行政院國家科學委員會補助專題研究計畫成果報告

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執行期間：　90 年 8 月 01 日至　91 年 7 月 31 日

計畫主持人：姚忠達　　淡江大學建築技術系

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中　華　民　國　九十一　　年　八　　月　十五　　日

行政院國家科學委員會專題研究計畫成果報告

高速列車通過連續橋之動力互制與減振分析

Vibration reduction and dynamic interaction of continuous bridges due to high speed trains

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1. 中文摘要

本研究旨在探討等跨連續橋受高速列車作用之車橋動力反應及減振分析，首先，藉由將連續橋簡化成連續梁、列車則視為集中型序列移動懸吊質量系統的力學模擬，吾人可以來了解連續橋跨數的變化與其頻率分佈的關係，進而推求出橋梁衝擊反應發生共振時的主要共振速度分佈情形。另一方面，為了抑制連續橋受高速列車作用可能引發的多個共振尖峰之衝擊反應的發生，本研究建議採用以連續梁受激發之自然振態為減振導向的「混合式最佳化調諧質量阻尼」減振分析方法，並藉由實例說明本研究的實用性及可行性。

關鍵詞：

調諧質量阻尼、連續鐵路橋、衝擊反應

Abstract

Concerning the minimization of cross sections for superstructures of bridges and the maximization of riding comfort for passengers, continuous bridges have the advantages of economy in construction materials and of less impact response caused by traveling vehicles. However, due to the existence of clustered frequencies of vibration, there may occur multi resonant peaks in the impact response of continuous bridges under the passage of high speed trains. To overcome this problem, multiple tuned mass dampers (MTMD) will be employed to reduce the train-induced vibrations of the continuous beam. To this end, a continuous bridge is modeled as a number of beam elements and a train as a sequence of sprung masses. The vehicle-bridge interaction (VBI) element derived previously by the writers based on the dynamic condensation technique is employed to simulate the interaction effect of the sprung masses. It is confirmed that the MTMD system can effectively suppress the main resonant peaks of the continuous bridge under the moving train loads.

Keywords:
Continuous Beams, Vehicle-Bridge Interaction, Multiple TMD, Impact Response

2. Introduction

The dynamic response of bridges caused by the passage of moving loads has been studied by many researchers. Recently, the writers have conducted a series of studies on the impact response of bridges to high-speed trains with emphasis on the vehicle-bridge interactions (VBI) [1-4]. By an analytical approach, the phenomena of resonance and cancellation of a simply supported beam subjected to a series of moving loads were investigated, along with the optimum design criteria for railway bridges proposed [2]. Aimed at computing the vehicle response, in addition to the bridge response, some VBI elements of various complexities were developed for simulating the interaction behavior between the bridge and traveling vehicles [1,3].

The dynamic interaction between the bridges and traveling trains is an issue of great concern in the design of high speed railways. From the impact factor (I) vs. speed parameter (S) plot of the continuous beam to moving loads, it is observed that there exist more than one resonant peaks, and that all these peaks relate to the higher frequencies of vibration of the beam, indicating that this is a multi-mode vibration problem [5]. For the sake of increasing structural safety and riding comfort of high speed railways, countermeasures should be considered to reduce the dynamic responses of the bridges and traveling vehicles. In order to meet the stringent design requirements for high speed railways, in certain situations it is necessary to install some vibration control devices, such as the tuned mass dampers (TMDs), on the bridge so that its dynamic response can be kept below the tolerance limit during the passage of high speed trains. To suppress the train-induced vibrations on the continuous bridge, multi tuned mass dampers (MTMD) of different arrangements aimed at counterbalancing the multi vibration modes are investigated.

The objective of this report is to present a procedure for analyzing the response of continuous bridges installed with TMD devices, using the VBI elements previously derived by the writers [1,4], and to investigate the effectiveness of different MTMD arrangements in reducing the multi mode vibrations [6].

