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Semiparametric Bayesian Inference of the Smooth-Coefficient Stochastic Frontier Models

Ho-Chuan (River) Huang [1]
Department of Banking and Finance
Tamkang University

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[1] **Corresponding author**: Ho-Chuan (River) Huang, Department of Banking and Finance, Tamkang University, 151 Ying-Chun Road, Tamsui 25137, Taipei County, **Taiwan**. Tel: +886-2-26215656 ext. 3337, Fax: +886-2-26214755, Email: **river@mail.tku.edu.tw**. Financial support from NSC, Taiwan under grant number NSC 93-2415-H-032-008 is highly appreciated. Any remaining errors are my own responsibility.

Abstract

This paper considers the measurement of country's specific (in)efficiency while allows for the possible heterogeneous technologies adopted by different countries. A novel semiparametric smooth-coefficient stochastic frontier model is proposed and the posterior inference of the model is made possible via the Gibbs sampling algorithm. The model is applied to a real data set consists of 82 countries with possible heterogeneous aggregate production functions. Empirical results show that the estimated coefficients vary moderately across countries, indicating the adoption of heterogeneous technologies by different countries. Thus, we argue that our novel modeling strategy considered in this study may allow for better understanding of country's (in)efficiency than do traditional methods.

Keywords: stochastic frontier, semiparametric, smooth-coefficient, Gibbs sampler

JEL classification: C13, C21, C25

1 Introduction

The stochastic frontier model, also known as the composed-error model, has been the primary tool in the measurement of technical (production or cost) inefficiencies ever since the pioneering work of Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). Within this framework, the observed output or cost is decomposed into three components — the actual frontier, which depends on a set of explanatory variables; a one-sided disturbance which denotes deviations of the individual unit from the frontier; and a symmetric disturbance, which captures other effects such as measurement error. Recent applications include Huang, Huang and Fu (2002) who implement a stochastic frontier regression with self-selection to study farmer's choice behavior and cost efficiency in field plowing arrangement; Greene (2004) analyzes panel data set on health care delivery from the World Health Organization by focusing on how to distinguishing between heterogeneity and inefficiency in a stochastic frontier framework; and Wang (2003) who models the investment under financing constraints by a one-sided deviation from a frictionless investment level as well as explicitly identifies and quantifies the effects of financing constraints, to name a few.

Existing extensions of the basic stochastic frontier approach include at least the following aspects. First, a more flexible distributional assumption of the one-sided disturbance is adopted for measuring inefficiencies. In contrast to the half-normal distribution of Aigner, Lovell and Schmidt (1977) and the exponential distribution of Meeusen and van den Broeck (1977), latter generalizations include the truncated-normal density of Stevenson (1980) and gamma density of Greene (1990, 2003). One step further, Griffin and Steel (2003) consider a (mixture of) gamma distribution(s) for powers of the inefficiency. Second, the distribution of technical inefficiency may depend on some exogenous variables. For example, Kumbhakar, Ghosh and McGuckin (1991), Huang and Liu (1994) and Battese and Coelli (1995) allow the mean of the distribution to depend on firm-specific characteristics whereas Caudill, Ford and Gropper (1995) and Hadri (1999) parameterize the variance of the distribution as a function of appropriate explanatory variables. Notably, Wang (2002) provides a flexible parameterization to allow exogenous influences on both the mean and variance of the technical inefficiency distribution and, in particular, accommodates non-monotonic efficiency effect. Please see Hadri, Guermat and Whittaker (2003) for a similar interesting extension. Third, alternative functional forms of the stochastic frontiers are examined. Those specifications include a variant of the Cobb-Douglas or translog models. Despite of the simplicity, it is well known that the primary objective of composed-error models, i.e., measurement of firms' inefficiencies, can be very sensitive to the choice of functional form of the frontier. As a result, Koop, Osiewalski and Steel (1994) propose the asymptotically ideal model whereas Zhu, Ellinger and Shumway (1995) and Giannakas, Tran and Tzouvelekas (2003) consider a generalized quadratic Box-Cox transformation of the stochastic frontiers.

Alternative modeling strategies and generalizations are the semiparametric or/and nonparametric analysis and inference. For example, Fan, Li and Weersink (1996) extend the linear stochastic frontier model to a semiparametric stochastic frontier model in which the functional form of the frontier is left unspecified but the distributions of the composite error terms are of known form. They propose semiparametric pseudolikelihood estimators based kernel estimation which are, as they argue, robust to possible misspecifications of the frontier as opposed to existing parametric estimators. Similarly, Huang and Fu (1999) also advocate a nonparametric specification of the frontier and adopt a parametric inefficiency distribution. In particular, they utilize the approach of average derivative to estimate slopes of a stochastic frontier function and the method of pseudolikelihood to infer inefficiency without making an assumption or approximation on the functional specification. In contrast, Park and Simar (1994) assume a parametric frontier and focus on the nonparametric inefficiency distribution. This setup is extended by Park, Sickles and Simar (1998) to allow for dependence between inefficiencies and regressors, and by Sickles, Good and Getachew (2002) to model the multiple output/multiple input technology. Similarly, Griffin and Steel (2004) implement Bayesian semiparametric inference on the stochastic frontier model in which the distribution of inefficiency is again modeled nonparametrically through a Dirichlet process prior but the functional form of the frontier still remains parametric.

