Genetic-based Sliding Mode Fuzzy Controller Design

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Abstract

In this paper, we proposed a genetic-based sliding mode fuzzy controller design method to avoid the chattering phenomena in traditional sliding mode controller (SMC), and shorten the hitting time of the controlled system. Chattering reduction is an important issue in the sliding mode controller that has been wildly discussed, and we know that a proper width Φ of the boundary layer of sliding surface can suppress chattering in sliding surface very well. In the other hand, the time that system state hits the sliding surface from initial state is also an important property that influences the performance of SMC. The time can also be shortened via a suitable determination of parameter K. In this paper, a genetic-based method is developed to obtain these two important parameters. An inverse pendulum system is utilized to perform the control effect of the designed sliding mode fuzzy controller.

Key Words: Fuzzy Control, Sliding Mode Control, Genetic Algorithm

1. Introduction

In various nonlinear system control issues, fuzzy controller is recently a popular method to combine with sliding mode control method that can improve some disadvantages in this issue. Comparing with the classical control theory, the fuzzy control theory does not pay much attention to the stability of system, and the stability of the controlled system cannot be so guaranteed. In fact, the stability is observed based on following two assumptions: First, the input/output data and system parameters must be crisply known. Second, the system has to be known precisely. For example, we can cite the Bode plot, Nyquist criterion of a linear system, the Lyapunov method, or the Popov & Cride criterion of the nonlinear system to observe the stability of system. These methods are always based on mathematical analysis. The fuzzy controller is weaker in stability because it lacks a strict mathematics model to demonstrate, although many researches show that it can be stabilized anyway [3,6,12]. Nevertheless, the concept of a sliding mode controller (SMC) can be employed to be a basis to ensure the stability of the controller. However, in SMC, the high frequency chattering phenomenon that results from the discontinuous control action is a severe problem when the state of the system is close to the sliding surface. To alleviate the chattering phenomenon, [13,19] used an idea of the boundary layer, called a modified sliding controller, to improve it. In this method, the control action was smoothed such that the chattering phenomenon can be decreased. Next, some researches [9,10,14,18] added a fuzzy structure to make the sliding mode controller more practical. The behavior of the sliding mode fuzzy controller (SMFC) is like a modified sliding mode controller. We will discuss details later.

Besides, one advantage of the sliding mode control is that when the system state reaches the sliding surface, the system state is insensitive to uncertainty or external disturbances of plant parameters [4,15]. However, some uncertainty that appeals before the system state reaches the sliding surface will still influence the stability of the system. If the system state can hit the sliding surface faster, the system stability and performance is more guaranteed. Some researches [11,16,20] were proposed to deal with this issue. These two issues, chattering reduction and hitting time shortening, in sliding mode control can be directly improved by choosing proper width of boundary layer and control gain. In this paper, the genetic algorithm (GA) is applied to determine the parameter set, consisting of the width of boundary layer and control gain. The hitting time and the chattering magnitude of the controlled system are considered as the performance index in GA and then an appropriate parameter set can be obtained, according to the guide of the determined fitness function, to reach the goal of hitting time shortening and chattering reduction.

2. Sliding Mode Controller

In the hyperplane of the state space, we use the dynamic character of system to obtain a sliding surface. Essentially, the conventional sliding mode controller uses a discontinuous control action to drive the state trajectory. When the state trajectory starts from an arbitrary point in the state space and moves toward the sliding surface, this process is called a "reaching mode". And we call it a "sliding mode" when the trajectory asymptotically tends to the origin of the hyperplane along the sliding surface. A detailed description of the sliding mode controller will be given as following.

