Optimal Mechanical Design with Robust Performance by Fuzzy Formulation Strategy

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Abstract

This paper presents an optimum design methodology for obtaining the highest robust performance using fuzzy multi-objective formulation strategy. The target performance and its variation, as functions of normally distributed variables with stochastic independence, are simultaneously minimized in this design process. A functional representation of the variability of the performance and the computational algorithm of the robust design process are presented in the paper. Two categories of design problems are examined: (1) the robust design with expected target of minimal variation. (2) The robust design with optimized target of minimal variation. The strength-based reliability behaves as the design objective that was merged in the formulation to extend the application of the proposed method. Three mechanical design examples further illustrate the presented integrated design methodology and successfully show its advantage.

Key Words **:** Robust Mechanical Design, Variability Optimization, Mean and Variance, Fuzzy Formulation, Engineering Design

1. Introduction

Conventional optimization minimizes the nominal value of the performance (objective) function and overlooks the deviation of the performance functions due to manufacturing and operation errors. In addition, the design variables and parameters often contain a range of uncontrollable variations or errors, as acknowledged by engineers. Those unavoidable variations, natural or artificial, will convey to the performance functions with uncertainties. These uncertainties considerably reduced the performance of the final design shown in Fig. 1 where the design point of *R* is more "robust" than the design of *P*. Accordingly, a robust design should optimize the both of the expected value and the deviation of the performance function simultaneously. Even though the control technique of Parkinson [1] of reducing the both of design parameter variability and the performance is available, however, this parameter

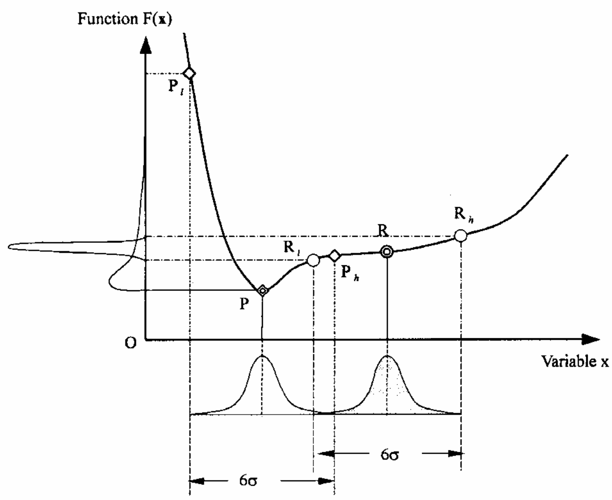

Figure 1. Nominal Optimum point and robust optimum point

variation cannot be eliminated in all the optimization problems.

The common employed strategy for obtaining the robust design is the Taguchi method in which two criteria are proposed: the average loss function and signal- to- noise ratios (S/N ratios). Each criterion contains three representations: lower-the-better, higher-the-better and nominal-the-better [2]. For example, the objective function of the S/N ratio for the case of nominal-the-better is written as:

$$\text{Maximize S/N}= 10\cdot\log_{10}\left(\frac{\overline{m}^2}{s^2}-\frac{1}{r}\right) \tag{1}$$

where r is the sample size, $\overline{m}$ and s is the mean value and the standard deviation of performance function, respectively. Taguchi optimized the S/N ratio and did not consider the actual mean and deviation of the performance that may result in a solution far away from the peak-optimum. Chang [3] proposed a two-stage optimization process for obtaining the minimum performance variability where the objective function of second stage is written as following:

$$\text{Min} \;\; S=\sum_{n=1}^{N}\left[F^{(n)}-\overline{F}\right]^2 \tag{2}$$

The above equation represents the sum of the worst combination of the design performance variability. In Eq. (2), $F^{(n)}$ is the performance function of the nth worst case, N is the number of the worst-cases, and $\overline{F}$ is the nominal value of the first stage optimization. Eggert & Mayne [4] suggested a performance function that consists of a weighting sum of the expected mean $\mu_f(X)$ and the standard deviation $\sigma_f(X)$ of the performance f:

$$F(X)=\omega\,\mu_f(X)+\gamma\sigma_f(X) \tag{3}$$

The minimization of the above equation is helpful for reducing the performance variability. However, the selection of the weighting factors, ω and γ, are somewhat arbitrary. Yu and Ishii [5] recommended another form of *Obj* based on the concept of statistical worst case:

$$Min\;\; Obj \;=\; \mu_f+\beta\cdot\sigma_f \approx EP+\beta\cdot QI \tag{4}$$

where EP represents the expected mean value of performance function f, quality index of QI is an estimate of performance deviation, and β is the quality coefficient. Parkinson, D. B. [6] developed a variability function in Lagrange form that is minimized as following:

$$L= F_u - F_L + \lambda(\, F_N - F(X)\,) \tag{5}$$

where F_u and F_L are defined as $\max_X F(X)$ and $\min_X F(X)$, respectively. F_N is the nominal value of the function $F(X)$ and λ is the Lagrange multiplier.