In the same spirit, measurements of the inefficiencies while allowing for possible diverse technologies among firms have been increasingly investigated via estimating a mixture of stochastic frontier regressions. For instance, Beard, Caudill and Gropper (1991, 1997) argue that firms in an industry may use different, but unobservable technologies. Since the technology employed by each firm is not observed, there is no sample separation information available. However, Beard, Caudill and Gropper (1991, 1997) and Caudill (2003) develop an EM algorithm to obtain maximum likelihood estimates of the mixture model and to separate firms probabilistically into different groups. More importantly, as they argue, if two or more underlying technologies are present and one single stochastic frontier is estimated, this mis-specification error can lead to misleading, and even incorrect, conclusions about (in)efficiency rankings. Similarly, in contrast to conventional specification which often assumes that all firms must share identical technology and differ only with respect to their degree of inefficiency, Tsionas (2002) and Huang (2004) propose a random-coefficient stochastic frontier model to distinguish technical inefficiencies from technological differences across firms. Greene (2004) also fins that there is considerable evidence of cross individual heterogeneity misqueraded as inefficiency using the World Health Organization's panel data.

In this study, we propose a novel semiparametric smooth-coefficient stochastic frontier (SPSC-SF) model in which the level of technology adopted by each country can be different and depends upon some observed variables, such as the initial GDP, education, literacy and so on. In particular, our model extends the semiparametric smooth-coefficient (SPSC) model of Li, Huang, Li and Fu (2002) and Chou, Liu and Huang (2004) to the stochastic frontier (SF) framework. As the stochastic frontier regression with random-coefficients in Tsionas (2002) and Huang (2004), our SPSC-SF model does not require each country to employ the same technology and, as a result, the frontier may not be common to all countries. In doing so, the country-specific efficiency can be separated from technological differentials across countries. In contrast to their studies, our approach explicitly models the possible sources of technological diversity as unknown smooth functions of an observable covariate. The intuition for making inference is to treat points on the regression lines as unknown parameters and priors are placed on differences between adjacent points to introduce the potential for smoothing the curve.

Koop and Tobias (2005) show how to estimate the unknown smooth function nonparametrically via the recent advances in the Bayesian Markov chain Monte Carlo (MCMC) approach, e.g., the Gibbs sampler with data augmentation (GSDA). Combined with the method used in Tsionas (2000, 2002) for estimating the stochastic frontier model, we are able to obtain the relevant full conditional densities needed for the implementation of the GSDA algorithm. All of those complete conditionals are in standard forms and, hence, are easy to simulate from. The resulting Gibbs output, after a transient stage, can be readily used for making posterior inference and model comparison.

This study is organized as follows. In section 2, the novel semiparametric smooth-coefficient stochastic frontier model is introduced and explained. Section 3 discusses how the estimation and inference can be carried out using Bayesian approach. Specifically, we briefly review the Gibbs sampler with data augmentation algorithm and obtain the full conditional densities needed in implementing the GSDA algorithm, given the chosen uninformative priors. Section 4 describes the data source and summarizes the estimation results. Final conclusions are provided in section 5.

2 The semiparametric smooth-coefficient stochastic frontier model

As shown below, our novel semiparametric smooth-coefficient stochastic frontier model synthesizes two interesting specifications from disparate branches of econometric models, namely the semiparametric smooth-coefficient model of Li, Huang, Li and Fu (2002) and Koop and Tobias (2005), and the stochastic frontier model of Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). For illustrative purpose, the SPSC-SF model can be simply described as,

$$y_i = \beta_1(z_i) + \beta_2(z_i)l_i + \beta_3(z_i)k_i + v_i - u_i, \; i = 1, 2, \cdots, n \tag{1}$$

where, for example, y_i represents the (natural) logarithm of the observed output for the 'i'th country, and the explanatory variables may include the (natural) logarithms of labor (l_i), capital (k_i), and country's initial condition (z_i) such as the education level (`iEduc`). [1] The measurement error v_i is assumed to be distributed as *iid* $\mathcal{N}(0, \sigma^2)$ and u_i is a non-negative error term indicating the extent of technical inefficiency. The model is termed as a "smooth-coefficient model" since the functions $\beta_2(z_i)$ and $\beta_3(z_i)$ serve as the coefficients on l_i and k_i, respectively, and we model these functions as depending in a '**smooth**' way on an observed variable z_i. It can be seen that their functional forms are left unspecified and can be estimated by the simulation-based Bayesian approach, e.g., the Gibbs sampling with data augmentation algorithm, discussed later.

