行政院國家科學委員會專題研究計畫　成果報告

梁式結構受高速移動載重之動力反應分析(II)

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計畫編號：NSC92-2211-E-032-016-
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共同主持人：林炳宏
計畫參與人員：林韻如

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※ 梁式結構受高速移動載重之動力反應分析(第二年) ※
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計畫類別：■個別型計畫 □整合型計畫

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共同主持人：林炳宏 淡江大學建築技術系

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行政院國家科學委員會專題研究計畫成果報告

梁式結構受高速移動載重之動力反應分析【第二年】

Dynamic response analysis of beam-type structures to moving loads with high speeds(II)

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共同主持人：林炳宏　　　淡江大學建築技術系
計畫參與人員：陳韻如　台灣大學土木工程研究所

中文摘要

本文旨在探討版式軌道在高速列車作用下的動態反應及輪軌間的互制行為，所謂的版式軌道是不用道碴的，與傳統的道碴型軌道系統有所不同，本文旨在研究版式軌道與列車的動力互制行為。文中先對版式軌道系統的模擬方式，作一簡單的介紹，並以 Yang 和 Yau(1997)所提之車橋互制理論作為基本架構來進行分析，而在列車系統方面，則是將介於車箱與車輪間之台車，模擬成剛性構件(rigid body)，以考慮其對相接輪軸之聯結效應。在分析列車-軌道反應的研究中，主要是利用 Newmark 差分技巧及動態濃縮法，將車體自由度濃縮至所對應的軌道元素自由度中，以得到車-軌互制元素，然後再以此元素為基礎，進行版式軌道系統之車-軌互制動力歷時分析。由本文結果顯示，當輪軌達到共振條件時，則鋼軌反應將會大幅放大。

關鍵詞：

無道碴軌道、高速鐵路、共振反應

Abstract

This report deals with the dynamic response and the contact phenomena of the slab track (ballastless track) systems of railways under the action of high-speed trains by considering the vehicle-track interaction. The vehicle-bridge interaction element developed by Yang and Yau (1997) is used. By modeling the train as a series of sprung masses, the track supported by the rail-pads as an Euler-Bernoulli beam supported by uniformly distributed springs, and the concrete slab with cement asphalt mortar (CA mortar) as a beam resting on an elastic (Winkler) foundation, the dynamic response of the track and the contact forces between the wheels and track can be computed. The result shows that when the vehicle speed meets the resonant condition, the dynamic response of the track and the wheel/track contact forces will be significantly amplified.

Keywords:

ballastless track, high speed trains, resonance.

1. Introduction

Since the commercial operation of the Japanese Shinkansen railway lines between Tokyo and Osaka in 1964, high-speed railway systems emerge as a convenient transportation tool for linking two long-distance cities. In the mean time, the dynamic interaction between the moving vehicles and tracks has brought a new challenge to the railway engineers. In the past two decades, a number of researchers devoted themselves to the investigation of the dynamics of rail mechanics (Newton and Clark, 1979; Grassie et al., 1982; Grassie and Cox, 1984; Fryba, 1987; Knothe and Grassie, 1993; Dong and Dukkipati, 1994; Luo et al., 1996; Igeland, 1996; Popp et al., 1999; Dukkipati and Dong, 1999; Zheng et al., 2000). Traditionally, a railway with ballasted tracks has been used for high-speed trains. However, because of the repeated action of high-speed trains moving at different speeds, the ballast layers often encounter permanent settlement due to the disaggregation and cracking of the constituting particles of the ballast and substrate. To overcome this problem, some new track systems, such as the ballastless track system, were developed in Germany (Pintag, 1989), France and Japan. Unlike the traditional ballast system, the balllastless track of the slab type is composed of a pair of rails, rail-pads, track slabs and cement asphalt mortar (CA mortar), as shown in Figure 1.