Consider a nonlinear time-varying system, described by a *n-th* order differential equation is:

$$y^{(n)} = f(X) + b(X)u + d(t) \tag{1}$$

where $X = [x_1(t), x_2(t), \cdots, x_n(t)]^T$ is the states vector, u is the control input, $f(X)$ and $b(X)$ are nonlinear functions, and $d(t)$ is an external disturbance which is defined as $|d(t)| \le Dis(t)$. We can present (1) in a canonical model of state space [3] as follows:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
&\vdots \\
\dot{x}_{n-1}(t) &= x_n(t) \\
\dot{x}_n(t) &= f(X) + b(X)u + d(t) \\
y(t) &= x_1(t)
\end{aligned} \tag{2}$$

Next, we define a time-varying sliding surface $S(X,t) = 0$ in the state space R^n by the scalar equation

$$S(X,t) = CX \tag{3}$$

where $C = [c_1, c_2, \cdots, c_{n-1}, 1]$ is a strictly positive real constant, and we get

$$S = c_1 x_1(t) + c_2 x_2(t) + \cdots + c_{n-1} x_{n-1}(t) + x_n(t) = 0 \tag{4}$$

From (2), we can eliminate disturbances by (4) and obtain

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
&\vdots \\
\dot{x}_{n-1}(t) &= -c_1 x_1(t) - c_2 x_2(t) - \cdots - c_{n-1} x_{n-1}(t)
\end{aligned} \tag{5}$$

It shows that the state will not be influenced by the external disturbances or a sudden change of system parameters when the state reaches the sliding surface. According to (5), which characterizes the dynamic behavior of system on the sliding surface we have

$$D^{(n-1)} x_1 + c_{n-1} D^{(n-2)} x_1 + \cdots + c_1 x_1 = 0 \tag{6}$$

where $D = \frac{d}{dt}$. We know that the system will be stable, if we can choose the values of $c_1, c_2, \cdots, c_{n-1}$ appropriately to make the roots of (6) in the open left-half complex plane, and the purpose of the sliding mode control will be reached.

From the theorem of Lyapunov, we can choose a Lyapunov function candidate as

$$V = \tfrac{1}{2} S^2 \tag{7}$$

Obviously, it is guaranteed that V is a positive real constant. From the theorem, if the $\dot{V} < 0$ is satisfied, the state trajectory of the system will be forced to approach the sliding surface. The condition of $\dot{V} < 0$ for the sliding mode controller is as followed:

$$\begin{aligned}
\dot{V} &= S\dot{S} \\
&= S(c_1\dot{x}_1 + c_2\dot{x}_2 + \cdots + c_{n-1}\dot{x}_{n-1} + \dot{x}_n) \\
&= S[c_1 x_2 + c_2 x_3 + \cdots + c_{n-1} x_n + f(X) + b(X)u + d(t)]
\end{aligned} \tag{8}$$

and we can obtain (9) from (8)

$$u = \hat{u} - K\,\mathrm{sgn}(sb(X)),\ K > \tfrac{Dis(t)}{|b(X)|} \tag{9}$$

where

$$\hat{u} = \frac{-c_1 x_2 - c_2 x_3 - \cdots - c_{n-1} x_n - f(X)}{b(X)} \tag{10}$$

and

$$\operatorname{sgn}(\varphi)=\begin{cases}1 & if\,\varphi>0\\ 0 & if\,\varphi=0\\ -1 & if\,\varphi<0\end{cases} \tag{11}$$

If we choose the control law u as (9), and we will guarantee that $\dot{V}$ is negative, and the system state will approach the sliding surface gradually. Although the control law (9) can force state to the sliding surface, a chattering phenomena will occur after the first time state hits the sliding surface. It is a drawback in the control behavior, and will lead to an unstable condition of the controlled system. That is owing to the discontinuousness of sgn function in (11), see Figure 1, and even the control action u changes slightly in (8), it might cause a dramatic oscillation of system state. The modified sliding mode controller invites an idea to restrict the width of boundary layer Φ, and uses a continuous function to smooth the control action:

$$sat(sb(X)/\Phi)=\begin{cases}\operatorname{sgn}(sb(X)/\Phi) & if\,\left|sb(X)/\Phi\right|\geq 1\\ (sb(X)/\Phi) & if\,\left|sb(X)/\Phi\right|<1\end{cases} \tag{12}$$

The $sat(sb(X)/\Phi)$ (see Figure 2) is substituted for the $\operatorname{sgn}(sb(X))$ in (9). Therefore, the problem of the discontinuousness of u can be solved, and the chattering phenomena will be decreased. The modified control behavior can be described as:

$$u=\hat{u}-Ksat(sb(X)/\Phi),\ \Phi>0 \tag{13}$$

where

$$\hat{u}=\frac{-c_1x_2-c_2x_3-\cdots-c_{n-1}x_n-f(X)}{b(X)} \tag{14}$$

and

$$sat(a)=\begin{cases}\operatorname{sgn}(a) & if\ |a|\geq 1\\ a & if\ |a|<1\end{cases} \tag{15}$$

It is called a modified sliding mode controller.