The SPSC-SF regression is flexible and nests some interesting models commonly used in the literature. For example, if $\beta_j(z_i) = \beta_j$ for $j = 1, 2, 3$, i.e., all coefficients are not affected by R&D, equation (1) reduces to the commonly-used parametric linear stochastic frontier specification,

$$y_i = \beta_1 + \beta_2 l_i + \beta_3 k_i + v_i - u_i \tag{2}$$

Alternatively, if only $\beta_2(z_i) = \beta_2$ and $\beta_3(z_i) = \beta_3$, i.e., the slope parameters β are invariant to `iEduc`, then equation (1) can be treated as a partially linear stochastic frontier regression,

$$y_i = \beta_1(z_i) + \beta_2 l_i + \beta_3 k_i + v_i - u_i \tag{3}$$

In this case, the `iEduc` can only shift the level of the production frontier via $\beta_1(z_i)$ and is regarded to have "neutral" effects on the production frontier. Similar specifications, although not in the stochastic frontier framework, can be found in Robinson (1988) and Stock (1989).

Notably, the semiparametric smooth-coefficient stochastic frontier model in equation (1) has the advantage that it allows for more flexibility in functional form than a parametric linear stochastic frontier counterpart as in (2) or a partially linear stochastic frontier specification of (3). This is achieved by allowing additionally the coefficients of labor and capital, i.e., $\beta(z_i)$, to vary directly with the country's `iEduc` values as well. Thus, both the output elasticities of labor and capital depend on the level of the country's `iEduc` and, as a result, the returns to scale may also be a function of `iEduc` as well. In such case, the `iEduc` affects the stochastic frontier in a "non-neutral" manner.

[1] Clearly, additional inputs can be included as explanatory variables in a straightforward way.

Intuitively, we can treat each point on the nonparametric regression lines as an unknown parameter to be estimated along with other model parameters. Specifically, let $\gamma_j = (\gamma_{j1}, \gamma_{j2}, \cdots, \gamma_{jn})' = [\beta_j(z_1), \beta_j(z_2), \cdots, \beta_j(z_n)]'$ for $j = 1, 2, 3$, and stack the observations into vectors and matrices, equation (1) can be re-written as,

$$\begin{aligned} y &= I_n\gamma_1 + L\gamma_2 + K\gamma_3 + v - u \\ &= (I_n \; L \; K) \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} + v - u \\ &\equiv X\gamma + v - u \end{aligned} \quad (4)$$

where $y = (y_1, y_2, \cdots, y_n)'$, $X = (I_n \; L \; K)$, I_n is an $n \times n$ identity matrix, $L\,(K)$ is an $n \times n$ diagonal matrix with i^{th} element given by $l_i\,(k_i)$, $\gamma = (\gamma_1', \gamma_2', \gamma_3')'$, $v = (v_1, v_2, \cdots, v_n)'$, and $u = (u_1, u_2, \cdots, u_n)'$.

As noted by Koop and Poirier (2004), without imposing any additional structure to our model, we are plagued by the problem of 'insufficient observations' in that we have more than three times as many parameters as observations. However, the problem can be resolved through the use of prior information about the degree of smoothness of the nonparametric regression lines.

3 Posterior analysis

3.1 The likelihood function

In order to make posterior inference, we need to specify the priors and write down the likelihood function. For the latter, we treat the inefficiency terms u as additional parameters to be estimated along with other model parameters. As a result, according to equation (4), the (augmented) likelihood function can be written as,

$$L(y|\gamma, \sigma^2, u) = (2\pi)^{-\frac{n}{2}}(\sigma^2)^{-\frac{n}{2}} \exp\left\{-\frac{1}{2\sigma^2}(y + u - X\gamma)'(y + u - X\gamma)\right\} \quad (5)$$

3.2 The priors

Without loss of generality, in the following analysis, we assume that all the data are ordered in an ascending way so that $z_1 \leq z_2 \leq \cdots \leq z_n$. Regarding to the prior of γ, we first follow Koop and Poirier (2004) and Koop and Tobias (2005) to assume,

$$R\gamma \sim \mathcal{N}\left(0, I_3 \otimes V(\eta)\right) \tag{6}$$

where the $3(n-2) \times 3n$ matrix R is,

$$R = \begin{bmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{bmatrix} = I_3 \otimes D$$

and the $(n-2) \times n$ second-difference matrix is,

$$D = \begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 \end{bmatrix}$$

In doing so, we can see that $D\gamma_j, j = 1, 2, 3$ is the second differences of points on the j^{th} nonparametric regression line, denoted by $\Delta^2\gamma_{ij}$. In particular, the mean of $D\gamma_j$ is set to be $0_{(n-2)\times 1}$ for $j = 1, 2$ and 3 so that the second differences of the regression functions are centered over a prior mean of zero. In addition, we assume that, for $j = 1, 2$ and 3, the prior covariance matrix for $D\gamma_j$ is identical as $V(\eta)$, a matrix with dimension $(n-2) \times (n-2)$. In latter application, we assume that $V(\eta) = \eta I_{n-2}$ where the scalar parameter η will act as a smoothing parameter. [2]

For future reference, we can re-write equation (6) in terms of the prior γ as,

$$\gamma \sim \mathcal{N}\left(0, \left[R' \left(I_3 \otimes V(\eta)\right)^{-1} R\right]^{-1}\right) \tag{7}$$

Since $V(\eta) = \eta I_{n-2}$, the prior covariance matrix of γ can be further simplified as $\eta(R'R)^{-1}$. As mentioned above, the prior centers the second differences of the functions γ_1, γ_2 and γ_3 around a mean of zero, and the scalar parameter η controls the tightness around this mean and thereby the degree of smoothness of these functions.

