Parameter Selection in the Sliding Mode Control Design Using Genetic Algorithms

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Abstract

In this paper, we propose a parameter selection method by using the genetic algorithms in the sliding mode control of the variable structure system. The proposed method can efficiently choose the appropriate gain parameters based on a proposed fitness function to increase the speed of system response in the reaching phase and reduce the chattering in the sliding phase so that a high performance can be achieved. Some simulation results prove the validity of the proposed Method.

***Keywords*:** Sliding mode control, hitting time, chattering, genetic algorithms.

1 Introduction

The sliding mode control theory of the variable structure system provides a method to design a system in such a way that the controlled system should be insensitive to parameter variations and external disturbances [6, 13]. Essentially, the sliding mode control uses discontinuous control action to drive the state trajectory toward a specific hyperplane in the state space, and then the state trajectory is maintained to slide on the specific hyperplane until the origin of the state space is reached. In the sliding mode control, the hitting time of the system state reaches the switching plane will affect the speed of the system with the desired dynamic behavior. One advantage of the sliding mode control is that when the system enters the sliding mode, it is insensitive to plant parameters uncertainty or external disturbance. If the hitting time is reduced, the time of the system with the desired dynamic behavior can be reduced and the uncertainty of the system can be attenuated. Furthermore, the chattering in the switching plane will affect the stability of the controlled system. Therefore, the hitting time reduction and the chattering attenuation are two important requirements in the slide mode control design. Hence, most effort is focus on the problem of minimizing the hitting time and the chattering phenomena [11, 14]. To minimize the hitting time, Young et al. [15] used a high-gain feedback to speed up the transient response toward the switching hyperplane, but this method causes high chattering along the switching hyperplane which is undesirable in the physical system. To alleviate the chattering phenomena, Slotine [12] and Yeung and Chen [16] used the boundary layer or sliding sector approach to cope with it. However, a time-varying boundary layer width may result in sophisticated systems and cause a difficult implementation. Bartolini [1] and Bengiamin and Kauffmann [2] suggested to insert an integrator to the system such that the chattering can be smoothed, but this slows down the system response. On the other hand, Hwang and Lin [5], Lin and Kung [9], and Lin and Chen [10] applied the fuzzy set theory to handle the chattering problem. The control scheme of the fuzzy sliding mode control can smooth the chattering, but this also slows down the system response. The hitting time reduction and the chattering attenuation are two essential requirements in the slide mode control. The problem of the mentioned methods is that most of them are difficult to simultaneously take into account both the hitting time and the chattering. To remedy this problem, we propose a selection method to choose the appropriate gain parameters so that the controlled system can simultaneously have small hitting time and small chattering in the sliding mode control design by taking the merit of the genetic algorithms that multiple objective functions can be considered.

In this paper, a parameter selection algorithm is proposed by GAs to select the gain parameters so that the controlled system can achieve a good overall performance in the slide mode control design. It is desirable to have the fast reaching velocity into the switching hyperplane during the reaching phase and herein slide to the origin with little chattering phenomena. Hence, the proposed technique is used to conquer the difficulty that how to simultaneously consider the hitting time and the chattering in the selection of the gain parameters. This paper is organized as follows. The system and problem is described in Section 2. The mechanism of GAs is presented in Section 3. The control parameters selected by GAs is described in Section 4. In Section 5, a plant is used to test the proposed method and some compared results demonstrate its feasibility. Finally, Section 6 concludes the paper.

2 Sliding Mode Control and Problem Description

In this section, the variable structure system with sliding mode control is briefly reviewed. Consider a linear time invariant single input control system represented by the following n-th order differential equation

$$y^{(n)}(t)+a_1 y^{(n-1)}(t)+...+a_{n-1}\ y^{(1)}(t)+\ a_n\ y(t)=u(t) \quad (1)$$

where u(t) is the input and y(t) is the output. Suppose the following state variable is selected

$$x_1(t)=y(t)$$
$$x_2(t)=\dot{x}_1(t)$$
$$\vdots$$
$$x_n(t)=\dot{x}_{n-1}(t) \quad (2)$$

then the matrix form of the system can be described as

$$\dot{x}(t)=Ax(t)+bu(t) \quad (3)$$

with

$$A=\begin{bmatrix} 0 & 1 & 0 & .. & 0 & 0 \\ 0 & 0 & 1 & .. & 0 & 0 \\ & \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & .. & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & .. & -a_2 & -a_1 \end{bmatrix} \quad b=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (4)$$

where $x=[x_1,\ldots,x_n]^T$ is an n-dimentional state vector, A is an nxn system matrix, and b is an n-dimensional input vector.

