行政院國家科學委員會專題研究計畫　成果報告

梁式結構受高速移動載重之動力反應分析(I)

計畫類別：個別型計畫
計畫編號：NSC91-2211-E-032-018-
執行期間：91 年 08 月 01 日至 92 年 07 月 31 日
執行單位：淡江大學建築技術系

計畫主持人：姚忠達

計畫參與人員：劉國瑞、陳韻如

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行政院國家科學委員會補助專題研究計畫成果報告

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※ 梁式結構受高速移動載重之動力反應分析(第一年）

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計畫類別：■個別型計畫　　□整合型計畫

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中　華　民　國　九十二　　年　八　　月　一　　日

行政院國家科學委員會專題研究計畫成果報告

梁式結構受高速移動載重之動力反應分析【第一年】

Dynamic response analysis of beam-type structures to moving loads with high speeds(I)

計畫編號：NSC 91-2211-E-032-018

執行期限：91 年 8 月 01 日至 92 年 7 月 31 日

主　持　人：姚忠達　　　　淡江大學建築技術系

計畫參與人員：劉國瑞、陳韻如　台灣大學土木工程研究所

中文摘要

根據過去數年的高鐵橋樑受高速列車作用的動態反應顯示，高鐵橋樑將會因爲列車輪軸距的軌則排列施力現象而產生共振反應，更進而影響到橋樑安全及列車行車品質。然而就鐵路設計而言，除了需考慮垂直自重、地震力、風力及車體活載外，也需要考量車行時所引起的衝擊反應，對此衝擊反應的涵蓋，在鐵路橋樑設計中，係以一衝擊係數(I)來作爲車體活載的額外放大倍數(1+I)。然誠如先前所言，當車速達到共振速度的情況，在現行大部分的鐵路橋（尤以高速鐵路橋樑）規範中，卻甚少提及。對此，本研究將利用含阻尼彈性結構系統在共振諧和力作用情況下，其穩態反應仍具有上限的特性，考慮在無限個移動力量情形，於共振車速時，進行鐵路橋衝擊反應的簡化公式推導。然後，並輔以有限元素之數值計算結果，以驗證本研究之鐵路橋衝擊公式的合理性。

關鍵詞：

鐵路橋衝擊公式、高鐵橋、衝擊反應

Abstract

In this report, the dynamic response of bridge girders with simple supports to moving train-loads is studied using an analytical approach. The present results indicate that the dynamic response of the beam at resonance remains rather bounded, if the effect of damping is taken into account. An envelope impact formula is proposed for the deflection of the girder with light damping, which serves as a useful and preliminary design aid to railway engineers.

Keywords:
impact envelope formula, high speed trains, resonance.

1. Introduction

The dynamic response of bridge structures to moving loads at high speeds is a problem of great concern in the design of high-speed railway bridges. In the literature, a larger number of analytical investigations have been carried out. Frequently, a bridge has been modelled as a beam-like structure and a vehicle as a moving load or moving mass [1]. Recently, Yang et al. [2] presented a closed form solution for the dynamic response of simple beams subjected to a series of moving loads at high speeds, in which the phenomena of resonance and cancellation have been investigated, along with optimal design criteria proposed. By considering the effect of damping, Li and Su [3], Yau et al. [4] investigated the fundamental characteristics and dominant factors for the resonant vibration of a girder bridge under high speed trains. The objective of this paper is to analytically investigate the dynamic behaviour of simple beams subjected to moving loads in the high speed range. Based on the analytical results, an envelope impact formula that takes into account the effect of damping will be proposed for the deflection of the beam. The accuracy of such a formula will be demonstrated in the numerical examples through comparison with the finite element solutions.