The prior for the smoothness parameter η^{-1} is assumed to be gamma distributed as,

$$p(\eta^{-1}) \sim \mathcal{G}(\underline{\nu}_0, \underline{\delta}_0) \tag{8}$$

[2] If preferably, it is straightforward to allow for different prior covariance matrices for $D\gamma_j$ by assigning η_j with different values so that $V(\eta_j) = \eta_j I_{n-2}$ for $j = 1, 2$ and 3. Please see Koop and Tobias (2005) for similar strategy.

where the mean $E(\eta^{-1}) = \underline{\nu}_0/\underline{\delta}_0$ and the variance $V(\eta^{-1}) = \underline{\nu}_0/\underline{\delta}_0^2$. [3]

Similarly, a natural conjugate (gamma) prior for the precision σ^{-2} is,

$$p(\sigma^{-2}) \sim \mathcal{G}(\underline{\nu}_1, \underline{\delta}_1) \tag{9}$$

where the mean $E(\sigma^{-2}) = \underline{\nu}_1/\underline{\delta}_1$ and the variance $V(\sigma^{-2}) = \underline{\nu}_1/\underline{\delta}_1^2$.

So far, we have been silent about the specification of the inefficiency term u_i. Following Tsionas (2002), we assume that the one-sided disturbance term u_i is *iid* exponentially distributed,

$$p(u_i) \sim \mathcal{E}(\theta) \tag{10}$$

where the mean $E(u_i) = 1/\theta$ and the variance $V(u_i) = 1/\theta^2$. However, with some additional effort, a more flexible specification such as the gamma distribution can be readily adopted, see Greene (1990, 2003) and Huang (2004).

The prior for θ is,

$$p(\theta) \sim \mathcal{G}(\underline{\nu}_2, \underline{\delta}_2) \tag{11}$$

As introduced in van den Broeck, Koop, Osiewalski and Steel (1994) and argued in Tsionas (2002), setting $\underline{\nu}_2 = 1$ will result in an exponential prior with parameter $\underline{\delta}_2 = -\ln r^*$ where r^* is prior median efficiency.

3.3 The full conditionals

Using standard Bayesian results, the full conditional density of γ can be shown to be normally distributed as,

$$\gamma | y, \Theta_{\backslash\gamma} \sim \mathcal{N}(\overline{\gamma}, \overline{G}) \tag{12}$$

where

$$\begin{aligned} \overline{\gamma} &= \overline{G}\left[X'(y+u)/\sigma^2\right] \\ \overline{G} &= \left(R'R/\eta + X'X/\sigma^2\right)^{-1} \end{aligned}$$

The full conditional density of η^{-1} is gamma distributed as,

$$\eta^{-1} | y, \Theta_{\backslash\eta^{-1}} \sim \mathcal{G}(\overline{\nu}_0, \overline{\delta}_0) \tag{13}$$

[3]The gamma density is $f(\eta^{-1}|\underline{\nu}_0, \underline{\delta}_0) = \frac{\underline{\delta}_0^{\underline{\nu}_0}}{\Gamma(\underline{\nu}_0)} \left(\eta^{-1}\right)^{\underline{\nu}_0 - 1} \exp\left\{-\underline{\delta}_0 \eta^{-1}\right\}$.

where

$$\begin{aligned}\overline{\nu}_0 &= \underline{\nu}_0 + 3(n-2)/2 \\ \overline{\delta}_0 &= \underline{\delta}_0 + (R\gamma)'(R\gamma)/2\end{aligned}$$

Given the prior in (9), it can be shown that the full conditional density of σ^{-2} is gamma distributed as,

$$\sigma^{-2}|y, \Theta_{\backslash \sigma^{-2}} \sim \mathcal{G}\left(\overline{\nu}_1, \overline{\delta}_1\right) \tag{14}$$

where

$$\begin{aligned}\overline{\nu}_1 &= \underline{\nu}_1 + n/2 \\ \overline{\delta}_1 &= \underline{\delta}_1 + (y+u-X\gamma)'(y+u-X\gamma)/2\end{aligned}$$

For $i = 1, 2, \cdots, n$, the full conditional density of the inefficiency measurement, u_i, is given by a truncated normal as,

$$u_i|y, \Theta_{\backslash u_i} \sim \mathcal{N}_{[0,\infty]}\left(x_i'\gamma_i - y_i - \theta\sigma^2, \sigma^2\right) \tag{15}$$

where $x_i = (1, l_i, k_i)'$ and $\gamma_i = (\gamma_{1i}, \gamma_{2i}, \gamma_{3i})'$.