In the sliding mode control design, the first step is the determination of the switching functions. Assume the sliding surface is given by

$$s = c_1x_1(t)+c_2x_2(t)+\ldots+c_nx_n(t)=c^Tx(t)=0 \quad (5)$$

where $c=[c_1,\ldots,c_n]^T$ is the sliding surface coefficient vector. Without loss of generality, let $c_n=1$, then

$$c_1y(t)+c_2y^{(1)}(t)+\ldots+c_{n-1}y^{(n-2)}(t)+y^{(n-1)}(t)=0 \quad (6)$$

i.e.

$$(D^{(n-1)}+c_{n-1}D^{(n-2)}+\ldots+c_1)y(t)=0 \quad (7)$$

where D=(d/dt). Suppose $\lambda_1, \lambda_2,\ldots, \lambda_{n-1}$ are the eigenvalues of (7), then the switching hyperplane can be represented by

$$s = (D - \lambda_1)(D - \lambda_2)\ldots(D - \lambda_{n-1})y(t) = 0 \quad (8)$$

Comparing (7) and (8), we have

$$c_1 = (-1)^{n-1} \lambda_1 \lambda_2\ldots \lambda_{n-1}$$
$$c_2 = (-1)^{n-2}(\lambda_2 \lambda_3\ldots \lambda_{n-1}+ \lambda_1 \lambda_3\ldots \lambda_{n-1}+ \lambda_1 \lambda_2\ldots \lambda_{n-2})$$
$$\vdots$$
$$c_{n-1}=-(\lambda_1+\lambda_2+\ldots+\lambda_{n-1}) \quad (9)$$

So the coefficient vector c can be suitably chosen by assigning the desired eigenvalues so that the controlled system is stable and the dynamic behavior of the sliding mode control can be dominated by the specific sliding surface.

The second step in the sliding mode control design is the choice of the control law. In general, the control law can be considered separately by the two control terms (u_h and u_{eq}) and is represented by

$$u(t) = u_h(t) + u_{eq}(t) = -k^T x(t) \tag{10}$$

with

$$k = k_{eq} + k_h \tag{11}$$

where the discontinuous feedback gain vector $k_h = [k_h^1, k_h^2, \ldots, k_h^n]^T$ of the hitting control part u_h is to transfer the state anywhere to hit the switching hyperplane, and the equivalent feedback gain vector k_{eq} of the equvalent control part u_{eq} is to keep the system state staying on the sliding surface s=0. When the feedback gain $k_h = [k_h^1, k_h^2, \ldots, k_h^n]^T$ is appropriate chosen as

$$k_h^i = \begin{cases} \alpha_i, & \text{for } s \bullet x_i > 0 \\ 0, & \text{for } s \bullet x_i = 0 \\ \beta_i, & \text{for } s \bullet x_i < 0 \end{cases} \qquad i=1,2,\ldots n \tag{12}$$

it is shown that if the gain parameters α_i *and* β_i, i=1,2,..,n are chosen so that

$$s \bullet \dot{s} < 0 \tag{13}$$

is satisfied, then the hitting will occur i.e., the state trajectory of the system will be attracted toward the sliding surface s=0. When the state is on the sliding surface, the purpose of the equivalent control is to keep the state staying on the sliding surface so it can be derived from setting the time derivative of s, $\dot{s}$, equal to zero, that is

$$u_{eq} = u\big|_{\dot{s}=0} \tag{14}$$

From (3) and (5), we have

$$\dot{s}(t) = c^T \dot{x}(t) = c^T(Ax(t) + bu_{eq}(t)) = 0, \tag{15}$$

i.e.,

$$k_{eq}^T = (c^T b)^{-1} c^T A \tag{16}$$

Therefore, under the sliding mode (s=0), the equivalent closed loop system can be obtained, by substituting (16) into (3), as follows:

$$\dot{x} = [A - b(c^T b)^{-1} c^T A]x \tag{17}$$

We see that the coefficient vector, c, will govern the dynamic behavior of the system in the sliding mode..

