

A Fast Global Optimization Approach to VAR Planning for the Large Scale Electric Power Systems

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Abstract

In this paper, an innovative fast global optimization technique, Hybrid Partial Gradient Descent/Simulated Annealing(**HPGDSA**), for optimal VAR planning is presented. The **HPGDSA** is introduced to search the global optimal solution considering both quality and speed at the same time. The basic idea of the **HPGDSA** is that partial gradient descent and simulated annealing alternate with each other such that it reduces the CPU time of the conventional Simulated Annealing(**SA**) method while retaining the main characteristics of **SA**, i.e., the ability to get the global optimal solution. The **HPGDSA** was applied to a practical power system, Taiwan Power System(Tai-Power System), with satisfactory results.

1. Introduction

The objective of VAR planning is to determine the minimum cost expansion pattern, in terms of locations, types and sizes, of new reactive power sources to be installed in power systems so as to insure secure and economic operation. The dispatch of reactive powers can be effectively used to maintain acceptable voltage levels throughout the system and to reduce overall real power loss of the system. Furthermore, the security margin of power systems can be enlarged to reduce the possibility of voltage collapse by providing enough reactive power[1].

Due to its goal, the VAR planning has commonly been formulated as a complicated constrained optimization problem with partially discrete, partially continuous and non-differentiable nonlinear objective function[2-9]. A survey of literature on the problem reveals that various numerical optimization techniques have been employed to approach the complicated VAR planning problem. More specifically, Opoku[4] has formulated the problem as a mathematical optimization problem based on a linearized

model (a restructured sparse admittance matrix) to reduce the dimensionality and computing time. Lebow[2], Granville[3] and Hong[5] have formulated the problem as a mixed-integer nonlinear programming problem with 0-1 integer variables representing whether new reactive devices should or should not be installed. In this formulation, however, both the number and value of capacitors were still treated as continuously differentiable variables. The Generalized Benders Decomposition (GBD) technique[10] was then employed to decompose the problem into a continuous subproblem and an integer subproblem. It should be noted that the above mentioned methods can be classified as greedy search technique. One main disadvantage of aforementioned techniques is that they often get stuck at local optimum rather than at global optimum. In order to circumvent this problem, Hsiao et al.[6,7] applied Simulated Annealing(**SA**) method to optimal VAR source planning in large scale power systems. **SA** is a powerful, general-purpose stochastic optimization technique, which can theoretically converges asymptotically to the global optimum solution with probability " 1 ". One main drawback, however, of **SA** is that it takes much CPU time to find the global optimum.

In this paper, we present a Hybrid Partial Gradient Dsecent/Simulated Annealing(**HPGDSA**) method to reduce the CPU time of **SA** while retaining the main characteristics of **SA**, i.e., the ability to get the global optimal solution. A new formulation of the optimal VAR planning as a constrained optimization problem is presented in Sec. 2. Then a detailed **HPGDSA** algorithm is described in Sec. 3. In section 4, the proposed algorithm is implemented in a software package and tested on a practical power system, Tai-Power System, with promising simulation results. Also, the performances of **HPGDSA** and **SA** methods are compared in section 4. Finally, a summary conclusion is given in section 5.

2. Problem Formulation

In this section, the VAR planning problem is formulated as a constrained optimization problem. First, we consider the objective function. The objective function is expressed as the sum of the cost of VAR source placement and the cost of real power loss. Mathematically, the function is as follows :

$$f(q_{ck}, q_{rk}, r_k) = C(q_{ck}, q_{rk}, r_k) + K_e D_u P_{loss}(x). \quad (1)$$

where

$C(q_{ck}, q_{rk}, r_k)$: total cost of the VAR sources, and

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$$C = \sum_{k \in \Omega} (d_k + s_{ck}q_{ck} + s_{rk}q_{rk}) r_k$$

Ω : a set of all candidate buses to install VAR sources,

d_k : installment cost at bus k ,

s_{ck} : unit costs of capacitors,

s_{rk} : unit costs of reactors,

q_{ck} : added capacitive compensation at bus k ,

q_{rk} : added inductive compensation at bus k ,

r_k : 0 don't placement reactive power source at bus k ,

1 placement reactive power source at bus k ,

$P_{loss}(x)$: real power loss in the system,

x : state vector of the system, which is also a function of the variables, q_{ck} , q_{rk} , and r_k ,

k_e : energy cost per unit(\$/kwh),

D_u : duration of the system operating time,

Note that the function $f(q_{ck}, q_{rk}, r_k)$ is a mixed-integer nonlinear function. The independent variables are q_{ck} , q_{rk} , and r_k . State vector, x , is a function q_{ck} , q_{rk} , and r_k .