Finally, the conditional distribution of θ is gamma distributed as,

$$\theta|y, \Theta_{\backslash \theta} \sim \mathcal{G}\left(\overline{\nu}_2, \overline{\delta}_2\right) \tag{16}$$

where

$$\begin{aligned}\overline{\nu}_2 &= \underline{\nu}_2 + n \\ \overline{\delta}_2 &= \underline{\delta}_2 + \sum_{i=1}^{n} u_i\end{aligned}$$

Given available those full conditionals, the Gibbs sampler can be implemented by drawing random variates from the full conditionals in (12), (13), (14), (15) and (16), respectively. The Gibbs output, after a transient stage, can be used to make posterior inference of the parameters.

4 Data and results

As argued in Durlauf, Kourtellos and Minkin (2001) and Kourtellos (2003), conventional analysis of the Solow growth model often unrealistically assumes that all countries under consideration share identical aggregate production functions. As a result, they examine a local generalization of the Solow growth model. By local, they mean that a Solow model applies to each country, but the model's parameters vary across countries. More specifically, they allow these parameters to change according to a country's initial conditions and characteristics.

For illustration of the practical use of our novel semiparametric smooth-coefficient stochastic frontier model, we apply it to the data collected by Duffy and Papageorgiou (2000). In particular, we average their panel data for twenty-eight years to obtain data on output (gross domestic product), capital and labor for 82 countries. Moreover, following the idea of Durlauf, Kourtellos and Minkin (2001) and Kourtellos (2003), we obtain the average years of education in 1960 ($\texttt{iEduc}_i$) as a measure of z_i in our model.

In order to let the data speak for themselves, all the priors are chosen to be very uninformative (diffuse) so that the estimation results represent the information from the data rather than that from our subjective priors. Specifically, the hyperparameters are chosen to be $\beta_0 = 0_{3\times 1}$, $B_0 = 10^6 \times I_3$, $\nu_0 = \delta_0 = \nu_1 = \delta_1 = 10^{-6}$. Then the algorithm is run for $20,000$ iterations. We discard the first $10,000$ draws to mitigate the effect of initial values and to assure the convergence of the chain. The last $10,000$ sample variates are collected and used for making posterior inference. The results are summarized in Table 1. All estimates are, as required (from gamma distributions), positive. The mean

Table 1: The SPSC-SF results[†]

	`mean`	`std`	`median`	2.5%	5%	95%	97.5%
$\eta^{\ddagger}$	0.2814	0.7602	0.0152	0.0016	0.0022	2.2585	3.0807
σ^2	0.0159	0.0127	0.0129	0.0002	0.0005	0.0399	0.0465
θ	3.4206	0.6762	3.3313	2.1027	2.4582	4.6534	4.9162

† The results are based on $10,000$ random variates from the Gibbs sampler after discarding the first $10,000$ iterations.

‡ All values are multiplied by 10^4.