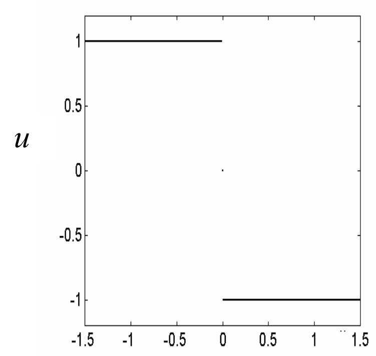


Figure 1. Control signal of conventional sliding mode controller

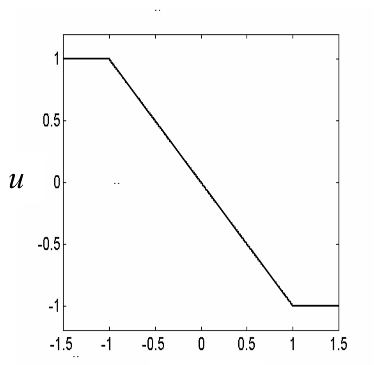


Figure 2. Control signal of modified sliding mode controller

3. Sliding Mode Fuzzy Controller

In this section, a fuzzy sliding surface is introduced to develop a sliding mode controller. The IF-THEN rules of fuzzy sliding mode controller can be described as [6]:

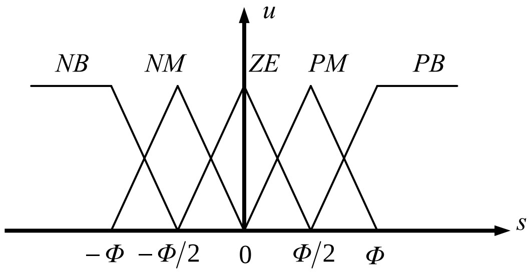
$$\begin{aligned}&R^1\!: If\ s\ is\ NB,\ then\ u_f\ is\ BIGGER\\ &R^2\!: If\ s\ is\ NM,\ then\ u_f\ is\ BIG\\ &R^3\!: If\ s\ is\ ZE,\ then\ u_f\ is\ MEDIUM\\ &R^4\!: If\ s\ is\ PM,\ then\ u_f\ is\ SMALL\\ &R^5\!: If\ s\ is\ PB,\ then\ u_f\ is\ SMALLER\end{aligned} \tag{16}$$

where *NB*, *NM*, *ZE*, *PM*, *PB* are linguistic terms of antecedent fuzzy set, they mean negative big, negative medium, zero, positive medium, and positive big, respectively. We can use a general form to describe these fuzzy rules:

$$R^i\!: If\ s\ is\ A_i,\ then\ u_f\ is\ B_i,\ i=1,\cdots,5 \tag{17}$$

where A_i is a triangle-shaped fuzzy number and B_i is a fuzzy singleton, see Figure 3 and Figure 4. Let *X* and *Y* be the input and output space, and *A* be an arbitrary fuzzy set in *X*. Then a fuzzy set, $A\circ R^i$ in *Y*, can be determined by each R^i of (17). We use the *sup-min* compositional rule of inference [6,7]:

$$\mu_{A\circ R^i}(u_f)=\sup_{s\in X}[\min[\mu_A(s),\min[\mu_{A_i}(s),\mu_{B_i}(u_f)]]] \tag{18}$$



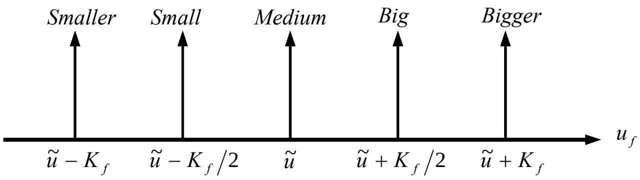


Figure 3. The input membership function of the sliding mode fuzzy controller

Figure 4. The output membership function of sliding mode fuzzy controller

If the antecedent fuzzy set A is a fuzzy singleton, we have

$$\mu_A(s)=\begin{cases}1 & if \;\; s=\alpha\\ 0 & otherwise\end{cases} \tag{19}$$

and the equation (18) becomes

$$\mu_{B_i}=\mu_{A\circ R^i}(u_f)=\min[\mu_{A_i}(\alpha),\mu_{B_i}(u_f)] \tag{20}$$