Next, we consider the constraint equations. Basically, there are two classes of constraint equations for our formulation. The first class consists of equality constraints stemming from conservation of power in a overall system. Mathematically, the equality constraint is expressed as follows :

$$L(x)=0, \quad (2)$$

where $L(\cdot)$ is a vector of power flow equations, and argument, x , is a state vector of the system, including the bus voltage magnitudes and angles. The other class comprise inequality constraints resulting from consideration of the following system security operation :

- (1) line flow limits,
- (2) voltage magnitude and phase angle difference limits,
- (3) voltage deviations,
- (4) transformer tap changing limits,
- (5) real and reactive power generation limits, and
- (6) reactive power compensation limits.

Mathematically, these inequality constraints can be expressed in terms of a single inequality vector as follows :

$$G(x) \leq 0 \quad (3)$$

Note that the inequality vector means that every component of vector is inequality equation.

In summary, combining Eqs. (1), (2) and (3) gives us a complete formulation of the VAR planning problem as the following :

<p>Minimize</p> $f(q_{ck}, q_{rk}, r_k) \quad (4)$ <p>subject to</p> $L(x) = 0,$ $G(x) \leq 0.$

Remark 1 : It is well recognized that formulating VAR planning as a constrained optimization problem has the advantage that techniques borrowed from nonlinear programming area can be

systematically applied to the problem instead of trial-and-error exhaustive planning.

Remark 2 : The constrained optimization problem is a complicated mixed-integer nonlinear problem. Previous experience shows that there are several local optima instead of a single.

Remark 3 : The complexity of the problem prevents the direct use of optimization techniques, for example, linear programming, simulated annealing, gradient descent, etc., without exploiting the nature of the VAR planning.

3. A Fast global Optimization Technique

We present a fast global optimization technique, namely Hybrid Partial Gradient Descent/Simulated Annealing(**HPGDSA**), for solving the constrained optimization problem, Eq. (4), in this section. The **HPGDSA** has an ultra fast search speed while retaining the characteristics of Simulated Annealing(**SA**), i.e., finding the near global optimum. The basic strategy of the **HPGDSA** is that using partial gradient vector guides the search point to local optimum and by using **SA** the search point escapes from the valley of the local optimum in order to arrive for global optimum. That is, partial gradient descent and simulated annealing alternate with each other in the **HPGDSA**. The **HPGDSA** algorithm is outlined in Fig. 1.

Remark 1 : In VAR planning, one only to decide whether one should place an unit of capacitor/reactor or not to decrease the objective function at a time during the Partial Gradient Descent(**PGD**) process. This is a binary decision problem. Moreover, the effect of capacitors is contrary to that of reactors for power systems. Therefore, it is sufficient that step 4 of the **HPGDSA** uses the signs of each component of partial gradient vector, **PGV**, instead of the numerical value of each component to guide the search point.

Remark 2 : A lot of CPU time savings of the **HPGDSA** result from incorporation of partial gradient descent into **SA**. We use the following Fig. 2 and Fig. 3 to illustrate the advantages of the **HPGDSA** over **SA**. From Fig. 2 and Fig. 3, we observe the fact that **SA** wonders for a while than **HPGDSA** before searching for the global optimum. Especially, in VAR planning problem, the difference of objective function between local optimum and global optimum is far less than that between initial point and local optimum. Therefore, **HPGDSA** improves CPU time by finding the first local optimum very soon. Although we, at this point, could not give a theoretical proof of the above observation, we show by heuristics and simulation results in Sec. 4 that **HPGDSA** is indeed superior to **SA** in VAR planning problem.

Remark 3 : The key element in the **HPGDSA** algorithm is that when and how the partial gradient descent and simulated annealing alternate with each other such that the full benefits can be achieved. The judgement of when the switch occurs is based on the following guidelines :

PGD \rightarrow **SA** : When the local optimum is searched.