One can observe that Equations (3) to (5) are trying to optimized nominal function and simultaneously minimize the variance of the performance function. These strategies have two defects: (1) since the expected mean and deviation of the performance function may not be proportional to each other. The minimization of simple addition of expected mean and deviation of performance function is not always suitable for constructing the utility objective function in the robust performance optimization. (2) The coefficient or weighting factor in the formulation is difficult to be determined in advance so that the designer will be in trouble for deciding the final design among several possible designs. Shih *et al* [7] adopted Eq. (2) to estimate the variance of the performance function and developed a reliability indicator representing the design performance robustness that can optimize the both of performance function and its variation simultaneously. However, this formula requires a fine-tuning parameter in the solution process by continuous trials and the errors exists in the approximation for the mean value of a performance function. On the other hand, the paper of Shih *et al* [8] is the preliminary study of the presenting paper.

The strength-based reliability of Rao's work [9] is commonly used to represent the reliability of a structural and mechanical system. The robust design of considering the system reliability is important especially for the industry with high precision, production and high technique requirements. In this paper, we employ the system reliability as the design objective to examine the robust optimal design. The design problem is formulated as the minimization of goal performance function and its variability at the same time. When each objective function is optimized, that result in other objective function has a corresponding value. Consequently, each objective function is in a fuzzy region between two extreme values. Several multi-criteria methods [10] were presented for dealing with such a problem, the fuzzy optimization technique is naturally suitable for solving current design problem due to the uncertain objective values. Fuzzy multi-objective optimization presented by Rao [11] and applied by Shih *et al* [7] has been adopted in this paper to deal with the uncertainty between the performance function and its variation. A functional representation for performance variability is presented that is equivalent to the function of the variance or standard deviation of the design performance. Three mechanical design examples were given to further illustrate the proposed robust optimization method and process. The use of fuzzy formulation strategy can result in a unique design with the highest design level in the sense of fuzzy degree of membership function.

2. Computation of Expected Mean and Variance of Performance

The approximated deviation of a response function *F*(*X*) can apply Taylor's expansion to get a closed-form as the following equation:

$$\sigma_F^2 = \sum_{i=1}^{n}\left(\frac{\partial F}{\partial x_i}\sigma_{x_i}\right)^2 \tag{6}$$

This simple system models are not available if the design involves either complexity or experiments, make the evaluations of response derivatives impractical. Taguchi [12] proposed 3-level method using uniform weighting for variables with normal distribution. He chose μ_i and $\mu_i \pm \sqrt{3/2}\sigma_i$ as three center points. This method works well for the estimation of expected mean, however, the estimation of variance becomes poor when dealing with the nonlinear effects. D'Errico & Zaino [13] presented an alternative method using Gauss-Hermite quadrature that provided better results particularly for nonlinear case. Fig. 2 illustrates the selection of two or three-point approximation and the corresponding weighting for normal variables. The expected value *EP* of a performance function *F(X)* and its deviation σ_F are written as following:

$$EP = E[F(X)] \approx \sum_{i=1}^{N} W_i \cdot F_i(X) \tag{7}$$

$$\sigma_F = \sqrt{Var[F(X)]} \approx \left\{\sum_{i=1}^{N} W_i[F_i(X) - E[F(X)]]^2\right\}^{1/2} \tag{8}$$

where $N=3^n$, *n* is the number of design variables, $W_i = w_1 \times w_2 \times \cdots \times w_n$, and w_i is the corresponding weighting of the *i*th variable x_i at the assigned level. Fig. 3 shows the weighting of the example of two variables with normal distribution. Eq. (9) shows an example of response surface function with the expected mean as -1.6366 and the standard deviation of 0.1767.

$$F(x_1, x_2) = 12.6 - 2.9x_2 + 0.5x_2^2 - 5.8x_1 + x_1^2 - 0.2x_1x_2 \tag{9}$$

where the mean value and the standard deviation of each design variables are $\mu_{x_1} = \mu_{x_2} = 2.9$ and $\sigma_{x_1} = \sigma_{x_2} = 0.2$, respectively. Table 1 gives the error percentages of the estimation results of mentioned methods and the 3-point approximation has the most accurate variance even though with