By using the center of area defuzzifier, we can obtain a crisp output u

$$u=\frac{\sum_{i=1}^{5}u_f\cdot\mu_{B_i}(u_f)}{\sum_{i=1}^{5}\mu_{B_i}(u_f)} \tag{21}$$

Figure 5 shows the result of a defuzzified output u of fuzzy input s, and we can use an equation to describe the relationship between u and s as follows:

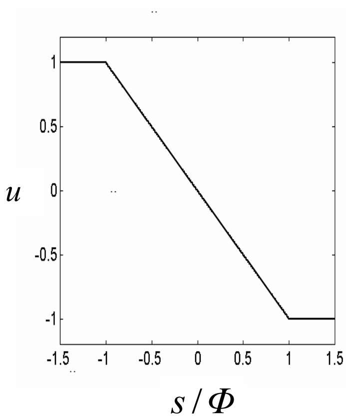


Figure 5. The control signal of sliding mode fuzzy controller

$$u=\tilde{u}-K_f sig(s/\Phi) \tag{22}$$

where

$$sig(a)=\begin{cases}1 & if \;\; a\ge 1\\ a & if \;\; -1<a<1\\ -1 & if \;\; a\le -1\end{cases} \tag{23}$$

From (22) and (13), we can find that the control signal in sliding mode fuzzy controller and modified sliding mode controller are totally the same. We can summarize the discussion of above. If we want to design a sliding mode fuzzy controller, the membership function of input and output can be obtained from the modified sliding mode controller. That is, the center of output fuzzy set $\tilde{u}$ can be substituted by $\hat{u}$ in the modified sliding mode controller, and the span of fuzzy set K_f can be substituted by K in the modified sliding mode controller. Then we can ensure the stability and robustness of fuzzy controller.

4. Genetic Algorithm Applied in Controller Design

In general, the performance of sliding mode controller is influenced by two important factors: chattering phenomenon and hitting time. The chattering phenomenon of sliding mode controller usually occurs when the system state gets close to the sliding surface, and it will affect the stability of the controlled system. Furthermore, if we can shorten the time that the state hit the sliding surface, the system with the desired dynamic character will be faster, and it can also decrease the uncertainty of the system. In order to improve the performance of sliding mode controller, we try to adjust the parameters Φ and K in equation of control law. We know that the width Φ of boundary layer will influence the chattering magnitude of sliding mode controller, and K will influence how soon the state reaches the sliding surface. We use GA to search the appropriate values of these two parameters and regard Φ and K as a parameter set

$$R_i=(\Phi,K) \tag{24}$$

that is going to form the chromosome in GA. We use the hitting time and chattering of the controlled system as the performance index of fitness function. The definition of the fitness function is defined as follows:

$$f(R_i)=g_1(HT(R_i))\times g_2(CH(R_i)) \tag{25}$$

where $HT(R_i)$ is the value of hitting time. It is obtained by calculating the time that the state first hits the sliding surface, and when the state reaches, $S(X, HT) = 0$. $CH(R_i)$ is chattering quantity that is defined by $CH(R_i) = \int_{t \geq HT} |S(X,t)| dt$. The definitions of g_1 and g_2 are

$$g_1(HT(R_i)) = e^{-(HT/W_1)^2} \quad (26)$$

$$g_2(CH(R_i)) = e^{-(CH/W_2)^2} \quad (27)$$

where W_1 and W_2 are weight factors that control the value of the hitting time and chattering quantity. From the definition of fitness function, the smaller chattering quantity and the shorter hitting time will make $f(R_i)$ a larger value and it means that the performance of the controlled system with the selected parameter set will be better.

In GA, we only need to select some suitable parameters, such as generations, population size, crossover rate, mutation rate, and coding length of chromosome [1,2,17], then the searching algorithm will search out a parameter set to satisfy the designer's specification or the system requirement. In this paper, GA will be included in the design of sliding mode fuzzy controller.