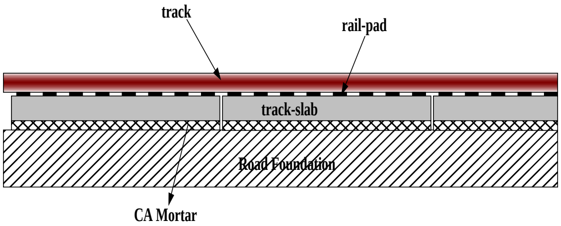


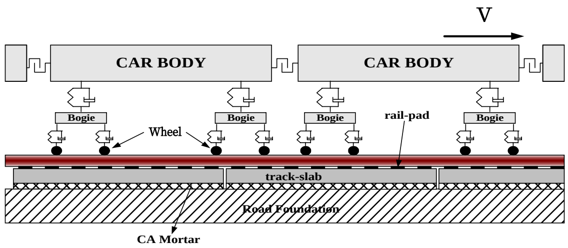
Figure 1 Ballastless track of the slab type

Conventionally, the ballast system has been modeled as an elastic (Winkler) foundation, and the rails in a pair supported by the ballast as an elastic beam resting on the elastic foundation. By so doing, numerous studies have been carried out for the dynamic response of a beam resting on an elastic foundation subjected to moving loads. To consider the contact effect between the moving wheels and the track using the finite element method, the track represented by a beam resting on a series of discrete pad-sleeper-ballast supports has been used to simulate the dynamic behavior of rails subjected to moving vehicles. According to the research results for the dynamic response of the ballasted track to moving trains, Dong and Dukkipati (1994) and Luo et al. (1996) pointed out that there exists a resonance peak related to the coupled wheel-rail resonance frequency.

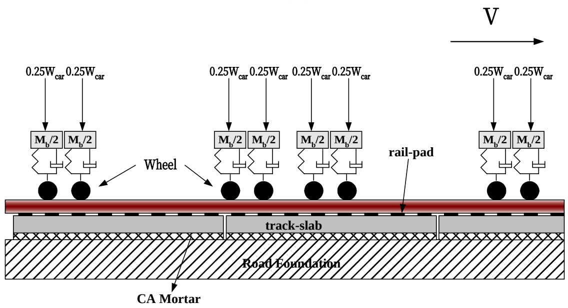
In contrast, concerning the dynamic response of railways with ballastless tracks due to moving trains, relatively few works have been conducted previously. In the present study, a finite element model taking into account the vehicle-track interaction developed by Yang and Yau (1997) will be employed to study the dynamic response of ballastless tracks of the slab type caused by high-speed trains. The numerical results indicate that the dynamic response of the track and the wheel/track contact forces will be significantly amplified when the speed of vehicle meets the resonant condition.

2. Formulation of the Problem

Because of the development of long-welded rails, researchers began to investigate the dynamic behavior of an infinite beam supported by an elastic foundation subjected to moving loads (Kenny, 1954; Keer, 1972; Chonan, 1975; Chen and Huang, 2000). Recently, to improve the passengers' comfort and to reduce the railway maintenance cost, some new ballastless track systems have been employed in high-speed rail transportation of Japan, Germany, and France. In this section, a finite element model of the track-slab type with long-welded rails lying on rail-pads will be used to study the dynamic response of ballastless track systems subjected to the action of moving vehicles that constitute a train.



(a)



(b)

Figure 2 Vehicle-track system:
(a) General model; (b) Lumped mass model

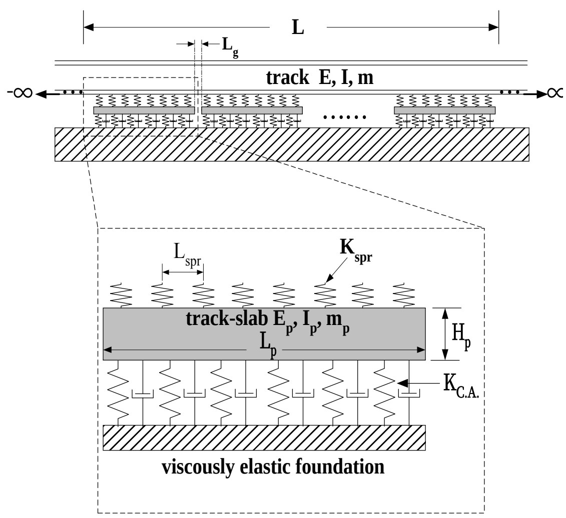


Figure 3 Ballastless track of the slab type

To simplify the finite element model used for the vehicle-track interaction problem, a vehicle is model as a sequence of moving sprung masses sustaining a concentrated load lumped from the weight of the car body. Only the vertical excitation of the track induced by the moving vehicles is considered herein. No consideration is made of rail irregularities and any nonlinear contact or separation between the wheels and the track.