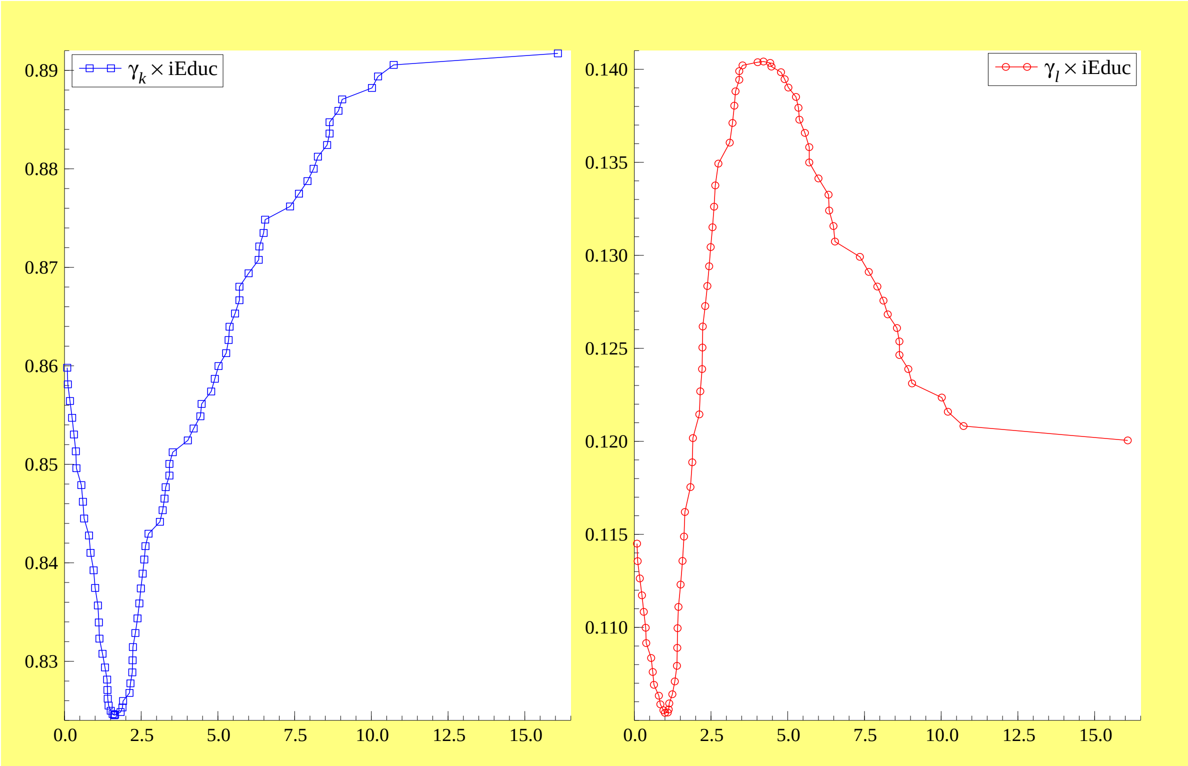

Figure 1: The mean values of $\beta_2(\texttt{iEduc})$ (left panel) and $\beta_3(\texttt{iEduc})$ (right panel) against `iEduc`, respectively.

estimate of the smoothness parameter, η, is 0.2014×10^{-4} and is larger than its median estimate 0.0152×10^{-4}, indicating the distribution of η is left-skewed. In addition, the mean value of σ^2 is 0.0159 while that of θ is 3.4206. More important, Figure 1 plots the relationship between the smooth coefficients of capital (labor) and the initial education, `iEduc`. It can be seen that both the estimated coefficients on capital and labor display moderate fluctuations. For $\beta_2(\texttt{iEduc})$, the value starts from 0.8598, decreases to 0.8245, and increases finally to 0.8917 as `iEduc` becomes higher while, for $\beta_3(\texttt{iEduc})$, the value starts from 0.1145, decreases to 0.1054, then increases to 0.1404 and finally reduces to 0.1200 as `iEduc` becomes larger. The results provide further evidence in support of the conjectures of Durlauf, Kourtellos and Minkin (2001) and Kourtellos (2003), in that we should explicitly allow for cross-country parameter heterogeneity in aggregate production functions. Failing to do so and imposing homogeneity assumption on the aggregate production functions may lead to inappropriate model specification and can result in misleading conclusions.

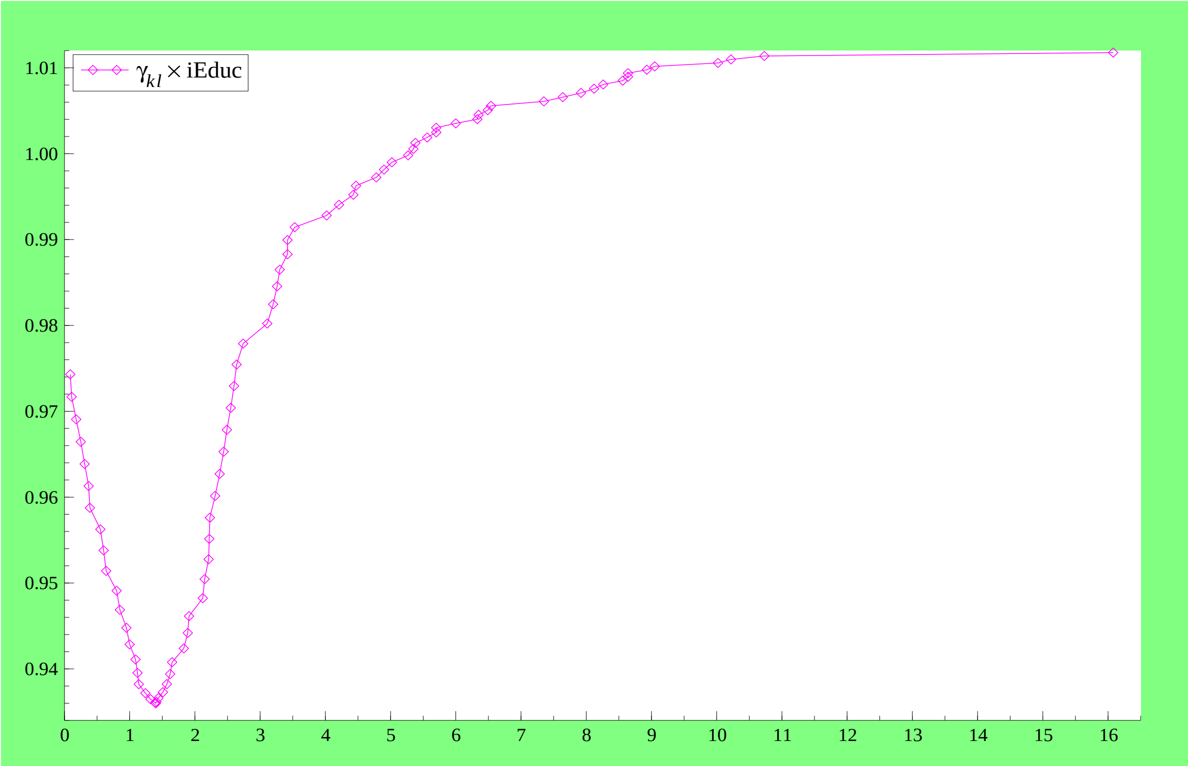

Figure 2: The returns to scale measured by the sum of β_2(`iEduc`) $+\beta_3$(`iEduc`) against `iEduc`.