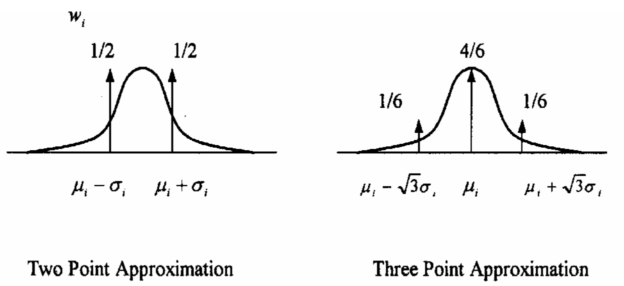

Figure 2. Selection of Gaussian points and weightings of normal variable

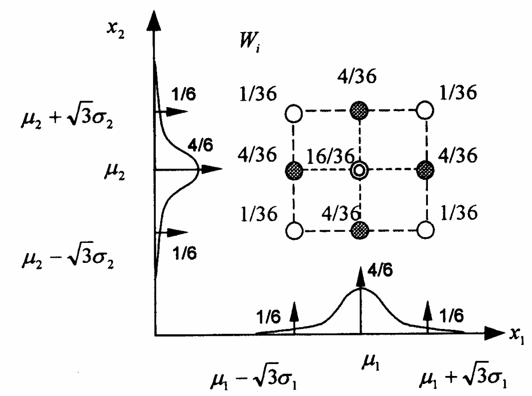

Figure 3. Values of W_i for two variables with normal distribution.

Table 1. Estimation errors of expected mean and variance

Method	Expected mean	Standard deviation	Error of expected mean(%)	Error of variance(%)	Number of experimental points.
Taylor's expansion	-1.6370	0.16405	--------	13.8068	--------
Taguchi 3-level method	-1.6370	0.16727	0.0244	10.3992	9
2-point approximation	-1.6370	0.16424	0.0244	13.6018	4
3-point approximation	-1.6370	0.17541	0.0244	0.7907	9

higher experimental points. Therefore, 3-point method is adopted for the computation of variance and standard deviation of design performance in this paper.

3. Robust Design with Expected Target of Minimal Variation

An optimization design problem may have specified beforehand a required design performance (or expected target). There are infinite different sets of final design as the optimal design with this expected target. However, only one set design has the minimal variation of the performance corresponding to the robust design. In this paper, we consider $\pm 3\sigma$ of a parameter as the limit range to find out the smallest value F_L and the largest value F_U of the performance function by the 2-level full factorial experiments. A variability representation between the largest and the smallest value of the performance function is taken as the objective function. At the end of the optimization, 3-point approximation is applied to compute the standard deviation of the final design performance. In this study, we applied the robust feasible direction method [14] for the numerical optimization. The algorithm is described in the following:

1. Initialize the design variables and the statistical information of parameters and variables. Define the predetermined expected mean value of *EP* as design performance function.
2. Compute the largest value of performance function indicated as F_U = Max $[F_i]$, $i=1,2^n$, and the smallest value of performance function indicated as F_L = Min $[F_i]$, $i=1,2,\ldots,2^n$, at the limit value of $\pm 3\sigma$.
3. Perform the optimization as following:
   Find the independent design variables of $X=[x_1, x_2,\ldots, x_n]^T$ by minimizing the largest variability in the following form:

$$\text{Minimize } \left|F_U - F_L\right| \tag{10}$$

$$\text{Subject to } F(X) - EP = 0 \tag{11}$$

$$g_i(X)\le 0, \quad i=1,2,\ldots,m \tag{12}$$

   where Eq. (11) indicates the equality constraint for that the optimum value of performance function is a pre-specified expected mean value. Eq. (12) represents other design constraints where *m* indicates the number of constreaints.
4. Check the convergence of the above optimization problem. If the problem is not converged yet, go back to step 2.
5. Compute the standard deviation of the performance of final design using 3-point approximation method.

Example 1: Slider-crank mechanism synthesis

A simple slider-crank mechanism consists of two connecting rods and a slider as shown in Fig. 4. The position of the slider is considered as the performance index. The operation starts with both rods lying horizontally and it stops when the crank rotates θ degrees. The optimization problem is to find the length of two rods x_1 and x_2, such that the deviation of the moving distance D of slider from the target is minimized, where the target of distance D is 1 cm when θ is 15°. The performance function from the kinematics analysis is written as:

$$F(x_1,x_2)=D=x_1+x_2-x_1\cos 15^{\circ}-(x_2^2-x_1^2\sin^2 15^{\circ})^{0.5} \tag{13}$$