2. Equation of Motion

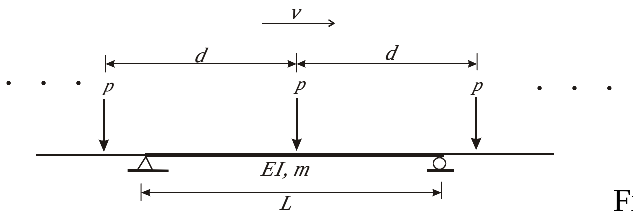

Figure 1 Simple Beams Subjected to Moving Train Loads

As shown in Figure 1, by modelling a bridge as a Bernoulli-Euler beam, a simple beam with length L and uniform cross section is considered. The train moving over the beam at speed v is modelled as a sequence of equidistant length d and moving loads with constant weight p. The equation of motion for the beam travelled by the moving loads can be written as [2]:

$$
\begin{aligned}
& m\ddot{u} + c\dot{u} + EIu'''' \\
& = p\sum_{k=1}^{N} ¢\left[x - v(t - t_k)\right] \times \left[H(t - t_k) - H(t - t_k - L/v)\right]
\end{aligned}
$$

(1)

where m = the mass per unit length of the beam, $u(x, t)$ = vertical displacement, c = damping coefficient, E = elastic modulus, I = moment of inertia of the beam, $¢$= Dirac's delta function, $H(t)$=unit step function, N = total number of moving loads, and t_k = (k!1)d/v = arriving time of the kth load at the beam. By assuming the displacement $u(x,t)$ in equation (1) to be $q(t)\sin(\pi x/L)$, according to the principle of virtual work, one obtains the equation of motion in terms of the generalized coordinate $q(t)$ as

$$
\begin{aligned}
& \ddot{q}(t) + 2¶\mu\dot{q}(t) + \mu^2 q(t) = \frac{2p}{mL}\sum_{k=1}^{N} F_k(v,t) \\
& F_k(v,t) = \sin\Omega(t - t_k)H(t - t_k) \\
& \qquad + \sin\Omega(t - t_k - L/v)H(t - t_k - L/v)
\end{aligned}
$$

(2a, b)

where ¶ is the modal damping ratio, ω = $(\pi/L)^2\sqrt{}(EI/m)$ = the fundamental frequency of the simple beam, Ω (= ®v/L) is the driving frequency by the moving loads, and $F_k(v, t)$ is the generalized forcing function.

3. Resonance and Cancellation

To simplify the derivation procedure, readers who are interested in derivation of the resonant response should refer to the paper by the writers [2] for further details, which will not be recapitulated herein. By neglecting the damping effect in equation (2), the closed form solution of the dynamic response $u(x, t)$ in equation (1) is expressed as:

$$
\begin{aligned}
& u(x,t) \square\ \Delta_{st}\sin(®x/L)\times \\
& \qquad \left[Q_1(v,t)H(t - t_N) + Q_2(v,t)H(t - t_{N-1} - L/v)\right]
\end{aligned}
$$

(3)

where $\Delta_{st} = 2pL^3/(\circledR^4 bf\text{-})$. the maximum static deflection of the simple beam, and

$$
Q_1(v,t) = \frac{\sin\Omega(t-t_N) - S\sin\mu(t-t_N)}{1-S^2}
$$

$$
Q_2(v,t) = \frac{-2S\cos(\circledR/2S)}{1-S^2} \times \left[\sin\mu\left(t-\frac{L}{2v}\right)\right.
$$

$$
\left. +\sin\mu\left(t-\frac{t_N+L/v}{2}\right) \times \frac{\sin\mu[(t_N-d/v)/2]}{\sin(\mu d/2v)}\right]
$$

(4a, b)

In equation (4), $S = \Omega/\mu$ = the speed parameter. From equations (4b), it can be seen that the response reaches a maximum when the denominator $\sin(\omega d/2v)$ equals zero. This is exactly the condition for *resonance* to occur, and a closed form solution for the resonant response $u_{res}(x,t)$ of the simple beam can be written as

$$
u_{res}(x,t) = \frac{\Delta_{st}}{1-S_{res}^2}\sin(\circledR x/L) \times
$$

$$
\left\{\left[\sin\Omega(t-t_N) - S_{res}\sin\mu(t-t_N)\right]H(t-t_N)\right.
$$

