行政院國家科學委員會補助專題研究計畫成果報告

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行政院國家科學委員會專題研究計畫成果報告

橋梁受高速列車作用之振動減振分析
Vibration reduction of bridges due to high speed trains

計畫編號：NSC 89-2211-E-032-018
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1. 中文摘要

採用調諧式質量阻尼(Tuned Mass Damper，簡稱 TMD)的減震裝置來降低主結構系統的振動量，是實務工程上可行的作法，本研究計劃將就 TMD 對高速鐵路橋梁於高速列車作用的減振效果進行探討。計劃中，將先就具單一 TMD 裝置的彈性支承梁受移動載重的動態反應理論解進行推導。然後，針對不同類型的減震裝置(如：單一 TMD 裝置與多個 TMD－稱 MTMD)來探討它們對高鐵橋梁減震程度的影響，進而從數值解中找出調諧式質量阻尼裝置對降低橋梁共振反應的最佳主控參數值，接著再配合有限元素法的車橋互制反應程式，從而評估減振裝置在高速鐵路橋梁的應用可行性。

關鍵詞：高鐵、隔震鐵路橋、衝擊反應

Abstract

In this report, the possibility of using the tuned mass damper (TMD) devices to reduce the vibration response of bridges subjected to high speed trains is investigated. Different arrangements of multiple tuned mass damper (MTMD) systems are evaluated in terms of the impact factor. The vehicle-bridge interaction (VBI) element derived by Prof. Yang and the writer (1997) based on the dynamic condensation technique will be employed to simulate the moving effect of the sprung masses. The numerical results indicate that the MTMD system mounted on the midpoint of the bridge is more effective than that distributed with equal spacing along the bridge.

Keywords: high speed railway, impact response, isolated railway bridges

2. Introduction

The utilization of TMD for controlling structural vibration has been considered in long-span bridges and high-rise buildings. Historically, a great number of researchers have contributed to the development of TMD. Frahm was perhaps the first to investigate the behavior of TMD as a dynamic absorber. In 1909, he used a vibration control device to transfer the vibration energy of the structure to the TMD. Assuming that the main system is undamped and is subjected to a harmonic excitation, Den Hartog (1956) derived a closed form solution for minimizing the dynamic response of the main mass with a single TMD device. Warburton (1980, 1981, 1982) has shown that there exists an optimum range for the absorber parameters of a TMD as far as the minimization of the vibration response of oscillatory systems is concerned. However, some disadvantages do exist with the TMD. For instance, if the acting time of the excitation force is so short, e.g., when subjected to the earthquakes or high speed trains, the effectiveness of a TMD diminishes gradually due to detuning of the optimal damping of the TMD. This has led to the use of more than one TMD to improve the effectiveness of vibration reduction. Recently, Xu and Igusa (1992) and Tamagushi and Harnpornchai (1993) have demonstrated that multiple tuned mass dampers (MTMD) with distributed natural frequencies are more effective than a single TMD.

In this report, the dynamic response of a simple with a single TMD attached at the midpoint due to a moving load will be first investigated by analytical method. Moreover, based on the finite element approach, the dynamic response of steel truss bridges installed with various TMD devices subjected to high speed trains will be studied using the vehicle-bridge interaction (VBI) element previously developed by the writers

(Yang and Yau, 1997; Yau et al., 1998). The effect of different arrangements of the TMD devices, i.e., a single TMD and MTMD systems, will be investigated to evaluate the effectiveness of vibration reduction of the steel-truss bridge under the passage of high speed trains. From the numerical results, it is concluded that the MTMD system attached at the midpoint of the bridge can significantly reduce the responses of the bridge and moving vehicles.

3. Equation of motion

In this study, a uniform beam that is elastically supported by two identical elastic bearings at the ends and attached a TMD ath the midpoint of the span is considered. The vehicle moving over the centerline of the beam is modeled as a moving load (Figure 1). Let v denote the speed and L the length of the beam and K the stiffness of the elastic bearings.