SA \rightarrow **PGD** : When the search point is in a valley of a new local optimum.

The nontriviality of the above guidelines is that how we know the search point is in a valley of a new local optimum without seeing the picture like Fig. 2. This difficulty is overcome by the following technique. Keep tracking the objective function of the search point and compare it with the stored objective function of the previous

The HPGDSA Algorithm

Step 1 : Input system data and control parameters.

Input system data and control parameters such as the initial temperature, frozen temperature, random number seed, cooling rate, the Boltzman factor K_b , and the number of moves at each temperature.

Step 2 : Run load flow program.

This obtains voltage magnitudes and angles of all buses in studied power system.

Step 3 : Calculate the partial gradient vector, PGV , of the objective function, $f(q_{ck}, q_{rk}, r_k)$, with respect to q_{ck} at each bus k .

Theoretically,

$$PGV = \left[\frac{\partial f}{\partial q_{c1}}, \frac{\partial f}{\partial q_{c2}}, \dots, \frac{\partial f}{\partial q_{cn}} \right]$$

However, in VAR planning, q_{ck} , $k=1, 2, \dots, n$, are discrete variables so that we use one unit of reactive power Δq_{ck} to approximate ∂q_{ck} , $k=1, 2, \dots, n$. Therefore, PGV is calculated as

$$PGV = \left[\frac{\Delta f}{\Delta q_{c1}}, \frac{\Delta f}{\Delta q_{c2}}, \dots, \frac{\Delta f}{\Delta q_{cn}} \right]$$

Step 4 : FOR $k=1$ to n DO

IF $PGV[k] < 0$ AND $V_k < \bar{V}_k$ (\bar{V}_k is upper limit of voltage magnitude on bus k)

THEN place an unit capacitor on bus k .

IF $PGV[k] > 0$ AND $V_k > \underline{V}_k$ (\underline{V}_k is lower limit of voltage magnitude on bus k)

THEN place an unit reactor on bus k .

END DO

IF no placement occurs,

THEN store the objective function by setting $L=f$, and cool down temperature by setting $T = \alpha_1(T) T$ ($\alpha_1(T)$: cooling rate), and go to step 5.

ELSE go to step 2.

Step 5 : Generate a new feasible point (a point corresponds to a configuration in VAR planning) using a perturbation mechanism.(see Appendix)

(1) To generate a new point j of the current point, i , using a perturbation mechanism.

(2) Check the point, and if any constraint is violated, then go to step 9. Otherwise, calculate the objective function, f_j , and go to step 6.

Step 6 : IF ($f_j \leq f_i$)

THEN go to step 7

ELSE go to step 8

Step 7 : Accept the new feasible points as the current point, and

IF ($f_i \leq L$)

THEN go to step 2.

ELSE go to step 9.

Step 8 : A random number r uniformly distributed in the interval $[0,1)$ is chosen. If $\exp(\frac{f_i - f_j}{K_b T}) > r$, then go to step 7. Otherwise, go to step 9.

Step 9 : If the moves are finished, then go to step 10.

Otherwise, go to step 5.

Step 10 : If the temperature is frozen, then go to step 11.

Otherwise, cool down temperature by setting $T = \alpha_2(T) T$ and go to step 5.

Step 11 : Print out the near global optimal solution.

