCH	4.1574	3.7550	1.6670	90.2878
<i>Correlation</i>				
GARCH	0.7270	0.1474	-0.0687	0.9018
CBP-GARCH	0.7278	0.1039	0.1258	0.8952

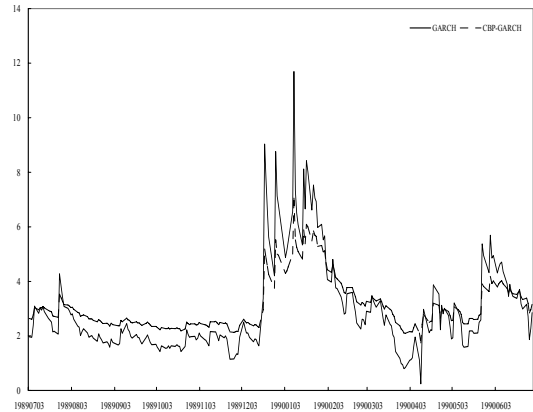
Notes: \*, \*\*, \*\*\* represent significance under 10%, 5% and 1% levels, respectively.

Table 4. Mean absolute percentage error in the variance of GARCH and CBP-GARCH model

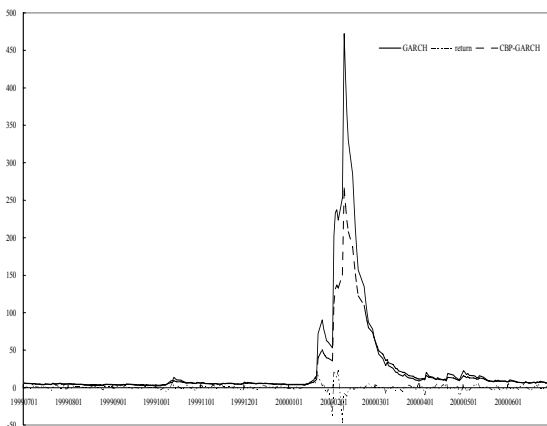
	<i>Crude oil</i>			<i>Heating oil</i>	
	GARCH	CBP-GARCH		GARCH	CBP-GARCH
All	26.8962	25.7473	All	29.2018	29.6286
Jan.	32.2035	29.4484	Jan.	34.9673	30.9817
Feb.	30.7024	30.5719	Feb.	43.9036	39.6531
Mar.	29.1938	27.0710	Mar.	37.7359	39.7053
Apr.	29.6359	28.1819	Apr.	24.6968	24.6642
May	30.9187	28.7809	May	30.8017	33.5456
Jun.	22.4268	21.5105	Jun.	24.4044	27.5511
Jul.	25.8333	25.0366	Jul.	26.9134	29.7519
Aug.	22.3410	22.5110	Aug.	24.6629	24.8305
Sep.	18.2282	18.7773	Sep.	28.0637	28.9339
Oct.	21.1115	23.4017	Oct.	24.8002	26.1859
Nov.	28.3120	26.3886	Nov.	28.1246	28.7220
Dec.	32.3882	27.7569	Dec.	22.3265	21.4858



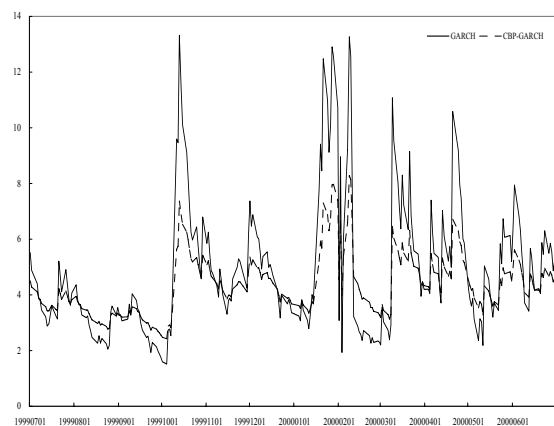
Part A1. Heating oil variance (July 1989-June 1990)



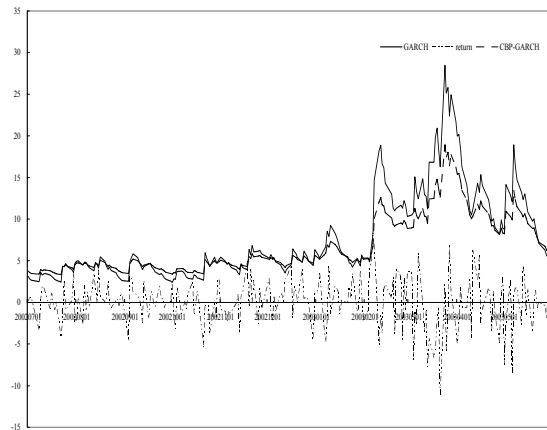
Part A2. Covariance (July 1989-June 1990)



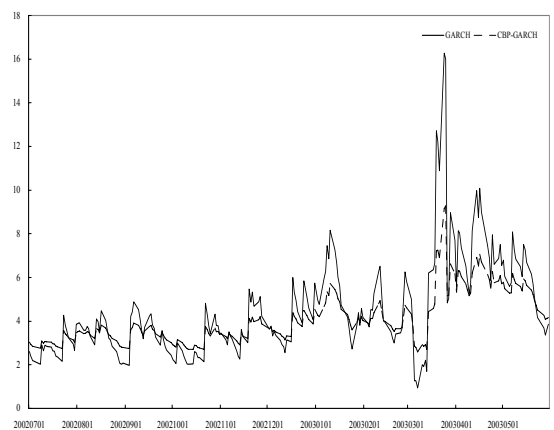
Part B1. Heating oil variance (July 1999-June 2000)