As we can see, the initial level of education has to cross some threshold values (1.62 for capital and 1 for labor) to increase the productivity of capital and labor. After that, the productivity of capital, measured by the coefficient (β(`iEduc`)) of the capital variable increases monotonically as the education level increases. In contrast, the labor productivity increases monotonically when the education level increases up to 4.21 but decreases monotonically thereafter as the level of education become higher. It seems to indicate that the main effect of the higher level of education is on the improvement of the capital productivity rather than on enhancing the labor productivity. In order to see if the returns to scale change with the level of education, we plot the sum of β_2(`iEduc`) $+\beta_3$(`iEduc`) against `iEduc` in Figure 2. Clearly, the returns to scale are decreasing when the level of education is lower. However, as education level getting larger than 1.4, the returns to scale increases approximately to 1, i.e., constant returns to scale.

5 Concluding remarks

This study proposes a novel semiparametric smooth-coefficient stochastic frontier model which allows for the different firms or countries to adopt different technologies. In particular, the coefficients measuring the technologies vary with some particular variables in an unknown but smooth way.

In order to make posterior inference, we explicitly derive all the relevant full conditional densities which are needed in the implementation of the Gibbs sampling algorithm. Following Durlauf, Kourtellos and Minkin (2001) and Kourtellos (2003), we estimate the aggregate production functions across countries and allow for parameter heterogeneity. In particular, we model the parameters as a nonparametric function of the initial education levels and find that the estimated coefficients display moderate fluctuation rather than fixed. The findings are in accord to those discovered in Durlauf, Kourtellos and Minkin (2001) and Kourtellos (2003). They suggest that the cross-country parameter heterogeneity should be allowed in aggregate production functions. Failing to do so by imposing homogeneity assumption will lead to misleading conclusions.

References

Aigner, D., Lovell, C. A. K. and Schmidt, P. (1977), "Formulation and Estimation of Stochastic Frontier Production Function Models." *Journal of Econometrics* 6, 21-37.

Battese, G. E. and Coelli, T. J. (1995), "A Model for Technical Inefficiency Effects in a Stochastic Frontier Production Function for Panel Data." *Empirical Economics* 20, 325-332.

Beard, T. R., Caudill, S. B. and Gropper, D. M. (1991), "Finite Mixture Estimation of Multiproduct Cost Functions." *Review of Economics and Statistics* 73, 654-464.

Beard, T. R., Caudill, S. B. and Gropper, D. M. (1997), "The Diffusion of Production Processes in the U.S. Banking Industry: A Finite Mixture Approach." *Journal of Business & Finance* 21, 721-740.

Caudill, S. B. (2003), "Estimating a Mixture of Stochastic Frontier Regression Models via the EM Algorithm: A Multiproduct Cost Function Application." *Empirical Economics* 28, 581-598.

Caudill, S. B., Ford, J. M. and Gropper, D. M. (1995), "Frontier Estimation and Firm-Specific Inefficiency Measures in the Presence of Heteroscedasticity." *Journal of Business & Economic Statistics* 13, 105-111.

Chou, S. Y., Liu, J. T. and Huang, C. J. (2004), "Health Insurance and Savings over the Life Cycle – A Semiparametric Smooth Coefficient Estimation." *Journal of Applied Econometrics* 19, 295-322.

Duffy, J. and Papageorgiou, C. (2000), "A Cross-Country Empirical Investigation of the Aggregate Production Function Specification." *Journal of Economic Growth* 5, 87-120.

Durlauf, S. N., Kourtellos, A. and Minkin, A. (2001), "The Local Solow Growth Model." *European Economic Review* 45, 928-940.

Fan, Y., Li, Q. and Weersink, A. (1996), "Semiparametric Estimation of Stochastic Production Frontier Models." *Journal of Business & Economic Statistics* 14, 460-468.

Giannakas, K., Tran, K. C. and Tzouvelekas, V. (2003), "On the Choice of Functional Form in Stochastic Frontier Modeling." *Empirical Economics* 28, 75-100.

Greene, W. H. (1990), "A Gamma-Distributed Stochastic Frontier Model." *Journal of Econometrics* 46, 141-163.

Greene, W. H. (2003), "Simulated Maximum Likelihood Estimation of the Normal-Gamma Stochastic Frontier Model." *Journal of Productivity Analysis* 19, 179-190.

Greene, W. H. (2004), "Distinguishing between Heterogeneity and Inefficiency: Stochastic Frontier Analysis of the World Health Organization's Panel Data on National Health Care Systems." *Health Economics* 13, 959-980.