As far as the dynamic response of the beam to vehicle loads moving at high speeds is concerned, it is basically a transient vibration problem because of the very short acting time. For this reason, the damping effect will be neglected in the present study. The equation of motion for the beam traveled by a series of moving loads can be written:

$$m\ddot{u} + EIu'''' + k_t(u - u_t) \times \delta(x - L/2) = p\delta(x - vt)$$
$$m_t\ddot{u}_t + k_t(u_t - u) = 0 \quad \text{at } x = L/2 \tag{1}$$

where m = the mass per unit length, $u(x, t)$ = displacement, x = axis, E = elastic modulus, I = moment of inertia of the beam, δ = Dirac's delta function and all the terms associated with the TMD devices have been indicated with a subscript "t". Correspondingly, the boundary conditions are

$$EIu''(0,t) = EIu''(L,t) = 0$$
$$EIu'''(0,t) = -EIu'''(L,t) = -Ku(0,t) \tag{2a-c}$$

and the initial conditions are

$$u(x,0) = \dot{u}(x,0) = 0 \tag{3}$$

To solve the equation of motion in equation (1), one can assume the deformation function $u(x, t)$ = q(t) [sin(πx/L) + κ] and substitute it into equation (1), by multiplying both sides of the equation by the shape function sin(πx/L) + κ, and then integrating the equation with respect to the beam length L. The result is the generalized equation of motion given below

$$\begin{Bmatrix} \ddot{q} \\ \ddot{u}_t \end{Bmatrix} + \mu^2 \begin{bmatrix} 1 + \frac{\alpha f^2 A^2}{Z} & -\frac{\alpha f^2 A}{Z} \\ -Af^2 & f^2 \end{bmatrix} = \begin{Bmatrix} \frac{2p}{mL}\left[\Theta + \sin\frac{\Phi vt}{L}\right] \\ 0 \end{Bmatrix} \tag{4a,b}$$

where κ = $EI\pi^3/KL^3$ = the stiffness ratio of the flexural rigidity of a beam to the elastic bearings, and

$$A = 1 + \Theta \quad Z = 1 + 8\Theta/\Phi + 2\Theta^2$$
$$f = \frac{\mu_t}{\mu} \quad \alpha = \frac{2m_t}{mL} \quad \mu^2 = \left(\frac{\Phi}{L}\right)^2 \frac{EI}{m} \quad \mu_t^2 = \frac{k_t}{m_t} \tag{5a-f}$$

In equation (5), f = frequency ratio, μ = mass ratio, ω = fundamental frequency of simple beam, and ω_t = frequency of TMD.

On the other hand, as can be seen from equation (4), when κ equals zero, the equation of motion of the elastically supported beam reduces to that of a simple beam.

4. Response Analysis

Considering the transient response q(t) of the beam due to a moving load, the solution to equation (4a) can be obtained as

$$q(t) = Q_1(t) + Q_2(t) \tag{6}$$

where

$$Q_1(t) = \frac{FZ}{G}\left\{\frac{f\bar{\mu}^2}{(\mu_2^2 - \mu_1^2)}\left[\frac{\mu_2}{\bar{\mu}}\left(1 - \frac{\mu_1^2}{\bar{\mu}^2 f^2}\right)\left(S^2 - \frac{\mu_2^2}{\bar{\mu}^2}\right)\sin\mu_1 t + \frac{\mu_1}{\bar{\mu}}\left(1 - \frac{\mu_2^2}{\bar{\mu}^2 f^2}\right)\left(\frac{\mu_1^2}{\bar{\mu}^2} - S^2\right)\sin\mu_2 t\right] + (f^2 - S^2)\sin\Omega t\right\} \tag{7a}$$

$$Q_2(t) = F\Theta\left\{\frac{\bar{\mu}^2}{(\mu_2^2 - \mu_1^2)}\left[\frac{\mu_2^2}{\bar{\mu}^2}\left(\frac{\mu_1^2}{\bar{\mu}^2 f^2} - 1\right)\cos\mu_1 t + \frac{\mu_1^2}{\bar{\mu}^2}\left(1 - \frac{\mu_2^2}{\bar{\mu}^2 f^2}\right)\cos\mu_2 t\right] + 1\right\} \tag{7b}$$

$$F = \frac{p}{\bar{\mu}^2} \tag{8a}$$

$$G = S^4 Z + Zf^2 - ZS^2 - ZS^2 f^2 - A^2 S^2 \alpha f^2$$
$$= Z(f^2 - S^2)(1 - S^2) - A^2 S^2 \alpha f^2 \tag{8b}$$

$$\mu_1^2 = \frac{\bar{\mu}^2}{2Z}\left[(Z + Zf^2 + A^2\alpha f^2) - \sqrt{(Z + Zf^2 + A^2\alpha f^2)^2 - 4Z^2 f^2}\right] \tag{8c}$$

$$\mu_2^2 = \frac{\bar{\mu}^2}{2Z}\left[(Z + Zf^2 + A^2\alpha f^2) + \sqrt{(Z + Zf^2 + A^2\alpha f^2)^2 - 4Z^2 f^2}\right] \tag{8d}$$

In equations (6) and (7), the term $Q_1(t)$ represents the contribution caused by the flexural vibration of the simple beam, and $Q_2(t)$ the rigid displacement of the elastic bearings.