In the sliding mode control of variable structure system, the switching gains will change the structure of the controlled system and affect the transient behavior in the switching structure and the dynamic characteristics of system. Due to the complexity of the combination of different switching gains ($\alpha_1, \beta_1, \ldots \alpha_n, \beta_n$) in k_h, the traditional sliding mode control design usually select these values by experiment. It is needed to study how to select the appropriate switching gains so that a high overall performance of small hitting time and small chattering can be achieved. In this paper, we introduce the GA to solve the selection problem of gain parameters.

3 Descriptions of Genetic Algorithms

GAs are search techniques using the mechanics of natural selection and natural genetics for efficient global searches [3, 4]. In comparison to the conventional searching algorithms, GAs has the following characteristics: (a) GAs work directly with the *discret*e points coded by finite-length strings (chromosomes), not the real parameters themselves; (b) GAs consider a group of points (called a population size) in the search space in every iteration, not a single point;
(c) GAs use fitness function information instead of derivatives or other auxiliary knowledge; and (d) GAs use probabilistic transition rules instead of deterministic rules. Generally, a simple GA consists of the three basic genetic operators: (a) Reproduction; (b) Crossover; and (c) Mutation. They are described as follows [7, 8].

(a) Reproduction:

Reproduction is a process to decide how many copies of individual strings should be produced in the mating pool according to their fitness value. The reproduction operation allows strings with higher fitness value to have larger number of while the strings with lower fitness values have a relatively smaller number of copies or even none at all. This is an artificial version of natural selection (strings with higher fitness values will have more chances to survive).

(b) Crossover:

Crossover is a recombined operator for two high-fitness strings (parents) to produce two offsprings by matching their desirable qualities through a random process. In this paper, the uniform crossover method is adopted. The procedure is to select a pair of strings from the mating pool at random, then, a mark is selected at random. Finally, two new strings are generated by swapping all characters correspond to the position of the mark where the bit is "1". Although the crossover is done by random selection, it is not the same as a random search through the search space. Since it is based on the reproduction process, it is an effective means of exchanging information and combining portions of high-fitness solutions.

(c) Mutation:

Mutation is a process to provide an occasional random alteration of the value at a particular string position. In the case of binary string, this simply means changing the state of a bit from 1 to O and vice versa. In this paper we provide a uniform mutation method. This method is first to produce a mask and select a string randomly, then complement the selected string value correspond to the position of mask where the bit value is “1”. Mutation is needed because some digits at particular position in all strings may be eliminated during the reproduction and the crossover operations. So the mutation plays the role of a safeguard in GAs. It can help GAs avoid the possibility of mistaking a local optimum for a global optimum.

The GA includes five fundamental parameters: (a) Population size, which influences amount of search points in every generation. The more population size in the Gas will increase the efficiency of searching, but it will time consuming; (b) Crossover probability, which influences the efficiency of exchanging information. In general, the crossover probability between 0.6 and 1; (c) Mutation probability, which occur with a small probability in the GAs, In general, the mutation probability under 0.1. A large mutation probability in GAs will eliminate the result of reproduction and crossover, which let GAs become a random search; (d) Chromosome length, which influences the resolution of the searching result. The GAs with longer chromosome length will have the higher resolution, but it will increase the search space; (e) Generations, which influences the searching time and searching result. The GAs with larger search space and less population size, it needs more generations for a global optimum.