5. Simulation of Inverse Pendulum

To prove the efficiency of the proposed method, we apply the designed controller to control an inverse pendulum system. The state equation of an inverse pendulum is described by [5,8]

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f + bu \quad (28)$$

where

$$f = \frac{g(m_p + m_c)\sin x_1 - m_p L x_2^2 \sin x_1 \cos x_1}{L \cdot \left(\frac{4}{3}(m_p + m_c) - m_p \cos^2 x_1 \right)} \quad (29)$$

$$b = \frac{\cos x_1}{L \cdot \left(\frac{4}{3}(m_p + m_c) - m_p \cos^2 x_1 \right)} \quad (30)$$

where $x_1 = \theta$ is the angle of pole, respect to the vertical axis, $x_2 = \dot{\theta}$ is the angular velocity of the pole, u is the force to move the car, $g = 9.8$, $L = 0.5$, $m_c = 1$, and $m_p = 0.05$. We define the sliding surface as follows:

$$s = c_1\theta + \dot{\theta} \quad (31)$$

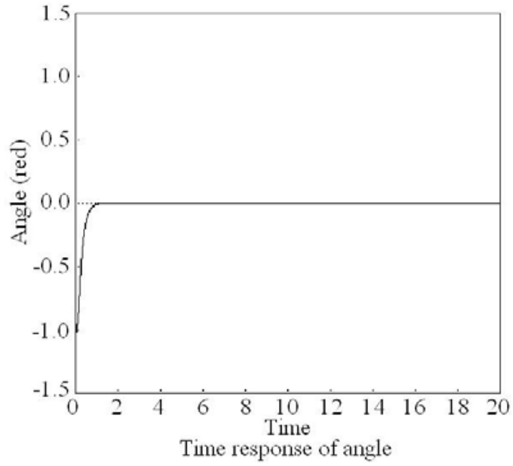
where $c_1 = 5$, and the control input u is chosen as

$$u = \hat{u} - Ksig(s/\Phi) \quad (32)$$

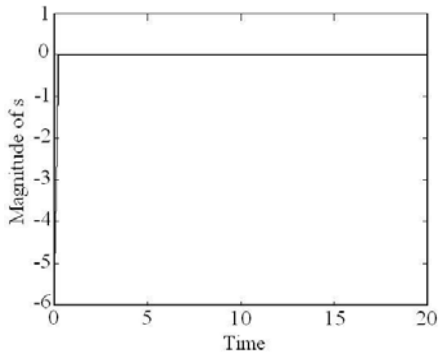
where

$$\hat{u} = \frac{-c_1\dot{\theta} - f}{b} \quad (33)$$

In GA's parameter setting, we define $W_1 = 1, W_2 = 5$ and set the population size to 50, the number of generations is 50, crossover rate is 0.8 and mutation rate is 0.4. Set $\Phi \in [0,20]$, $K \in [0,20]$ as the searching range of Φ and K. Based on the proposed method, the selected value of Φ and K are 0.30311 and 19.961, respectively. Figure 6 shows the result of response. Figure 7 and Figure 8 are the results of the conventional sliding mode control and sliding mode fuzzy control, respectively. Their boundary layers Φ and K are set to 5 and 10. The results of chattering and hitting time by the three control methods are showed in Table 1.



(a)



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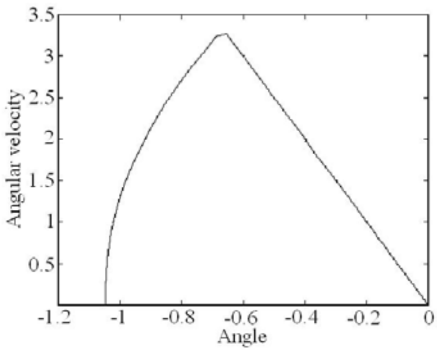
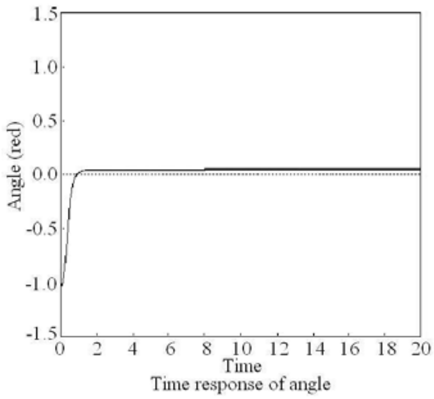
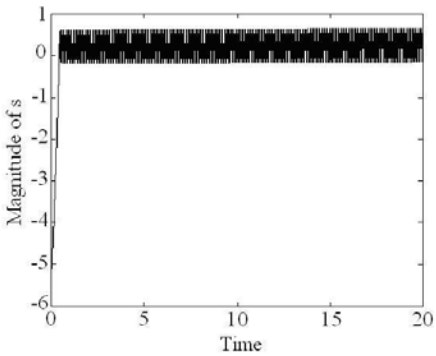