Griffin, J. E. and Steel, M. F. J. (2003), "Flexible Mixture Modelling of Stochastic Frontiers." Working paper.

Griffin, J. E. and Steel, M. F. J. (2004), "Semiparametric Bayesian Inference for Stochastic Frontier Models." *Journal of Econometrics* 123, 121-152.

Hadri, K. (1999), "Estimation of a Doubly Heteroscedastic Stochastic Frontier Cost Function." *Journal of Business & Economic Statistics* 17, 359-363.

Hadri, K., Guermat, C. and Whittaker, J. (2003), "Estimation of Technical Inefficiency Effects using Panel Data and Doubly Heteroscedastic Stochastic Production Frontiers." *Empirical Economics* 28, 203-222.

Huang, H. C. (2004), "Estimation of Technical Inefficiencies with Heterogeneous Technologies." *Journal of Productivity Analysis* 21, 277-296.

Huang, C. J. and Fu, T. T. (1999), "An Average Derivative Estimation of Stochastic Frontiers." *Journal of Productivity Analysis* 12, 45-53.

Huang, M. Y., Huang, C. J. and Fu, T. T. (2002), "Cultivation Arrangements and the Cost Efficiency of Rice Farming in Taiwan." *Journal of Productivity Analysis* 18, 223-239.

Huang, C. J. and Liu, J. T. (1994), "Estimation of a Non-neutral Stochastic Frontier Production Function." *Journal of Productivity Analysis* 5, 171-180.

Koop, G., Osiewalski, J. and Steel, M. F. J. (1994), "Bayesian Efficiency Analysis with a Flexible Form: The AIM Cost Function." *Journal of Business & Economic Statistics* 12, 339-346.

Koop, G. and Poirier, D. (2004), "Bayesian Variants of Some Classical Semiparametric Regression Techniques." *Journal of Econometrics* 123, 259-282.

Koop, G. and Tobias, J. L. (2005), "Semiparametric Bayesian Inference in Smooth Coefficient Models." Forthcoming in *Journal of Econometrics*.

Kourtellos, A. (2003), "Modeling Parameter Heterogeneity in Cross-Country Growth Regression Models." Working Paper.

Kumbhakar, S. C., Ghosh, S. and McGuckin, J. T. (1991), "A Generalized Production Frontier Approach for Estimating Determinants of Inefficiency in U.S. Dairy Farms." *Journal of Business & Economic Statistics* 9, 279-286.

Li, Q., Huang, C. J., Li, D. and Fu, T. T. (2002), "Semiparametric Smooth Coefficient Models." *Journal of Business & Economic Statistics* 20, 412-422.

Meeusen, W. and van den Broeck, J. (1977), "Efficiency Estimation from Cobb-Douglas Productions with Composed Errors." *International Economic Review* 8, 435-444.

Park, B. U., Sickles, R. C. and Simar, L. (1998), "Stochastic Panel Frontiers: A Semiparametric Approach." *Journal of Econometrics* 84, 273-301.

Park, B. U. and Simar, L. (1994), "Efficient Semiparametric Estimation in a Stochastic Frontier Model." *Journal of the American Statistical Association* 89, 929-936.

Robinson, P. M. (1988), "Root-N-Consistent Semiparametric Regression." *Econometrica* 56, 931-954.

Sickles, R. C., Good, D. H. and Getachew, L. (2002), "Specification of Distance Functions using Semi- and Nonparametric Methods with an Application to the Dynamic Performance of Eastern and Western European Air Carriers." *Journal of Productivity Analysis* 17, 133-155.

Stevenson, R. E. (1980), "Likelihood Functions for Generalized Stochastic Frontier Estimation." *Journal of Econometrics* 13, 57-66.

Stock, J. H. (1989), "Nonparametric Policy Analysis." *Journal of the American Statistical Association* 84, 567-575.

Tsionas, E. G. (2000), "Full Likelihood Inference in Normal-Gamma Stochastic Frontier Models." *Journal of Productivity Analysis* 13, 183-205.

Tsionas, E. G. (2002), "Stochastic Frontier Models with Random Coefficients." *Journal of Applied Econometrics* 17, 127-147.

van den Broeck, J., Koop, G., Osiewalski, J. and Steel, M. F. J. (1994), "Stochastic Frontier Models: A Bayesian Perspective." *Journal of Econometrics* 61, 273-303.

Wang, H. J. (2002), "Heteroscedasticity and Non-Monotonic Efficiency Effects of a Stochastic Frontier Model." *Journal of Productivity Analysis* 18, 241-253.

Wang, H. J. (2003), "A Stochastic Frontier Analysis of Financing Constraints on Investment: The Case of Financial Liberalization in Taiwan." *Journal of Business & Economic Statistics* 21, 406-419.

Zhu, S., Ellinger, P. N. and Shumway, C. R. (1995), "The Choice of Functional Form and Estimation of Banking Inefficiency." *Applied Economics Letters* 2, 375-379.