3. Optimization of TMD Parameters

Let us consider a train that is crossing a railway bridge, as shown in Figure 1. Because of their regular, repetitive nature, the moving wheel loads of the train can be regarded essentially as a set of harmonic forces. In Reference [2], it has been demonstrated that whenever any of the excitation frequencies of the moving loads coincides with any of the frequencies of the bridge, resonance will occur on the bridge, in the sense that the response of the bridge will increase as there are more wheels passing the bridge. To reduce the resonant response of the bridge due to vehicles moving at high speeds, vibration absorbers such as the TMD devices or MTMD systems have been used [6, 7].

For the present purposes, consider a TMD attached to a single-DOF mass *M*, as shown in Figure 2, that is subjected to a sinusoidal force of amplitude *P* and frequency ø u The equations of motion for this 2-DOF system can be expressed as

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} C+c & -c \\ -c & c \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} + \begin{bmatrix} K+k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} P\sin \mu t \\ 0 \end{Bmatrix} \quad (4)$$

where y_1 and y_2 are the displacements of the main mass and absorber, respectively, *M*, *C*, and *K* are the mass, damping coefficient, and spring stiffness of the main mass, and *m* , *c*, and *k* are the mass, damping coefficient and spring stiffness of the TMD. Let Ù and $ù_t$ denote the frequencies of vibration of the main mass and absorber, respectively, i.e., $\Omega = \sqrt{K/M}$ and $\mu_t = \sqrt{k/m}$. Also, define the following parameters: *f* = the tuning frequency ratio $ù_t$/Ù, *j* = the exciting frequency ratio ù/Ù, ì = the tuning mass ratio *m/M*, $\P = C/(2M\Omega)$ = the damping ratio of the main mass, and $s = c/(2m\mu_t)$ = the damping ratio of the TMD. The dynamic amplification factor (*DAF*) for the steady-state response of the main mass is given by [8-10]

$$DAF = \frac{y_1}{P/K} = \sqrt{\frac{A^2+B^2}{C^2+D^2}}$$

$$A = \left(1 - \frac{j^2}{f^2}\right) \quad B = \left(\frac{2s\,j}{f}\right)$$

$$C = \left[\frac{j^4 - j^2}{f^2} - j^2(1+\text{«}) - 4\frac{s\P j^2}{f} + 1\right]$$

$$D = 2j \times \left[\frac{j^2[\P + s(1+\text{«})] - s}{f} - \P\right] \quad \text{(5a-e)}$$

In a practical design, a small mass ratio ì (from 0.01 to 0.04) is usually adopted, and the primary damping ratio î of the structure is selected according to the material used. To obtain optimal absorber parameters for the TMD, the minimum-maximum dynamic amplification factor criterion [8-10] is adopted in this study, by which a TMD is optimized such that the maximum response amplitude of the main mass is minimized. Given the damping ratio î of the main mass and the tuning mass ratio ì , by fixing a tuning damping ratio ζ of the TMD, one may plot a curve for the *DAF* values for a specific tuning frequency ratio *f* throughout the entire range of the exciting frequency ratio *j* considered, from which the maximum peak response $(DAF)_{max}$ can be obtained. By changing the tuning damping ratio ζ, another maximum peak response for the $(DAF)_{max}$ value can be constructed. By comparing all these maximum peak responses for different tuning frequency ratios *f* and tuning damping ratio ζ and selecting the minimum value of all the maximum peak responses $(DAF)_{max}$, the optimal absorber parameters (f_{opt}, æ$_{opt}$) can be identified for a given set of structural damping ratio î and tuning mass ratio ì .