Numerical Model of Track-Slab Systems

Figure 3 shows a physical model of the slab tracks used in this paper, in which a pair of long-welded rails supported by the rail-pads laid on the track-slabs is modeled as an infinite Euler-Bernoulli beam with flexural rigidities *EI* and mass *m* per unit length supported by discrete springs of equal spacing L_{spr} and linear stiffness K_{spr}, and the track-slab on the CA mortar as a beam element with flexural rigidity E_pI_p resting on a continuous viscoelastic foundation of stiffness $K_{C.A.}$ and viscous damping coefficient *C*. On the other hand, to simulate the boundary conditions of the long-welded rails, the two infinite ends of the ballastless track-slab system will be represented by the semi-infinite beam elements supported by viscoelastic foundations, as derived by Yin (1997). The following is a summary of the positive and negative semi-infinite elements derived by Yin (1997).

Negative semi-infinite element of track

Mass matrix: $$\left[m_r^-\right]=\begin{bmatrix}\frac{3m}{4\lambda} & \frac{-3m}{4\lambda^2}\\ \frac{-3m}{4\lambda^2} & \frac{m}{8\lambda^3}\end{bmatrix} \quad (1a)$$

Damping matrix: $$\left[c_r^-\right]=\begin{bmatrix}\frac{c_d}{4\lambda} & \frac{-c_d}{4\lambda^2}\\ \frac{-c_d}{4\lambda^2} & \frac{c_d}{8\lambda^3}\end{bmatrix} \quad (1b)$$

Stiffness matrix:

$$\left[k_r^-\right]=\begin{bmatrix}\frac{3k}{4\lambda}+EI\lambda^3 & -\left(\frac{k}{4\lambda^2}+EI\lambda^2\right)\\ -\left(\frac{k}{4\lambda^2}+EI\lambda^2\right) & \frac{k}{8\lambda^2}+\frac{3}{2}EI\lambda\end{bmatrix} \quad (1c)$$

Positive semi-infinite element of track

Mass matrix: $$\left[m_r^+\right]=\begin{bmatrix}\frac{3m}{4\lambda} & \frac{3m}{4\lambda^2}\\ \frac{3m}{4\lambda^2} & \frac{m}{8\lambda^3}\end{bmatrix} \quad (2a)$$

Damping matrix: $$\left[c_r^+\right]=\begin{bmatrix}\frac{c_d}{4\lambda} & \frac{c_d}{4\lambda^2}\\ \frac{c_d}{4\lambda^2} & \frac{c_d}{8\lambda^3}\end{bmatrix} \quad (2b)$$

Stiffness matrix:

$$\left[k_r^+\right]=\begin{bmatrix}\frac{3k}{4\lambda}+EI\lambda^3 & \left(\frac{k}{4\lambda^2}+EI\lambda^2\right)\\ \left(\frac{k}{4\lambda^2}+EI\lambda^2\right) & \frac{k}{8\lambda^2}+\frac{3}{2}EI\lambda\end{bmatrix} \quad (2c)$$

where c_d = the viscous damping coefficient, *k* = the stiffness coefficient of the elastic foundation, and $\lambda = (k/4EI)^{1/4}$.

Vehicle-Track Interaction Element

A typical vehicle model is shown in Figure 2(a), in which a vehicle of weight W_{car} is represented as a car body resting on two bogies. In practice, concerning the riding comfort of trains moving at high speeds, there always exists an isolation device between the bogies and the car body, so as to reduce the feedback response exerted by the bogies. Therefore, for the vehicle-track interaction problem shown in Figure 2(b), a train traveling over the rails in pair is idealized as a series of lumped masses sustaining one quarter of the car body weight and supported by the suspension units interconnected by an un-sprung mass to represent the wheel load. To analyze the dynamic response of the rails caused by the moving train, the track is represented by a number of beam elements. At certain instant during the passage of the train over the track, some elements of the track will be directly acted upon by the sprung masses, while the others are not. The number of vehicles directly acting on the track changes as the train moves, so do the contact points between the track and moving vehicles. Following Yang and Yau (1997), a beam element that is directly under the action of a sprung mass and dashpot system is referred to as the vehicle-track interaction element in this study (see Figure 4). By the concept of dynamic condensation, the degrees of freedom (DOFs) of the sprung mass are condensed to the associated DOFs of the beam element directly in contact, after the former were discretized by Newmark's finite difference formulas. The will result in a vehicle-track interaction element which possesses the same number of DOFs as the parent element, while the properties of symmetry and bandedness are preserved.