$$
-2(N-1)S_{res}\cos(\mu L/2v_{res})
$$

$$
\left.\times\sin\mu(t-L/2v_{res})H(t-t_{N-1}-L/v_{res})\right\}
$$

(5)

where the subscript *res* means resonance. As can be seen, under the condition of resonance, larger response will be induced on the beam, as there are more loads passing the beam. On the other hand, whenever the cancellation condition is met, that is, $\cos(\omega L/2v) = 0$, the dynamic response for cancellation $u_{can}(x,t)$ becomes

$$
u_{can}(x,t) = \Delta_{st}\sin\left(\frac{\circledR x}{L}\right) \times
$$

$$
\left[\frac{\sin\Omega(t-t_N) - S_{can}\sin\mu(t-t_N)}{1-S_{can}^2}\right]H(t-t_N)
$$

(6)

where the subscript *can* means resonance. This implies that the excitation effects of all the previous N-1 moving loads sum to zero.

4. Derivation of Impact Envelope Formula

In order to derive the impact envelope formula of simple beam subjected to moving loads at high speeds, first, consider the case when only a single moving load is crossing the bridge. The equation of motion (2) becomes

$$\ddot{q}(t)+2\P\mu\dot{q}(t)+\mu^{2}q(t)=\frac{2p}{mL}\sin\frac{\circledR vt}{L} \qquad (7)$$

For most of the vehicle-bridge problems encountered in practice, the speed parameter S is less than 0.3. In this study, only simple beams with light damping ($\P$ <0.05) are considered, which implies that terms involving $\P^2$ and $\P S$, can be neglected. As a result, the response in equation (7) can be reduced to

$$q(t)=\Delta_{st}\left(\frac{\sin\Omega t-Se^{-\P\mu t}\sin\mu t}{1-S^{2}}\right) \quad 0\le vt\le L \quad (8)$$

Further, when a series of moving loads of constant intervals d are crossing the bridge as shown in Figure 1, the response in equation (8) can be extended as follows:

$$\begin{aligned} q(t) \square \frac{\Delta_{st}}{1-S^{2}}\sum_{k=1}^{N}\Big[&G_1(v,t-t_k)H(t-t_k) \\ &+G_1\left(v,t-t_k-\frac{L}{v}\right)H\left(t-t_k-\frac{L}{v}\right)\Big] \quad (9a, b) \end{aligned}$$
$$G_1(v,t)=\sin\Omega t-Se^{-\P\mu t}\sin\mu t$$

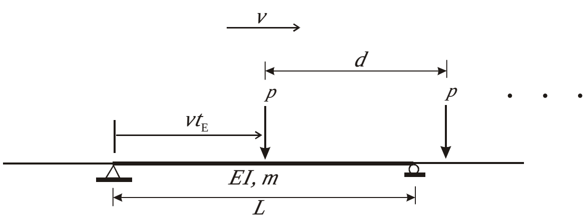

Figure 2 The Nth moving load is acting at position vt_E

Consider the resonance condition in equation (4b), i.e., $\sin(\mu d/2v_{res}) = 0$, as shown in Figure 2, when the Nth moving load is acting on the beam at time $t = t_E + t_N$, i.e., $t = t_E + (N\,!1)d/v_{res}$. The resonant response in equation (9) for the beam under the action of the Nth moving load can be expressed as

$$q_{res}(t_E)=\frac{\Delta_{st}}{1-S_{res}^2}\Big\{\left(\sin\Omega t_E - S_{res}e^{-\xi\mu t_E}\sin\mu t_E\right)H(t_E)$$
$$-S_{res}e^{-\xi\mu(t_E+t_N)}\sum_{k=1}^{N-1}e^{\xi\mu t_k}\left[\sin\mu t_E + e^{\xi\pi/S_{res}}\sin\mu\left(t_E-\frac{L}{v_{res}}\right)\right]\Big\}$$

(12)