4. Properties of Continuous Beams

Multi-span continuous bridges of uniform spans are one type of structures commonly used in practice, because they can be constructed in a rather systematic way. As shown in Figure 3(a), a three-span continuous bridge with uniform spans and simple supports is considered in this study. Based on an eigenvalue analysis, the first three vibration modes of the continuous beam have been plotted in Figure 3(b). In Table 2, the natural frequencies ø a solved for the three-span continuous beam have been compared with those of a simple beam that has a length equal to the span length of the continuous beam, both with reference to the fundamental frequency ø a of the simple beam, i.e.,

$$\mu_1 = \left(\frac{®}{L}\right)^2 \sqrt{\frac{EI}{m_0}} \qquad (6)$$

where L is the span length, EI the flexural rigidity, and m_0 the mass per unit length of the beam. As can be seen from Table 2, the *fundamental* frequency of the uniform three-span continuous beam is the same as that of the simple beam. However, the distribution of frequencies becomes much denser than those of the simple beam. Such a phenomenon implies that the higher modes of vibration of the continuous beam can be more easily excited by vehicles of the same driving frequencies or cruising speeds, compared with those of the simple beam.

Table 1 Comparison of natural frequencies

Beam Type	Modes of Vibration			
	1st	2nd	3rd	4th
Simple Beam (ω/ø 1)	1	**4**	9	16
3-Span Continuous Beam (ω/ø 1)	1	**1.28**	**1.87**	**4**

5. Vehicle-bridge System with MTMD Devices

Consider a bridge structure that is in resonance under the passage of a train. Due to the resonance effect, the bridge may undergo rather large vibrations, which may affect the safety of the bridge itself and the riding comfort and controllability of the travelling train [4]. To reduce the dynamic response of the bridge due to high speed trains, different arrangements of the TMD devices, such as those shown in Figure 5, will be considered in this study. A vehicle-bridge interaction system installed with one TMD device will have one DOF more than that of the original structure. The following are the equations of motion for the combined system:

$$[M']\{\ddot{U}\} + [C']\{\dot{U}\} + [K']\{U\} = \{P\} \qquad (7)$$

where the mass, damping, and stiffness matrices of the combined system are

$$[M'] = \begin{bmatrix} [M] & [0] \\ [0] & [m_t] \end{bmatrix},\quad [C'] = \begin{bmatrix} [C]+[c_t] & -[c_t]^T \\ -[c_t] & [c_t] \end{bmatrix}, \qquad (8a\text{-}c)$$

$$[K'] = \begin{bmatrix} [K]+[k_t] & -[k_t]^T \\ -[k_t] & [k_t] \end{bmatrix}$$

and the corresponding displacement and force vectors are

$$\{U\} = \begin{Bmatrix} \{U_b\} \\ \{u_t\} \end{Bmatrix},\quad \{P\} = \begin{Bmatrix} \{P_b\} \\ \{0\} \end{Bmatrix} \qquad (9a,\ b)$$

Here, all the terms associated with the TMD devices or the MTMD system have been indicated with a subscript "t". The matrices $[M]$, $[C]$ and $[K]$ in Eq. (9) should be interpreted as those assembled for the vehicle-bridge system in Eq. (4), with the moving action of the sprung masses taken into account by the VBI elements. In this study, the equations of motion as given in Eq. (9) will be solved using the Newmark â method with constant average acceleration, i.e., with â = 1/4 and ã = 1/2, for its unconditional stability.

6. Impact Factor and Speed Parameter

In design practice, the impact factor I is used to account for the amplification effect of the bridge due to the passage of moving vehicles through increase of the design forces and stresses. The impact factor I to be used in this study is [11]

$$I = \frac{R_d(x) - R_s(x)}{R_s(x)} \qquad (10)$$

where $R_d(x)$ and $R_s(x)$ respectively denote the maximum dynamic and static responses of the bridge computed at position x. Let v denote the speed of the moving vehicles and L_c the characteristic length of the bridge, i.e., the length between two adjacent inflection points of the first mode of vibration of the bridge [6]. The speed parameter S is defined as the ratio of the first excitation frequency of the moving vehicles, i.e., ðv/L_c to the fundamental frequency ø of the bridge,

$$S = \frac{®v}{\mu L_c} \qquad (11)$$

As can be seen, both the impact factor I and speed parameter S are dimensionless. It is more rational to relate the impact factor I to the speed parameter S, rather than to other physical parameters, such as the bridge length, as is the case with some design codes [12].