5. Vehicle-Bridge System with TMD

Consider a plane truss that is simply

supported at both ends and is made up of four equal panels, as shown in Figure 2(a). Because of its relative flexibility, the truss bridge will undergo rather large deflections and vibrations when it is traveled by the high speed train, which may adversely affect the safety of the bridge itself and the riding comfort and controllability of the train. To reduce the dynamic response of the bridge due to high speed trains, different arrangements of TMD devices are considered in this study, including the three cases shown in Figure 2(b)-(d), which indicate respectively a single TMD attached to the midpoint, MTMD mounted on the bridge along the longitudinal axis, and MTMD installed only at the midspan of the truss. The vehicle-bridge system installed with TMD devices will have one or more DOFs than the original structure. The following are the equations of motion for the combined system:

$$\underset{\sim}{M}\{\ddot{U}\} + \underset{\sim}{C}\{\dot{U}\} + \underset{\sim}{K}\{U\} = \{P\} \tag{9}$$

where the coefficient matrices are the mass, damping, and stiffness matrices of the combined system:

$$\underset{\sim}{M} = \begin{bmatrix} [M_b] & [0] \\ [0] & [m_t] \end{bmatrix} \quad \underset{\sim}{C} = \begin{bmatrix} [C_b]+[c_t] & -[c_t]^T \\ -[c_t] & [c_t] \end{bmatrix} \quad \underset{\sim}{K} = \begin{bmatrix} [K_b]+[k_t] & -[k_t]^T \\ -[k_t] & [k_t] \end{bmatrix} \tag{10a-c}$$

and the corresponding displacement and force vectors are

$$\{U\} = \begin{Bmatrix} \{U_b\} \\ \{u_t\} \end{Bmatrix} \quad \{P\} = \begin{Bmatrix} \{P_b\} \\ \{0\} \end{Bmatrix} \tag{11}$$

Here, all the terms associated with the TMD devices have been indicated with a subscript "t".

6. Impact Factor

In design practice, the impact factor I is used to account for the amplification effect of the bridge due to the passage of moving vehicles through increase of the design forces and stresses. The impact factor is defined as follows (Yang et al., 1995)

$$I = \frac{R_d(x) - R_s(x)}{R_s(x)} \tag{12}$$

where $R_d(x)$ and $R_s(x)$ = the maximum dynamic and static response, respectively, of the bridge calculated at position x.

7. Numerical Examples

Figure 2(a) shows a simple truss bridge with a span length L = 36 m and height H = 6 m, which is made of steel with elastic modulus E = 204 GPa , and Poisson's ratio ¬ = 0.3. The railroad mass per unit length is m = 7.6 t/m. A proportional damping ratio of 2 % is adopted for the truss bridge. The member properties of the truss are given in Table 1. The fundamental frequency ω and effective mass M_{eff} of the steel truss bridge are 27.25 rad/s and 203 t, respectively. In the present study, the train is modeled as a sequence of 15 sprung masses equally spaced at 18 m. The dynamic properties of the train model are given in Table 2. Two examples will be analyzed to demonstrate the effect of TMDs in suppressing the structural vibrations of the VBI system.

Table 1. Properties of the steel-truss bridge.

Member Properties	A (m^2)	I (m^4)	Density ∘ (t/m^3)
(A)	0.228	0.032	7.85
(B)	0.02	-	7.85
(C)	0.114	0.016	7.85

Table 2. Dynamic properties of the train.