Further, by the approximation in expansion for the exponential function, $\exp(\xi\pi/S_{res}) \approx 1+\xi\pi/S_{res}$ for $\xi\pi/S_{res} \leq 0.3$, and series sum, and let us assume that there is an infinite number of moving loads crossing the beam, i.e., $N \to \infty$, the dynamic response in equation (12) can further be expressed as

$$q_{res}(t_E)=\frac{\Delta_{st}}{1-S_{res}^2}\Big\{\left(\sin\Omega t_E - S_{res}e^{-\xi\mu t_E}\sin\mu t_E\right)H(t_E)$$
$$-\frac{S_{res}L}{d}e^{-\xi\mu t_E}\times\left[\left(\frac{2S_{res}}{\xi\pi}+1\right)\cos\frac{\pi}{2S_{res}}\sin\left(\mu t_E-\frac{\pi}{2S_{res}}\right)\right.$$
$$\left.+\sin\frac{\pi}{2S_{res}}\cos\left(\mu t_E-\frac{\pi}{2S_{res}}\right)\right]H\left(t_E-\frac{L-d}{v_{res}}\right)\Big\}$$

(13)

Since $\sin(\omega t - \pi/2S)$ and $\cos(\omega t - \pi/2S)$ are out of phase, when the function $\sin(\omega t - \pi/2S)$ reaches the maximum, the function $\cos(\omega t - \pi/2S)$ is at its minimum. Also, since $2S/\xi\pi+1>2$ and we are interested only in the maximum response, the preceding expression can be approximated by dropping the term in the third line as follows:

$$q_{res}(t_E)\approx\frac{\Delta_{st}}{1-S_{res}^2}\times\Big\{\left(\sin\Omega t_E - S_{res}e^{-\xi\mu t_E}\sin\mu t_E\right)H(t_E)$$
$$-\frac{S_{res}L}{d}e^{-\xi\mu t_E}\left(\frac{2S_{res}}{\xi\pi}+1\right)\cos\frac{\pi}{2S_{res}}$$
$$\times\sin\left(\mu t_E-\frac{\pi}{2S_{res}}\right)H\left(t_E-\frac{L-d}{v_{res}}\right)\Big\}$$

(14)

At this point, we have derived the maximum response for the simply supported beam in equation (14) considering the effect of damping. To obtain the maximum response in equation (14), by letting ωt_E be equal to $\pi(1+1/S_{res})/2$, the maximum response in equation (14) can be reduced as

$$q_{res,\max}\approx\frac{\Delta_{st}}{1-S_{res}^2}\times\Big\{\left|\cos\left(\frac{S_{res}\pi}{2}\right)\right|+S_{res}\left|\cos\frac{\pi}{2S_{res}}\right|$$
$$\times\left[\frac{L}{d}\left(\frac{2S_{res}}{\xi\pi}+1\right)+1\right]e^{-\xi\frac{\pi(1+S_{res})}{2S_{res}}}\Big\}$$

(15)

For the case of light damping considered in this study, implying that ¶®/S_{res} < 0.3, the maximum response in equation (15) can be further approximated as

$$q_{res,\max} \approx \Delta_{st} \times \left(1 + \frac{2L}{\text{¶®}d}\left|\cos\frac{\text{®}}{2S_{res}}\right|\right) \qquad (16)$$

which is applicable for the case where the resonance condition is met, but the cancellation condition is not. To consider such a situation, when both the resonance condition and the cancellation condition given in equation (16) are satisfied, i.e., $S_{res} = S_{can}$, the response in equation (6) becomes

$$q_{res}(t_E) = \Delta_{st} \times \left[\frac{\sin\Omega t_E - S_{can}e^{-\text{¶}\mu t_E}\sin\mu t_E}{1 - S_{can}^2}\right] H(t_E)$$

(17)

By letting sinΩt_E=1 and using the relations exp(-⁄_/2S_{can}).1-⁄_/2S_{can} and S_{can}/(1- $S^2{}_{can}$) . S_{can}, the maximum response in equation (17) can be represented as

$$q_{res,\max} \approx \Delta_{st} \times \left[\frac{1}{1 - S_{can}^2} + \left(S_{can} - \frac{\text{¶®}}{2}\right)\right] \qquad (18)$$

This formula is valid for the case when both the condition of resonance and the condition of cancellation are satisfied.