Fig. 1 Algorithm of the HPGDSA

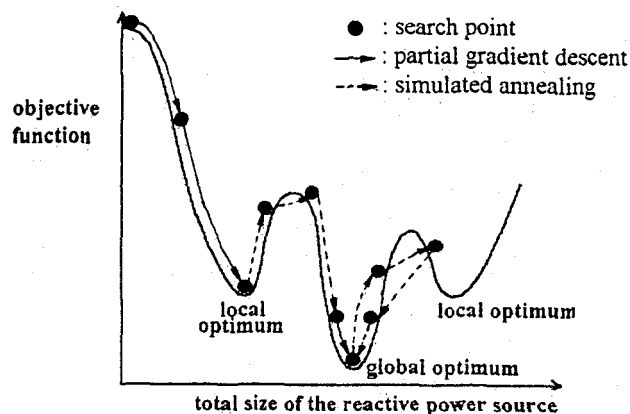


Fig. 2 Search process of the HPGDSA

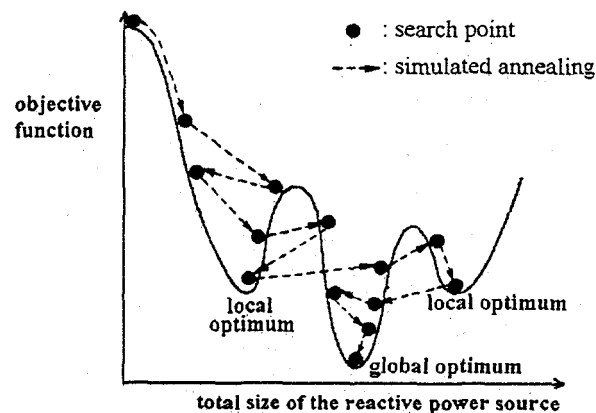


Fig. 3 Search process of the SA

searched local optimum. If the objective function of the search point is less than that of the searched local optimum, then we conclude that the search point is in a valley of a new local optimum. The implementation of the above technique is in the Step 4 and Step 7 of Fig. 1.

Remark 4 : In the HPGDSA algorithm, control parameter values are chosen by systematic approaches. For example, cooling rates, $\alpha_1(T)$ and $\alpha_2(T)$, are functions of temperature with range values smaller than 1. $\alpha_1(T)$ is used as cooling rate when partial gradient descent switches to SA search. When the solution is close to global optimum, then $\alpha_1(T)$ is adjusted to a higher value in order to settle down a global optimal solution. $\alpha_2(T)$ is used as cooling rate in SA search. When the accepted ratio is lower or the sampled mean and variance of objective function at current temperature have big drops, then $\alpha_2(T)$ is adjusted to a higher value in order to avoid getting stuck at a local optimal point.

Remark 5 : The overall process of the HPGDSA is a finite sequence (PGD, SA, PGD, SA, ..., PGD, SA). Note that the final stage of HPGDSA is SA. This arrangement allows us to find the solution, which is feasible(satisfying all constraints) and near global, at the final stage.

4. Simulation Results

In this section, a practical power system, Tai-Power system, was tested by using the **HPGDSA**. The practical power system of Tai-Power Company has 358 buses with 439 transmission lines. A total of 56 buses are selected as candidate buses for installing new VAR sources. The candidate buses include weaker buses[11] and heavier loaded buses. In this test system, the following parameters are used : power loss per unit $K_e = NT\$2.5 / kwh$, q_{ci}^{max} and q_{ri}^{max} are all limited to 30 banks. One bank of the VAR source is set at 28.8 MVAR, and the VAR source cost per unit are: $s_{ci} = s_{ri} = NT\$3,278,880 / bank$, and $d_i = NT\$520,000 / location$.

In order to evaluate the performance of various load level of Tai-Power System by using **HPGDSA** and **SA** methods, respectively, two cases are considered :

case 1 : The Tai-Power system operates under normal load condition.

case 2 : The Tai-Power system operates under peak load condition.

The results of case 1 and case 2 using the **HPGDSA** technique are shown in Table 1 and Table 2, including the minimum power loss, cost, VAR source locations with size and the CPU time on a SUN SPARC 2 workstation in the computer center of National Taiwan University. The performance of **HPGDSA** compared with the performance of **SA** for case 1 and case 2 are shown in Table 3 and Table 4. Here, we would like to emphasize that all the simulation results are typical based on many simulations.

Table 1 : Results of **HPGDSA** before and after planning for case 1 (Tai-Power system)

	before planning	after planning	reduce rates
power loss(MW)	202.286	179.312	11.36 %
total cost *	4,430.0625	4,280.6191	3.37 %
total locations/ total size(bank) **	37 / 102		
CPU time(m : s)	4 : 32		

* : million dollars, NT\$.
** : one bank=28.8MVAR.

Table 2 : Results of **HPGDSA** before and after planning for case 2 (Tai-Power system)

	before planning	after planning	reduce rates
power loss(MW)	302.0719	246.8956	18.27 %
total cost *	6,615.364258	5,834.754883	11.80 %
total locations/ total size(bank) **	48 / 126		
CPU time(m : s)	5 : 00		

* : million dollars, NT\$.
** : one bank=28.8MVAR.