7. Numerical Examples

In this study, a three-span uniform continuous beam is considered, which is made of prestressed concrete with elastic constant E = 28.25 GPa, moment of inertia I = 31.6 m^4, and per-unit mass m_0 = 3.76 t/m. The beam has a span length of L = 50 m and a damping ratio of 2.5%. Because the continuous beam considered here is of equal span lengths, the characteristic length L_c of the beam is equal to the span length L. As shown in Figure 3, the frequencies solved for the first three modes of the beam are: ø = 19.23, 24.64, 35.98 rad/s, and the corresponding modal masses are: $M_{mod,1,2,3}$ =

282, 186, 128.

The train travelling over the bridge is simulated as a sequence of 20 sprung masses equally spaced at d = 18 m. Each sprung mass has the following properties: mass M_v = 30.6 t, spring constant k_v = 1,700 kN/m and damping coefficient c_v = 90 kN-s/m. The surface of the rails is assumed to be smooth and rigid.

Example 7.1 Multiple Resonant Peaks

For uniformly three-span continuous bridges, the impact response of the middle span is generally larger than that of the other two side spans [5]. In the present study, only the impact responses of the middle span and departure span of the continuous beams will be considered. The impact factor I solved for the midpoint of the middle span and the departure span of the continuous beams have been plotted against the speed parameter S (referred to as I-S plot) in Figure 4. As can be seen, there exist multiple resonant peaks for the impact response of the departure span and the middle span of the continuous beam. This is mainly due to coincidence of some of the excitation frequencies implied by the moving wheel loads at different speeds with the fundamental or higher frequencies of the continuous beam. According to the analytical formulas of Reference [2], the resonant response for a simple beam subjected to a series of wheel loads will occur at $S_{res} = d/2L$, where d is the wheel load interval. For the present case, the first resonance speed parameter for the departure and middle spans of the continuous bridge is $S_{res,1}$ = 18/(2e50) = 0.18, or equivalently v = 198 km/h. The second resonance speed parameter of the departure span is $S_{res,2}$ = $S_{res,1}$ù$_2$/ù$_1$ = 0.23, which is equivalent to 254 km/h. And the third resonance speed parameter of the middle span is $S_{res,3}$ = $S_{res,1}$ù$_3$/ù$_1$ = 0.34, which is equivalent to 370 km/h. Evidently, all the three resonance speeds can be identified from Figure 4.

Example 7.2 Application of MTMD System

In Figure 4, we have identified three resonant peaks. The first two resonant peaks (at $S_{res,1}$ = 0.18, $S_{res,1}$ = 0.23) affect the departure span of the continuous beam, but only the first resonant peak (at $S_{res,1}$ = 0.18) affects the middle span. The reason is that the inflection point of the second mode is located right at the midpoint of the middle span, as indicated by Figure 3.

Because the response contribution of the first mode of the continuous beams is greater than that of the second mode, the same modal mass ratio with respect to the fundamental mode, ¥$_{m,1}$ = 0.01, as that of the preceding example will be adopted in this example, that is, P $m_{t,1}$ = ¥$_{m,1}$ $M_{\mathrm{mod},1}$ = 2.82 t. And this is the total mass that will be imposed on the bridge. By dividing the total mass P $m_{t,1}$ of the MTMD system into three equal masses, the tuning mass $m_{t,1}$ selected for each TMD mounted on each of the three spans is 0.94 t.

In order to suppress the contribution of not only the first mode, but also the second mode on the side spans, a MTMD system is considered by splitting the tuning mass $m_{t,1}$ on each side span into two tuning masses $m_{\mathrm{h},1}$ and $m_{\mathrm{h},2}$. Based on the modal mass ratio of the first mode (0.94 t) and second mode (1.86 t/2 = 0.93 t) on the side span, which equals 1:1, the mass allocated for the side span is divided into two equal masses as $m_{\mathrm{h},1}$ = $m_{\mathrm{h},2}$ = 0.47 t. The corresponding optimum parameters and properties of the hybrid MTMD model are listed in Table 2.