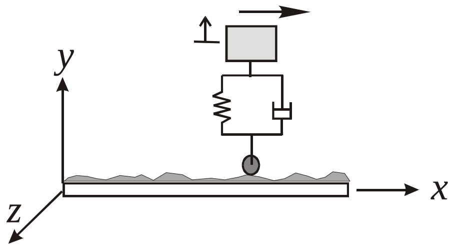


Figure 4 Vehicle-track interaction element

Typically, a beam element will be acted upon by a sprung mass, as shown in Figure 4. For this element, two sets of equations of motion can be written, one for the track element and the other for the sprung mass:

$$[m_b]\{\ddot{u}_b\}+[c_b]\{\dot{u}_b\}+[k_b]\{u_b\}=\{p_b\}-\{f_c\}$$
$$[m_v]\{\ddot{u}_v\}+[c_v]\{\dot{u}_v\}+[k_v]\{u_v\}=\{p_v\}+\{f_c\} \quad (3a, b)$$

where $[m_b]$, $[c_b]$, $[k_b]$ = the mass, damping, and stiffness matrices of the beam element, and $\{p_b\}$ and $\{f_c\}$ = the external nodal loads and the contact forces existing between the sprung mass and the beam element; $[m_v]$, $[c_v]$, $[k_v]$ = the mass, damping, stiffness matrices of the sprung mass, and $\{p_v\}$ = the weight lumped from the car body of the vehicle. The preceding two equations (3a, b) are coupled through the contact forces $\{f_c\}$, while the coefficient matrices of the sprung mass vary according to its acting position on the track. To overcome the time-varying nature of the problem, Yang and Yau (1997) proposed a method for condensing the DOFs of the sprung mass into those of the element in contact, after the sprung mass equations are discretized in advance by Newmark's finite difference formulas. The result is the vehicle-track interaction element desired. Such an element is particularly suitable for analyzing the dynamic response of the vehicle-track interaction problems concerning both the track and vehicle responses. Readers who are interested in derivation of this element should refer to the paper by Yang and Yau (1997) for further details.

Because the vehicle-track interaction element and its parent element are fully compatible, conventional element assembly process can be applied with no difficulty to forming the equations of motion for the entire vehicle-track system, that is,

$$[M]\{\ddot{U}_b\}+[C]\{\dot{U}_b\}+[K]\{U_b\}=\{P_b\} \quad (4)$$

where $[M]$, $[C]$, $[K]$ respectively denote the mass, damping, and stiffness matrices of the entire vehicle-track system, $\{U_b\}$ the track displacements, and $\{P_b\}$ the external loads acting on the track. The preceding equations are typical second-order differential equations, which can be solved by a number of time-marching schemes. In this study, the Newmark β method with constant average acceleration is employed to render the preceding equations into a set of equivalent stiffness equations, from which the track displacements $\{U_b\}$ can be solved for each time step. Once the track displacements $\{U_b\}$ are made available, the track accelerations and velocities can be computed accordingly. By a backward procedure, the response of the sprung masses can be computed as well on the element level, which serves as an indicator of the riding comfort (Yang and Yau, 1997).

3. Loaded Track Resonant Speed

According to the research result of Dong and Dukkipati (1994), the loaded track frequency $f_{w/t}$ of a coupled wheel-track system on an elastic foundation of uniform stiffness can be approximately estimated by

$$f_{w/t} = \frac{1}{2\pi}\sqrt{\frac{k_{tr}}{m_{tr}+m_w}}, k_{tr} = 2\sqrt[4]{4EI \times k_f^3},$$ (5a-c)

$$m_{tr} = 3m \times \sqrt[3]{EI / k_{tr}}$$

where k_{tr} and m_{tr} are the effective stiffness and mass respectively, and k_f is the equivalent stiffness per unit length of the elastic foundation. For a moving wheel-load system with constant speed V travelling along a track supported by discrete rail-pads of constant intervals, the excitation frequency f_{ext} to the wheel-load system due to the discrete rail-pads is

$$f_{ext} = V / L_e \quad (6)$$

wehre L_e = the effective spacing between two adjacent rail-pads.

When the excitation frequency equals the loaded track frequency, resonance can be excited between the bogies and the rails. By setting Equation (5a) equal to Equation (6), the resonant speed V_{res} can be solved as

$$V_{res} = \frac{L_e}{2\pi}\sqrt{\frac{k_{tr}}{m_{tr}+m_w}} \quad (7)$$

It is expected that under the condition of resonance, the response of the track will be built up as there are more vehicles passing the track.