Part B2. Covariance (July 1999-June 2000)

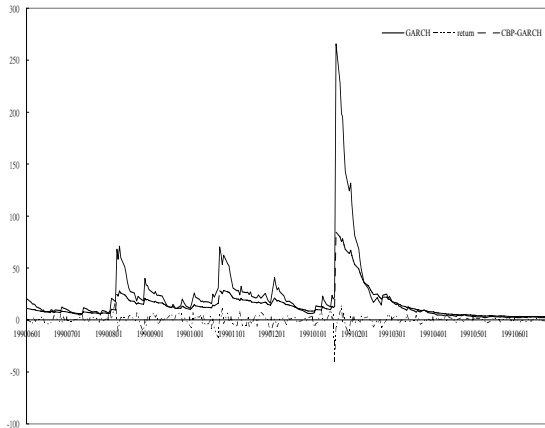


Panel C1. Heating oil variance (July 2002-May 2003)

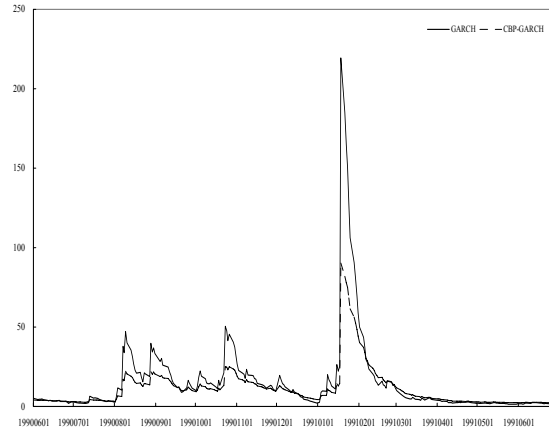


Panel C2. Covariance (July 2002-May 2003)

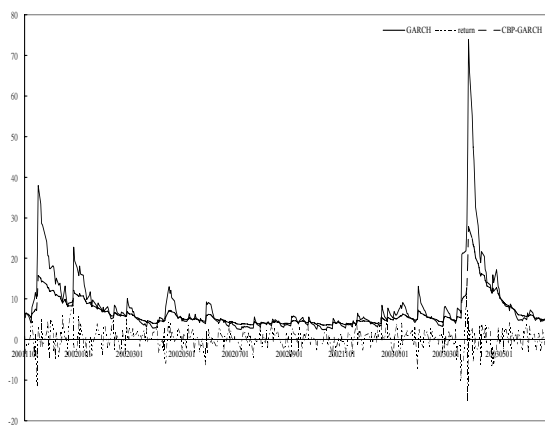




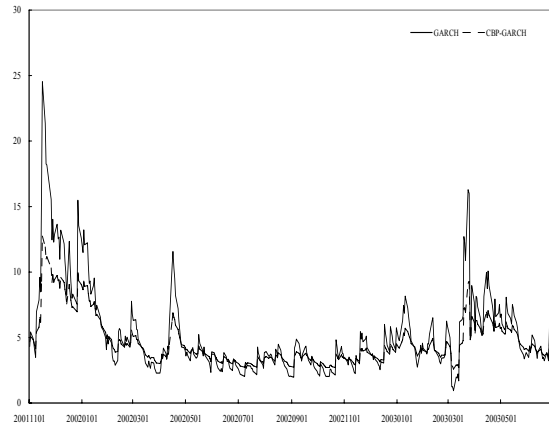
Panel D1. Crude oil variance (Gulf War I)



Panel D2. Covariance (Gulf War I)



Panel E1. Crude oil variance (Gulf War II)



Panel E2. Covariance (Gulf War II)

Figure 7. Variance and covariance in each model during the specific periods

## CONCLUSIONS

This paper investigates the price volatility of crude oil and heating oil over past 20 years using the CBP-GARCH model. Both features of jump and bivariate are considering in CBP-GARCH model, and we argue that the performance is better than traditional model, especially during the jump periods. The empirical results show that in crude oil, the mean of jump size is significant negative and the jump variance is higher than heating oil. Relatively, huge jumps in heating oil always appear in winter months, that is, stronger seasonal effects exist in heating oil price volatility. On average, jump intensity of heating oil is highest in February, January and December in sequence. As to the crude oil, jump intensity is more stable and higher than heating oil most of time. Moreover, we find that the variance and covariance under GARCH model resemble to CBP-GARCH model in peacetime, but higher in the specific periods of high volatility.

The variances are overestimation in traditional GARCH model around high

volatility periods causing by the continuous assumption. Relative to the CBP-GARCH model, assuming the specific shocks are independent with normal volatility and lowering the persistence of the abnormal volatility. We further compare the volatility with realized volatility and find that the CBP-GARCH model performs better in crude oil with higher jump intensity, and just performs identically to GARCH model in heating oil during the extremely low jump intensity month. Therefore, the CBP-GARCH model with concerning the jumps is appropriate and necessary in high volatility markets. For the reason that the overestimation in variance and covariance will bias the further application in finance, this paper is useful to traders, speculators and any participators for lowering the transaction costs and maximizing the profits.

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