Table 2 Properties of the hybrid MTMD system

	Optimal Parameters	m_t (t)	k_t (kN/m)	c_t (kN-s/m)
Middle Span	TMD1 f_{opt} = 0.997 æ$_{opt}$ = 0.040	0.94	368	1.49
Two Side Spans	TMD1 f_{opt} = 0.998 æ$_{opt}$ = 0.028	0.47	173	0.51
	TMD2 f_{opt} = 0.998 æ$_{opt}$ = 0.034	0.47	284	0.79

As shown in Figure 5, only the TMD device with lumped mass of $m_{t,1}$ = 0.94 t is installed at the midpoint of the central span, since the central span is governed primarily by the first mode. In the meantime, two equal lumped masses $m_{h,1}$, $m_{h,2}$ each of 0.47 t are mounted on the side span, since the side span is affected both by the first and second modes. One advantage with the present strategy is its simplicity, with no recourse to the sophisticated theories of optimization.

The impact response of the departure and central spans of the continuous bridge has been plotted in Figures 6 and 7, respectively, with reference to the one with no TMDs. As can be seen, the MTMD system proposed using the present simple idea considering only the contribution of the first and second modes is quite effective for suppressing the bridge response.

For the present interest, the maximum acceleration of the moving vehicles that comprise a high speed train, each with a sprung mass M_v, computed for the two kinds of arrangement considered in this paper, including the ones with and with no TMDs, has been plotted in Figure 8. It is observed that the reduction in vehicle vibrations through installation of the TMDs on the bridge is generally small, due to the relatively high flexural rigidity of the bridge (compared with the vehicles) and the relatively short acting time of the vehicles on the bridge, which makes the MTMD system not so effective for reducing the vehicles' response, although it remains effective for the bridge response.

8. Concluding Remarks

In this study, a simple idea for designing the MTMD devices for suppressing the train-induced vibrations on uniform continuous bridges has been proposed by considering the contribution of only the first and second modes. It has been demonstrated that such a system is effective for reducing the bridge response to high speed trains, even with no recourse to the sophisticated theories of optimization. However, the reduction in the vehicles' response is only marginal, due to the relative high flexural rigidity of the bridge and short passing time of the train over the bridge.

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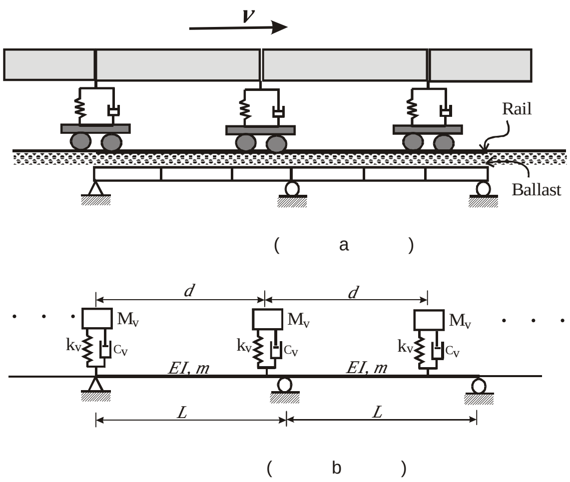

Figure 1 Train-Bridge System:

(a) General Model; (b) Sprung Mass Model

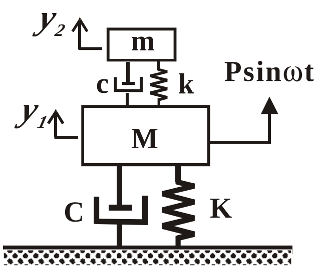

Figure 2 Vibration Absorber System

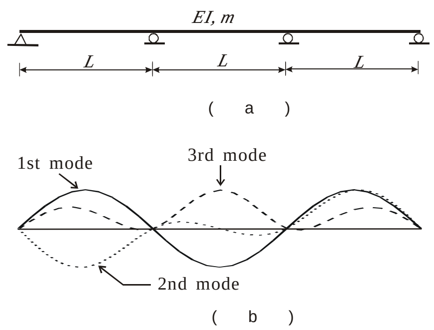

Figure 3 Continuous Bridge:

(a) Model; (b) First Three Vibration Modes

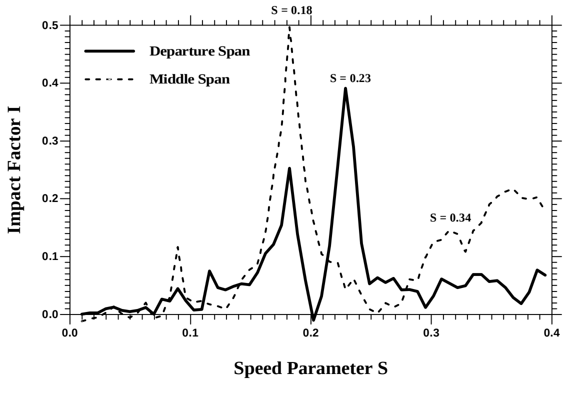

Figure 4 Impact Response of the Continuous Bridge

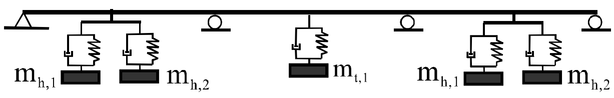

Figure 5 continuous beams with MTMD System

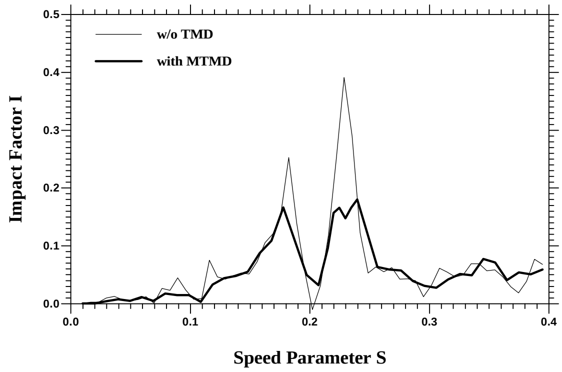

Figure 6 Impact Response of the Departure Span of the Bridge Using MTMD System

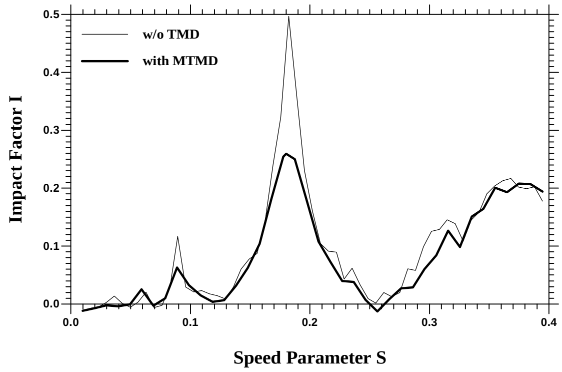

Figure 7 Impact Response of the Middle Span of the Bridge Using MTMD System

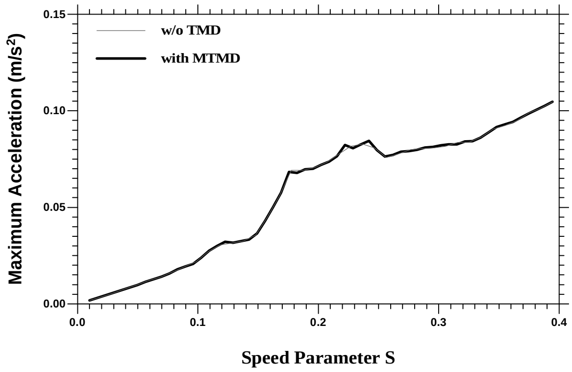

Figure 8 Maximum Acceleration of Sprung Masses