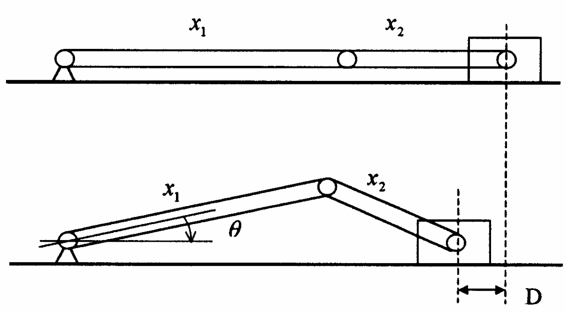

Figure 4. A slider-crank mechanism and its variables

The optimization formulation of this problem is written as following:

$$\text{Min } \left|F_U(x_1,x_2)-F_L(x_1,x_2)\right| \quad (14)$$

$$\text{Subject to } g_1: \; -x_2^2+x_1^2\sin^2 15^\circ \le 0 \quad (15)$$

$$h_1: \; F(X)-1.0=0 \quad (16)$$

where $0 \le x_i \le 10cm$ (i=1,2) and the standard deviation of each x_i is $\sigma_{x1}=\sigma_{x2}$= 0.03 cm. Table 2 shows the optimum result in which the deviation of final design computed by 3-point approximation method that is much smaller than that of initial design with 32.9% deduction. The final design using the proposed algorithm is identical to the Monte Carlo simulations and Chang's work [15].

Table 2. Designs of slider-crank mechanism

Item	x_1	x_2	Deviation of performance σ_F
Initial design	8.3673	3.6374	0.011104
Final design	10.00	5.4111	0.007446
Chang, 1989 [15]	10.0	5.41	0.00745

4. Robust Performance Optimization for Mechanical Design

When simultaneously minimizing the design objective and its variation, the problem becomes a multi-objective optimization problem. As described in the paper, the fuzzy characteristics existing in uncertain objective functions so that a fuzzy optimization problem can be constructed. The solution of this fuzzy formulation strategy can results in a unique design with the highest design level among the fuzzy environment. The design process including several optimization stages are stated in the following:

Step 1: Find the independent variables of X by minimizing $F(X)$ subject to $g_i(X)\le 0$, $i=1,2,\ldots,m$. This nominal design obtained in this formulation is the ideal value of performance function indicated as F_{ideal}.

Step 2: Find X by minimizing the largest variability of $\left|F_U-F_L\right|$ subject to $g_i(X)\le 0$, $i=1,2,\ldots,m$. The output of the performance function is defined as F_a. The variability of the performance function associated this output is defined as V_{ideal}.

Step 3: Find X by maximizing the variability of $\left|F_U-F_L\right|$ subject to $g_i(X)\le 0$, $i=1,2,\ldots,m$. The output performance function of this sub-problem is defined as F_b. The variability of the performance function associated this output is defined as V_{max}.

Step 4: Select the larger one between F_a and F_b as F_{max}. i.e. F_{max}= Max [F_a , F_b].

Step 5: Then a fuzzy formulation can be stated as following:

$$\text{Find } X \text{ by Maximize } \lambda \quad (17)$$

$$\text{Subject to } \lambda-\mu_F \le 0 \quad (18)$$

$$\lambda-\mu_V \le 0 \quad (19)$$

$$g_i(X)\le 0, \quad i=1,2,\ldots,m$$

where

$$\mu_F=\begin{cases} 1 & if \quad F(X)\le F_{ideal} \\ \dfrac{F_{\max}-F(X)}{F_{\max}-F_{ideal}} & if \;\; F_{ideal}<F<F_{\max} \\ 0 & if \quad F_{\max}\le F(X) \end{cases} \quad (20)$$

$$\mu_V=\begin{cases} 1 & if \quad \left|F_U-F_L\right|\le V_{ideal} \\ \dfrac{V_{\max}-\left|F_U-F_L\right|}{V_{\max}-V_{ideal}} & if \;\; V_{ideal}<\left|F_U-F_L\right|<V_{\max} \\ 0 & if \quad V_{\max}\le\left|F_U-F_L\right| \end{cases} \quad (21)$$

where the parameter λ is a scalar as well as an extra design variable with a meaning of the highest design level.

Step 6: Check the convergence of the above optimization problem. If the problem is not converge, go back to step 2.

Step 7: Compute the standard deviation σ_F of the performance function using the three-point approximation method described in section 3.