Table 3 : Performances of **HPGDSA** and **SA** for case 1(Tai-Power System)

method	power loss (MW)	total cost *	total locations /total size(bank) **	CPU time (m : s)
HPGDSA	179.312	4,280.619141	37 / 102	4 : 32
SA	187.3268	4,281.278809	16 / 52	251 : 58

* : million dollars, NT\$.

** : one bank=28.8MVAR.

Table 4 : Performances of **HPGDSA** and **SA** for case 2(Tai-Power System)

method	power loss (MW)	total cost *	total locations /total size(bank) **	CPU time (m : s)
HPGDSA	246.8956	5,834.754883	48 / 126	5 : 00
SA	253.5412	5,860.650391	25 / 90	286 : 36

* : million dollars, NT\$.

** : one bank=28.8MVAR.

From these results, we have the following observations:

- (1) The **HPGDSA** method can handle various load level condition of a practical power system, Tai-Power System, with satisfactory results(see Table 1, Table 2).
- (2) The solution time of **HPGDSA** is much lesser than that of the conventional **SA** method and the solution quality is also improved slightly(see Table 3, Table 4).
- (3) All parameters of the problem formulation used in the algorithm are industry specifications. Therefore, the solutions of the **HPGDSA** algorithm are ready for industry use.
- (4) In the above testing systems, we can obtain very close results in many timing tests. Therefore, these results are near global optimal solutions.

Conclusion

In this paper, a relatively realistic problem formulation for the optimal VAR source planning problem is presented. The new problem formulation is treated as a constrained and non-differentiable optimization problem. The formulation includes the cost of system operation, investment cost of VAR sources, load and operational constraints. They are the factors of most concern in the VAR planning problem.

We have developed a novel global optimization approach, hybrid partial gradient descent/simulated annealing, for solving VAR planning problem in the large scale electric power systems. The solution algorithm is that partial gradient descent method is used to search fast a local optimal solution. Then the simulated annealing method is used to jump out of the local optimal solution toward a global optimal solution. The solution algorithm is efficiently applied to a practical power system, Tai-Power System, with satisfactory results. The prominent feature of the **HPGDSA** is that its solution time is much lesser than that of the conventional simulated annealing method and the solution quality is also improved slightly. Therefore, the **HPGDSA** algorithm has the

potential to be a practical global optimal solution to VAR planning problem in the large scale electric power systems.

Appendix

In step 5 of the HPGDSA, the new system configuration is generated from current system configuration via a perturbation mechanism. Four types of moves are devised to implement the perturbation mechanism. These four types of moves are described as follows[12]:

(1) add/subtract move:

To add or subtract a preset realistic an unit size of reactive power source into a bus which was chosen from the candidate bus set by using a random number generator. The action of addition or subtraction is also determined by a random number(see Fig. A1).

(2) multiplicative move:

To add or subtract a reactive power source which is of positive integer multiple of an unit size of reactive power source into a bus which was chosen from the candidate bus set by using a random number generator. The action of addition or subtraction is also determined by random number(see Fig. A2).

(3) interchange move:

To interchange the reactive power sources of the two different buses based on a random number generator(see Fig. A3).

(4) combinative move:

To move reactive power sources of one bus into the other bus, and the selection is based on a random number generator(see Fig. A4).

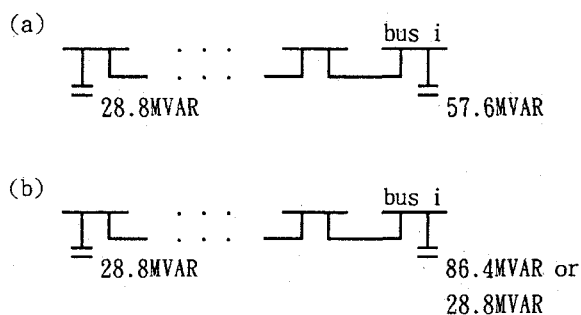


Fig. A1 The add/subtract move: (a) is current configuration and (b) is new configuration after the move.