Axle dist. d (m)	M_v (t)	k_v (kN/m)	c_v (kN-s/m)
| 18 | ...| 18 | (Total = 15)	30.6	1700	90

Example 7.1

The optimum absorber parameters proposed by Den Hartog (1956) for the TMD are adopted in this study, that is,

$$\text{TMD's mass ratio } \varepsilon_m = \frac{m_t}{M_{eff}}$$

$$\text{TMD's frequency ratio } \varepsilon_f = \frac{\mu_t}{\mu} = \frac{1}{1+\varepsilon_m} \qquad (13)$$

$$\text{TMD's damping ratio } \eta_t = \sqrt{\frac{3\varepsilon_m}{8(1+\varepsilon_m)^3}}$$

A small mass ratio ε_m = 0.01 is selected for the TMD, that is, m_t = 2 t, the spring stiffness is selected as k_t = 1,456 kN/m and damping coefficient as c_t = 6.54 kN-s/m. To search for the optimal damping ratio of the TMD, the damping ratio is allowed to vary in from 0 to 0.4. From the result plotted in Figure 3, one observes that the impact response of the bridge will become minimal at the resonant speed, when the damping ratio is near 0.06. Therefore, a damping ratio of 6 % is adopted for the TMD device to be considered in the example to follow.

Example 7.2

To investigate the effect of the MTMD devices on the impact response of truss bridges due to vehicles moving at high speeds, different numbers of TMDs of constant intervals 4.5 m are mounted on the bottom chords of the truss shown in Figure 2(c). The positions and properties of TMD devices for three different arrangements shown in Figure 2(c) are listed in Table 3.

Table 3. Properties of MTMD.

	Position of TMDs	m_t (t)	k_t (kN/m)	c_t (kN-s/m)
3 TMDs	|s||s|	0.667	486	2.16
5 TMDs	|s|s||s|s|	0.4	291	1.3
7 TMDs	|s|s|s||s|s|s|	0.286	7.71	1

Note: ||: midpoint of the bridge; s = spacing of TMD = 4.5 m

In Table 3, the mass ratio of the entire MTMD to the effective mass of the truss bridge is set to be 0.01, while the frequency ratio and damping ratio of each TMD in the MTMD system still satisfy the optimum conditions given in Eq. (9). The impact response of the truss bridge and the maximum acceleration of the moving vehicles have been plotted in Figures 4 and 5, respectively. As can be seen, the installation of MTMD in the middle of the bridge does not result in further significant reduction of the response, compared with the case of a single TMD mounted on the midpoint of the bridge.

Alternatively, we shall try using the same MTMD system as previously described, but having them installed at the midpoint of the bridge, as shown in Figure 3(d). For this case, the results have been plotted in Figures 6 and 7 respectively for the bridge and vehicle response. As can be seen, slightly better results have been achieved, owing to the fact that the deflection response of the bridge induced by high speed trains is controlled mainly by the fundamental mode (Yang et al., 1997). Therefore, the MTMD mounted on the midpoint of the bridge is considered more effective than those distributed along the bottom chords of the bridge.

8. Concluding Remarks

In this study, the dynamic response of a simple with a single TMD attached at the midpoint due to a moving load will be first investigated by analytical method. Moreover, based on the finite element approach, the TMD is used to suppress the structural vibrations of a steel-truss bridge induced by high speed trains. The numerical results have confirmed that the TMD is an effective device for suppressing the vibration response of the truss bridge under the passage of high speed trains. The optimal damping ratio of the TMD obtained from the finite element analysis agrees well with Den Hartog's result. On the other hand, the MTMD system installed at the middle cross section of the bridge is observed to be more effective than that distributed along the bottom chords of the bridge.

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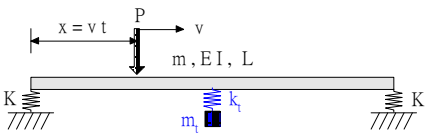

Figure 1
Elastically Supported Beam Model and TMD

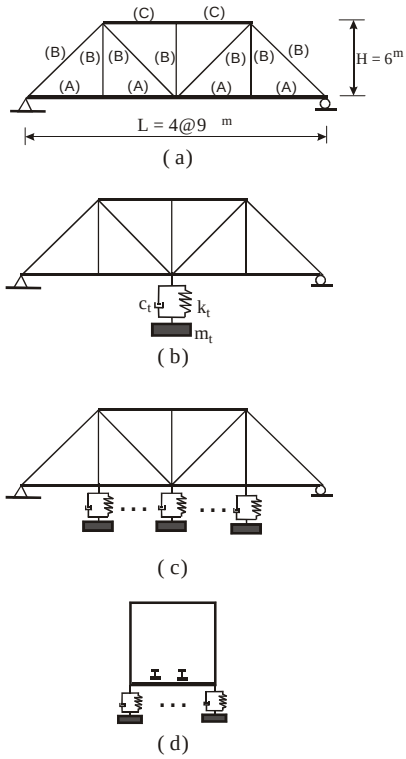

Figure 2
Truss bridge model and TMDs:
(a) a simple truss, (b) truss with a single TMD,
(c) truss with MTMD distributed along the bridge,
(d) truss with MTMD mounted at midpoint