The impact factor for the deflection of a simply supported beam subjected to the moving loads is defined as the ratio of the maximum dynamic to the maximum static response of the bridge under the same load minus one. By the use of equations (16) and (17), the deflection impact formula for the simple beam subjected to a sequence moving loads can be expressed as:

$$I = \begin{cases} \dfrac{L}{d}\dfrac{2S_{res}^2}{\text{¶®}}\left|\cos\left(\dfrac{\text{®}}{2S_{res}}\right)\right| & \textit{for resonance} \\ \dfrac{S_{can}^2}{1 - S_{can}^2} + \left(S_{can} - \dfrac{\text{¶®}}{2}\right) & \textit{for}\ (\textit{resonance} + \textit{cancellation}) \end{cases}$$

(19)

This is exactly the envelope impact formula for the deflection of the simple beam subjected to the moving loads.

5. Illustrated Example

As shown in Figure1, the bridge length is 23m and the train moving over the bridge is assumed to have interval length d = 25m. To investigate the effect of damping on the resonant response of an elastically supported beam due to an infinite series of moving loads, 30 moving loads are considered in this example. A damping ratio ξ of two percent and the resonant speed parameter S_{res} of $d/4L$ are assumed for the beam. As can be seen from Figures 3, due to the presence of damping, the vibration of the beam remains rather bounded, in a steady state manner, even when the resonance condition is met. This is very different from the undamped case, in which the response amplitude tends to grow increasingly when the resonance condition is met, as there are more loads passing the beam.

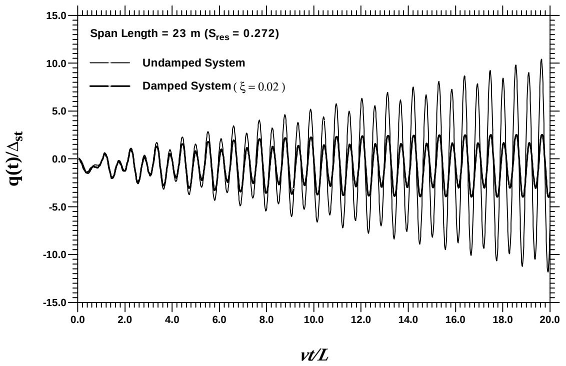

Figure 3 Comparsion of time history responses at resonant speed $S_{res} = d/4L$

The envelope impact formula of equation (19) has been plotted in Figure 4 for the simple beam with two different damping ratios, i.e., */* = 0.02, 0.04, in comparison with the more accurate impact factors I computed using the finite element method. As can be seen, the envelope impact formula shows a trend in good consistency with the impact response for different ratios of damping for the entire range of speed parameters considered.

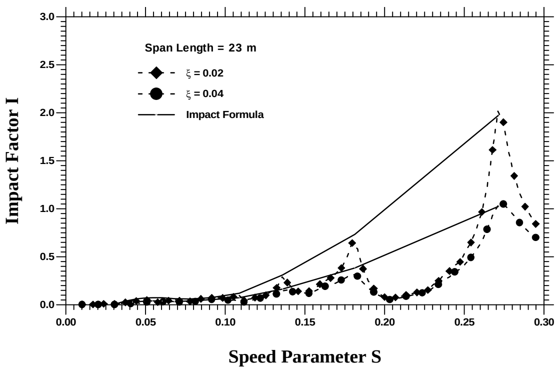

Figure 4 Envelope Impact Formula for Different Damping Ratios

6. Conclusions

In this study, an analytical approach is adopted to investigate the envelope impact formula of simple beams subjected to a sequence of moving loads. Light damping is assumed for the beam. Both the conditions of resonance and cancellation are identified. It is observed that the resonant responses remain very well bounded due to the presence of damping. For the case of infinite number of moving loads, an envelope impact formula is derived for the simple beam with the effect of damping taken into account.

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