4. Impact Factor

In design practice, the impact factor I is used to account for the amplification effect on the response of the track or rails due to the passage of the moving vehicles through increase of the design forces and stresses. The impact factor is defined as follows (Yang and Lin, 1995):

$$I = \frac{R_d(x) - R_s(x)}{R_s(x)} \quad (8)$$

where $R_d(x)$ and $R_s(x)$ respectively denote the maximum dynamic and static responses of the beam at position x due to the action of the moving loads.

5. Numerical Example

To investigate the dynamic response of the ballastless track system of the slab type due to moving vehicles, let us consider a single sprung mass carrying one quarter of the vehicle weight, that is $W_{car}/4$, and moving along the ballastless track of the slab type, as shown in Figure 5. The following are the properties adopted for this example:

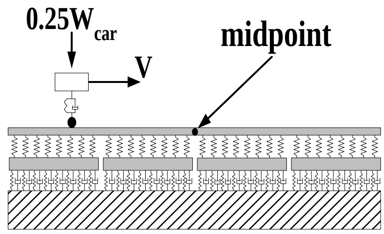


Figure 5 Single sprung mass moving over ballastless track

Sprung mass system:
W_{car}=40.75t, M_b=3.04t, L_b=2.5m, k_s= 1180 kN/m, c_s=39.2kN-s/m, m_w=1.78 t.

Ballastless track system:
L=81.55m, EI=1.26x10^{4}kN-m^2, m= 60kg/m, L_{spr}=0.6m, K_{spr}=1.177x10^5 kN/m/m, E_pI_p=2.06x10^{7}kN-m^2, m_p=1067 kg/m, L_p=4.8 m, $K_{C.A}$=2.823x10^6 kN/m^2, C=0.

Semi-infinite element:
$k = k_{tr}$, $m = m_{tr}$, $c_d = 0$.

Based on the finite element analysis of the vehicle-rail interaction problem as mentioned in the previous section, the impact factor I solved for the midpoint of the track and the increase rate of the contact force between the wheel and the track have been plotted with respect to the train speed V in Figures 6 and 7, respectively. Here, the increase rate of the contact force between the wheel and the track is defined as the ratio of maximum dynamic to static contact force minus one. As can be seen, there exist multiple resonant peaks for the impact response and contact force of the track. This is mainly due to the coincidence of some of the excitation frequency f_{ext} implied by the moving sprung mass model at different speeds with the coupled frequency $f_{w/t}$ of the wheel/track/rail-pad system. In the present study, the effective spacing L_e of the rail pads can be expressed as $nL_{spr}|_{n=1,2,3...}$ as they are regularly distributed. By substituting the data assumed for the sprung mass and the ballastless track system into Equations (5) and (7), the resonant speeds can be computed as 37, 74, 111 m/s, which are consistent with the resonant peaks shown in Figures 6 and 7.

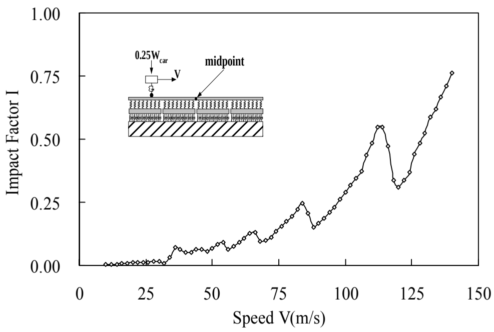


Figure 6 Impact response of track

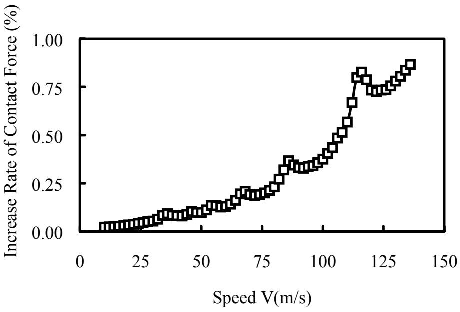


Figure 7 Increase rate of wheel/track contact force

6. Concluding Remarks

Based on the vehicle-track interaction analysis by the finite element procedures, the impact response and the wheel/track contact force of ballastless track of the slab type due to high-speed trains is investigated. The numerical results indicate that there exist multiple resonant peaks for the impact response and wheel/track contact force of the ballastless track system of slab type, due to coincidence of the inherent frequencies of the constituting subsystems.

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