(c)

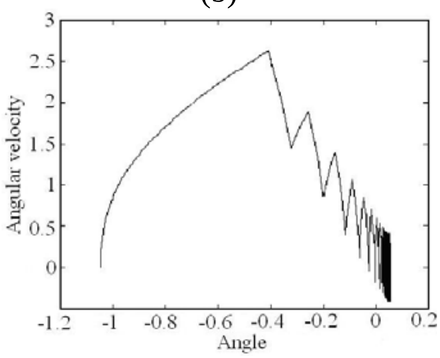
Figure 6. The simulation results by GA method: (a) The time response of angle, (b) The time response of *S*, and (c) The state trajectory in the phase plane



(a)



(b)



(c)

Figure 7. The simulation results of the conventional sliding mode control method: (a) The time response of angle, (b) The time response of *S*, and (c) The state trajectory in the phase plane

Table 1. The comparison of results by three control methods

	Φ	K	Hitting	Chattering
SMC	×	1	3.5	4.1603
	×	5	0.79	4.9935
	×	10	0.43	5.4048
	×	15	0.31	6.368
SMFC	1	10	0.71	0.000043146
	5	10	2.32	0.00024903
	7	10	3.23	0.00034047
	10	10	4.58	0.00049063
	5	3	7.57	0.00083025
	5	7	3.29	0.00035297
	5	10	2.32	0.00024903
	5	17	1.39	0.00014348
GSMFC $K \in [0,20]$ $\Phi \in [0,20]$	0.30311	19.961	0.24	0.000004363

From the result of simulation we can find that the control result of conventional sliding mode controller produces a serious chattering phenomena. On the contrary, the chattering phenomenon of controlled system is suppressed in the sliding mode fuzzy controller, but the hitting time becomes longer. This result is due to the smoothness of control force in the sliding mode fuzzy controller. The smooth control force decreases the sudden change in the sliding surface, but provides a smaller force to speed the state to the sliding surface. However, this issue can be improved by applying GA into the sliding mode fuzzy controller to determine a proper parameter set, and minimize the hitting time and chattering, simultaneously.

6. Conclusion

In this paper, the issue of the improvement of the sliding mode control design is investigated. We wish to have a fast reaching velocity to the sliding surface in the reaching phase and herein slide to the origin with little chattering phenomena in the sliding phase. The main objective is to propose an efficient method to choose an appropriate parameter set by using GA to reduce the hitting time and attenuate the chattering such that a high performance of small hitting time and small chattering can be achieved. The advantage of GA is that they do not need extra professional knowledge or mathematics analysis but fitness function as a guide to find a better parameter set. The performance surface does not need to be derivative with respect to the change of control parameters and no derivatives, gradient calculations or other environment knowledge are necessary by GA. Therefore, GA is more suitable for this design problem than other searching method such as gradient-based algorithms. Since GA can consider multiple objective problem and the selected control parameters by GA are based on the direction of the fitness function, we choose the hitting time and the chattering of the controlled system's response as the performance measures for selecting the parameters. The proposed fitness function is defined in such a way that the selected parameters can drive the state to hit the sliding surface fast and then keep the state slide along the surface with less chattering. Finally, the performance of the proposed method is compared with that of the other control structure. From the results, we find that the control parameters can be easily and efficiently selected from the proposed method and the selected control parameters can provide the controlled system with a high global performance where the hitting time is small and the chattering is small.

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