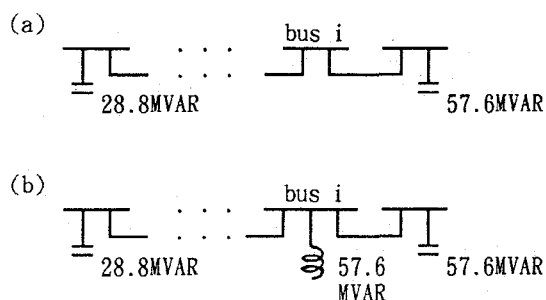


Fig. A2 The multiplicative move: (a) is current configuration and (b) is new configuration after the move.

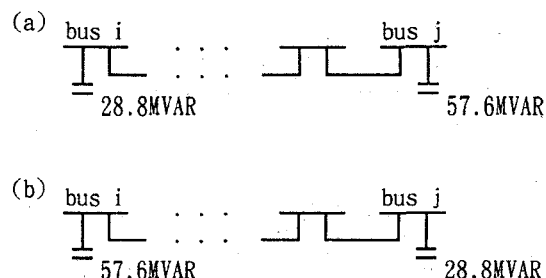


Fig. A3 The interchange move: (a) is current configuration and (b) is new configuration after the move.

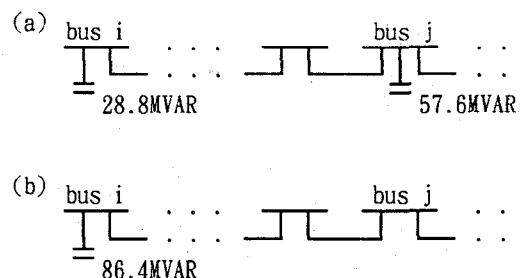


Fig. A4 The combinative move: (a) is current configuration and (b) is new configuration after the move.

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Biographies

Chih-Wen Liu was born in Taiwan in 1964. He received the B.S. degree in electrical engineering from National Taiwan University in 1987, Ph.D. degree in electrical engineering from Cornell University in 1994. Since 1994, he has been with National Taiwan University, where he is associate professor of electrical engineering. His research interests include application of numerical methods to power system and motor control.

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Chun-Chang Liu was born in Taiwan in 1925. He received his B.S. degree in electrical engineering from National Taiwan University in 1951. Since 1951, he has been with National Taiwan University, where he is professor of electrical engineering. From 1987 to 1988, he was a visiting professor at the Kyushu University in Japan. His current research interests include power system analysis and electrical machine analysis.

Ying-Tung Hsiao was born in Taiwan in 1959. He received his B.S. degree in electrical engineering from National Taiwan Institute of Technology in 1986 and Ph.D. degree in electrical engineering from National Taiwan University in 1993. After that, he joined the faculty of Tamkang University, Taiwan. His research interests include power system analysis, optimal theory and software engineering.

Discussion

L L Lai & J T Ma (Energy Systems Group, City University, Northampton Square, London EC1V 0HB, UK) The authors are to be commended for an interesting paper. It would be helpful if the authors could respond to the following comments:

The authors claimed to have developed a global optimization approach based on the partial gradient descent and simulated annealing. It seems to the discussers that it would take a very long time to search for the problem space before any conclusion, if any, could be drawn on whether the solution is a global one or not. The discussers cannot also understand the reason why the solution quality is improved with this

approach as compared to the one obtained with simulated annealing alone.

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C.-W. Liu, W.-S. Jwo, C.-C. Liu, & Y.-T. Hsiao: The authors thank the discussers for their interest on this paper.

In response to the first comment, the proposed Hybrid Partial Gradient Descent / Simulated Annealing (HPGDSA) can search the global optimal solution considering both quality and speed at the same time due to the following reason. Basically, the overall process of the HPGDSA is a finite sequence (PGD, SA, PGD, SA, ..., PGD, SA). Note that the final stage is SA. Moreover, SA is a general-purpose stochastic optimization technique, which can theoretically converge asymptotically to the global optimum solution with probability "1" when the search time is infinite. Therefore, HPGDSA can theoretically find the global optimum. In response to the second comments, the solution quality of HPGDSA is better than that of SA in the paper due to the following reason. The practical implementation of HPGDSA and SA in simulation is a finite-step algorithm. Moreover, both HPGDSA and SA are stochastic in nature. Therefore, both solutions are near global solutions in practice and differ from each other at each test. However, the simulation results in the paper are based on large number of simulation tests.

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