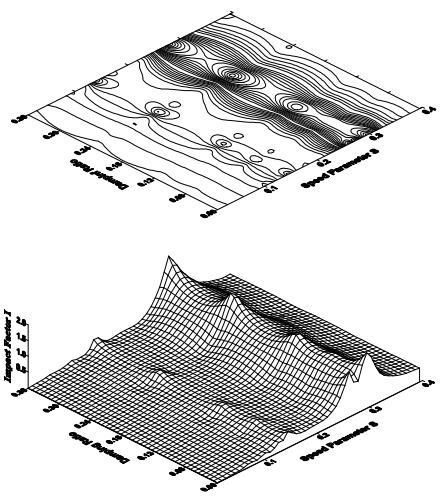

Figure 3
Effect of TMD's damping ratio on impact factor

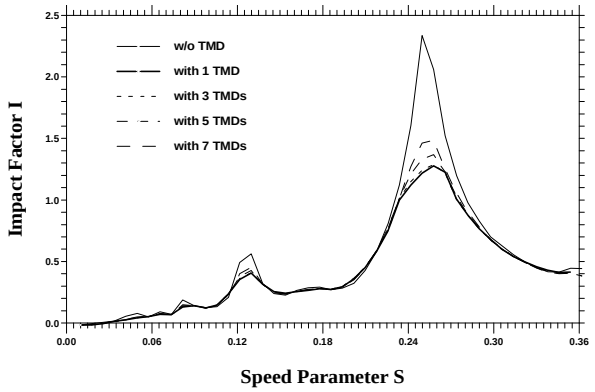

Figure 4

Impact response of bridge:
effect of MTMD distributed along the bridge

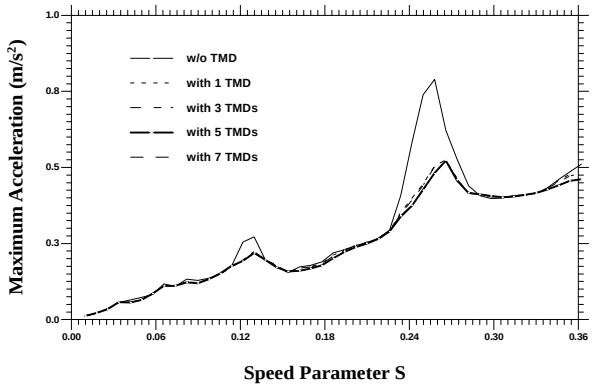

Figure 7

Max. acceleration of sprung mass:
effect of MTMD mounted on midpoint of the bridge

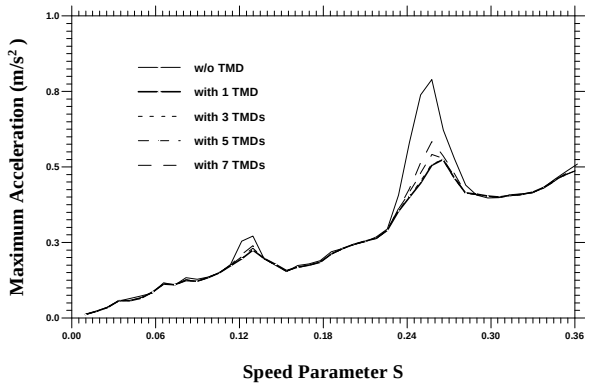

Figure 5

Maximum acceleration of sprung mass:
effect of MTMD distributed along the bridge

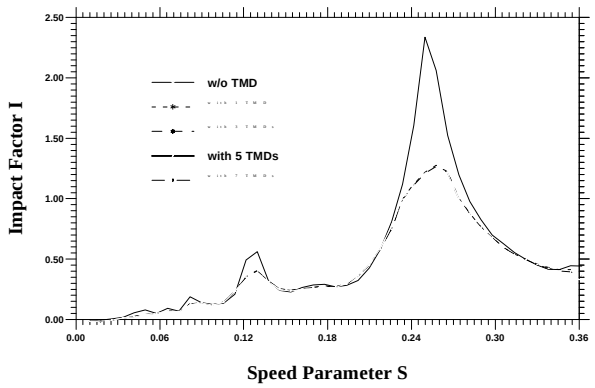

Figure 6

Impact response of bridge:
effect of MTMD mounted on midpoint of the bridge