4 Parameters Selected by GAs

In this section, a genetic based sliding mode control method is proposed so that the parameters of switching gain are self-generated by means of GAs based on the direction of a proposed fitness function. In order to select the set of control parameters $R=(\alpha_1,\beta_1,\alpha_2,\beta_2,\ldots,\alpha_n\beta_n)$ by using Gas, first, we select R as a parameter set and code it as a finite-length string, then choose a fitness function so that GAs can be used to search for a better solution in the parameter space. If we define a function, the search direction of GAs will depend on the requirement of fitness function. So it is a key role on the defined fitness function so that the controlled system can achieve a desired performance. In this paper, we want to find the gain parameters of the sliding mode control to reduce the hitting time and the chattering of the controlled system, so we propose the following fitness function:

$$fi = g_1(HT) \bullet g_2(CH) \quad (18)$$

where HT is the value of the hitting time denoted by the first time so that $s(HT)=0$ and CH is the value of the chattering denoted by $CH=\int_{HT\le t}|s(t)|dt$. By taking the merit of the genetic algorithm, two major performance measures (the hitting time and chattering) of the controlled system’s response in the slide mode control design can be considered simultaneously in the proposed fitness function so that the selected controller by GAs has the ability to consider the hitting time and the chattering of the controlled system simultaneously. The function of g_1 and g_2 are described by

$$g_1(HT)=\left[\frac{1}{1+(\frac{HT}{\delta_1})}\right]^2 \quad (19)$$

$$g_2(CH)=\left[\frac{1}{1+(\frac{CH}{\delta_2})}\right]^2 \quad (20)$$

where the membership function of the fuzzy concept is used to express the grade of goodness of each performance. Note that, the representation of membership function (δ_1 and δ_2) can be adjusted according to the designer's specification or the system requirement. The proposed fitness function contains multiple objective, which is different from the conventional performance criterion expression. In this way, the selected control parameters based on the direction of the proposed fitness function will provide the system with a good overall performance of small hitting time and small chattering. That is, as *fi* increases as greatly as possible, the global performance of the controlled system corresponding to the string will work as well as possible. Therefore, the selection problem becomes the following optimization problem

$$\underset{R \in P}{MAX} \{ f_i(R) \} \tag{21}$$

where *R* is a string which represents a point located in the search space *P*. Hence, three basic genetic operators can be applied to select the parameters ($\alpha_1, \beta_1, \alpha_2, \beta_2, \ldots, \alpha_n, \beta_n$) to maximize the performance index in the parameter space P. If the final string is obtained, it can be selected as the gain parameters of the sliding mode control so that a high performance can be achieved. The procedure of selecting the control parameters by GAs is summarized as follows:

Step 1 : Determine the set of selecting parameters and select the GA's parameters.
Step 2 : Construct an initial population randomly.
Step 3 : Decode each string in the population and evaluate the performance of the response.
Step 4 : Evaluate the fitness value for each string.
Step 5 : Reproduce strings into the mating pool according to the fitness value calculated in Step 4.
Step 6 : Create the offspring and replace old strings by the offspring through the crossover and mutation operations.
Step 7 : Go to Step 3 until the maximum number of iterations is met.

5 Simulation Results and Discussion

In this section, we illustrate the performance of the proposed method by applying it to deal with the following system:

$$\dot{x}(t) = Ax(t) + bu(t) \tag{22}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad x^T(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The sliding surface is chosen as

$$s = c^T x(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(t)$$

(23)

From (16), we have

$$k_{eq}^T = (c^T b)^{-1} c^T A = [-1 \ -1] \tag{24}$$

i.e.,

$$u_{eq}(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(t) \tag{25}$$

The equivalent control will keep the stste to slide slong the specific surface. If the feedback gain is appropriately chosen as

$$k_h^i = \begin{cases} \alpha_i, & if\ s \bullet x_i > 0 \\ 0, & if\ s \bullet x_i = 0 \\ \beta_i, & if\ s \bullet x_i < 0 \end{cases} \qquad i=1,2 \tag{26}$$

then the control law becomes

$$u = -(k_{eq}^T + k_h^T)x = (1 - k_h^1)x_1 + (1 - k_h^2)x_2 \tag{27}$$

The required work is to find a appropriate combination of gain parameters ($\alpha_1, \beta_1, \alpha_2, \beta_2$). In this problem, the proposed method is compared with a traditional method to demonstrate its feasibility in the selection problem so that a better performance with a small hitting time and a small chattering can be achieved. They are described as follows:

(i) A traditional sliding mode control (SMC) method:

If the feedback gain is chosen according the extreme values of the control, then it can be represented by

$$k_h^i = \begin{cases} \alpha_i = 10, & if\ s \bullet x_i > 0 \\ 0, & if\ s \bullet x_i = 0 \\ \beta_i = -10, & if\ s \bullet x_i < 0 \end{cases} \qquad i=1,2 \tag{28}$$

And the responses of this method are shown in Figure 1.

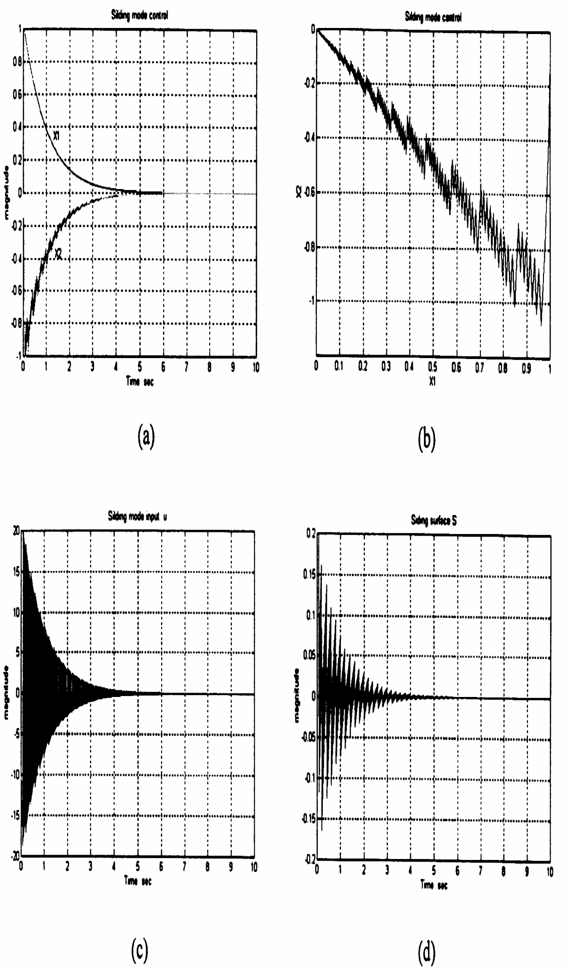

Figure 1. The simulation results of an traditional sliding mode control method. (a) The time response of state variables (b) The state trajectory in the phase plane. (c) The control signal u (d) The time response of s(t).

(ii) The genetic based sliding mode control (GSMC) method:

The required work is to find an appropriate combination of gain parameters by the proposed method so that a better performance with a small hitting time and a small chattering can be achieved. For the comparison, the searching spaces of the gain parameters $\alpha_1, \beta_1, \alpha_2$ and β_2 by GAs are all limited to [-10,10] according to the values chosen in equation (30). The sampling period is 0.01 sec, $\delta_1 = 0.5$, $\delta_2 = 4.5$, and the following parameters of GAs are considered.

population size = 1500

crossover probability = 0.6

mutation probability = 0.05

chromosome length = 48 (12 bit for each parameter)

generations = 20.

Following the proposed procedure in Section 3, a set of the control parameters ($\alpha_1, \beta_1, \alpha_2, \beta_2$) which maximizes the performance is automatically and efficiently selected by $\alpha_1 = 9.742400$, $\beta_1 = 9.685020$, $\alpha_2 = 9.685020$ and $\beta_2 = -6.877060$. Figure 2 shows the final results of the controlled system where the gain parameters are selected by the GSMC method. The prognun for the proposed GSMC is written in C language and simulated by PC 486-50. The time to find the parameters by the proposed method only need a few minutes, so the time consuming trial-and-error method is prevented by this method. The results show that the appropriate gain parameters can be efficiently determined by our method. In Figure 3, the best-of-generation fitness value against the generation number is plotted. This result shows that the selected parameter set has converged to a stable solution with a high performance.

Comparing the simulation results of the two methods from Figure 1 and Figure 2, we find that the overall performance of the proposed method (GSMC) is more better than that of the traditional method (SMC). That is, the objective that the gain parameters can be automatically selected so that the controlled system has a better performance of small hitting time and small chattering is satisfied.

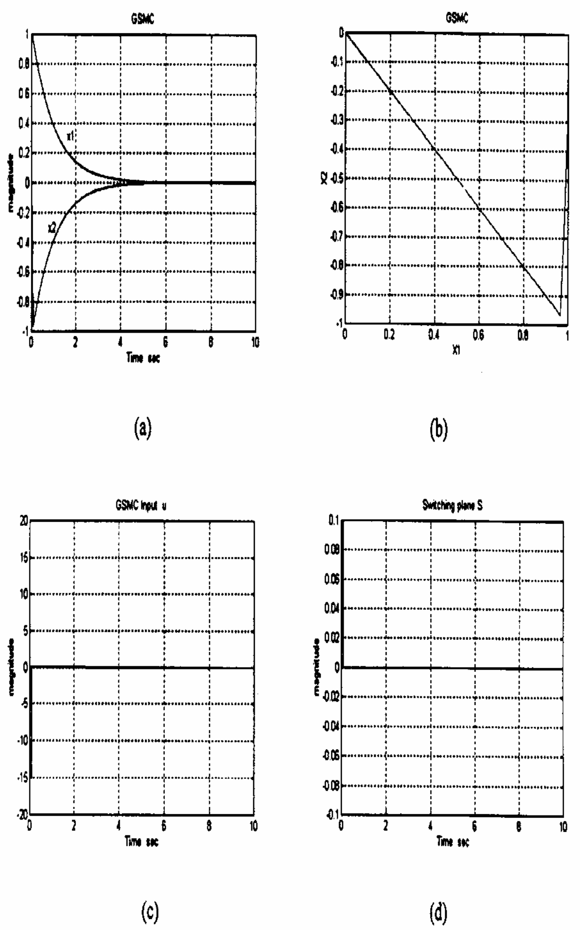

Figure 2. The simulation results of the genetic based sliding mode control method. (a) The time response of state variables (b) The state trajectory in the phase plane. (c) The control signal u (d) The time response of s(t).

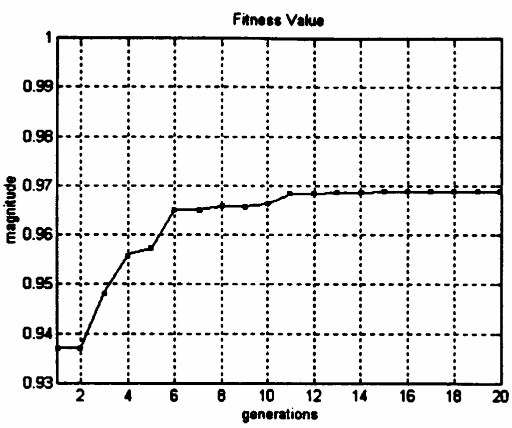

Figure 3. The fitness value convergence diagram

6 Conclusion

In this paper, the problem about the improvement of sliding mode control design is investigated. It is desirable to have the fast reaching velocity into the switching hyperplane in the hitting phase and herein slide to the origin with little chattering phenomena in the sliding phase. The main objective is to propose an effective method to choose an appropriate parameter set by using GAs to reduce the hitting time and attenuate the chattering so that a high overall performance of small hitting time and small chattering can be achieved. The advantage of the GAs is that they don't need extra professional knowledge or mathematics analysis. During the execution of the GAs, only the fitness function of the strings is evaluated. The performance surface doesn't need to be differentiated with respect to the change of control parameters and no derivatives, gradient calculations or other environment knowledge is necessary by GAs. Therefore, GAs are more suitable for this design problem than other searching methods such as gradient-based algorithm and random searching algorithm. Since GAs can consider multiple objective problem and the selected control parameters by GAs is based on the direction of the fitness function, we choose the hitting time and the chattering of the controlled system's response as the performance measures for selecting the parameters. The proposed fitness function is defined in such a way that the selected parameters can drive the state to hit the sliding surface fast and then keep the state slide along the surface with less chattering. Finally, the performance of the proposed method is compared with that of the other control structure. From *the* results, we find that the control parameters can be easily and efficiently selected fiom the proposed method and the selected control parameters can provide the controlled system with a high global performance where the hitting time is small and the chattering is small.

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