Example 2: A helical spring design

A mechanical helical spring design has the number of coil n, the wire diameter d of spring and the outside diameter can not exceed 26 mm. The design variable n and d are independently normal distribution with the standard deviation as σ_n = 0.015 and σ_d = 0.1 mm, respectively. An external load F applies on the spring that deforms an original height of h_f to the height of h_0. Another fluctuating load F_0 is applied on it to yield a fluctuating displacement δ_0. The problem is to optimally design this spring that has a fixed δ_0 to sustain the maximum amplitude of fluctuating load F_0 (Fig. 5). The parameter G represents the shear modulus of spring material that is equal to 8.4 (10^3) Mpa. The other values of related parameters are: h_f = 68 mm, h_0= 60 mm, D = 20 mm, δ_0= 5 mm. The mathematical formulation of this optimization problem can be written as the following:

$$\text{Find} \quad X = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} n \\ d \end{Bmatrix}$$

$$\text{Maximize}\, F_0 = \left(\frac{Gd^4}{8nD^3} \right)(h_f - h_0) \qquad (22)$$

$$\text{subject to} \quad \left(\frac{\tau_m - \tau_a}{\tau_w} + \frac{\tau_a}{\tau_e} \right) \le \frac{1}{S_f} \qquad (23)$$

$$nd \le (h_0 - \delta_0) \qquad (24)$$

$$D + d \le 26 \qquad (25)$$

where τ_m represents the mean stress, τ_a represents the alternating stress, τ_e and τ_w are 42 Mpa and 84 Mpa, represents the endurance limit and working stress of spring, respectively. The notation S_f represents the safety factor with value 1.1. Consequently, one can substitute all design parameters into the above formulation and the formulation can be simplified as follows:

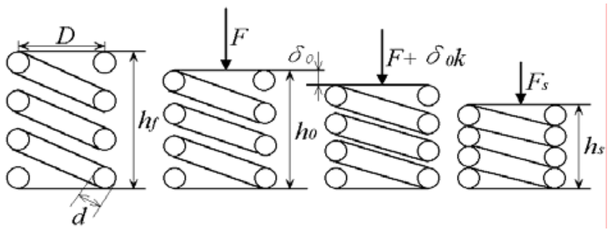

Figure 5. A helical spring with loading

$$\text{Maximize}\, F = \frac{1.05d^4}{n} \qquad (26)$$

subject to

$$1.356d\left(\frac{80-d}{80-4d} + 0.03075d \right) - n \le 0 \qquad (27)$$

$$nd - 55 \le 0 \qquad (28)$$

$$d - 6 \le 0 \qquad (29)$$

This problem was solved step-by-step using the previous describing algorithm. The result of nominal design and robust design is shown in Table 3 where the design range are $1 - d \le 0$ and $n - 30 \le 0$. The last two columns show the robust performance design and the modified robust design.

One can see that although the nominal design can sustain a high loading (performance function) with 80.982 kg, unfortunately, the deviation of the loading (6.1068 kg) is also large. Through the robust design process presenting in this paper, the deviation of the loading is reduced considerably with 43.38%. However, the sustaining loading reduces 57.1% that is relatively too low to be usable if it is considered by the designer. In this particular case, if the designer considered that a useful sustaining load could not be less than 55 kg. Thus, an additional constraint has to add on the formulation of Eq. (26) to (29) as following:

$$\frac{1.05d^4}{n} - 55 \le 0 \qquad (30)$$

The last column of Table 3 shows the modified robust design using the presenting algorithm where

Table 3. Design of a helical spring

Item	Nominal design	Robust design	Modified robust design
Number of coils n = x_1	10.348	7.987	9.388
Diameter of spring wire d = x_2 (mm)	5.315	4.032	4.937
Loading F (kg)	80.982	34.733	56.31
Deviation of performance σ_F (kg)	6.1068	3.4575	5.388

not only the deviation of the sustaining load reduced 11.77% compared to the nominal design, but also the sustaining load is usable. This modified design is able to means a relatively robust performance design.

5. Robust Design with maximizing the System Reliability of a Mechanical System

As described before that the system reliability is important for robust design, the formulation of reliability can be treated as a design target performance. This is better using the following example for further illustration.

Example 3: A mechanical shoe brake design

A single-shoe brake, shown in Fig. 6, has been designed to have a braking capacity of $T_C \pm \Delta T_C$ = (350± 35) lb-in. This problem is adopted and modified from Rao's work (1992) that a robust optimum design problem is created to find the mean value of b and r by maximizing the reliability of torque (T_b) acting on the brake drum without exceeding a mean pressure of 120 lb/in^2 on the brake pad. The applied force F, the coefficient of friction f between the brake drum and the brake shoe, the dimensions a, b, c, and r are known to follow the normal distribution as F = N(50, 5) lb,

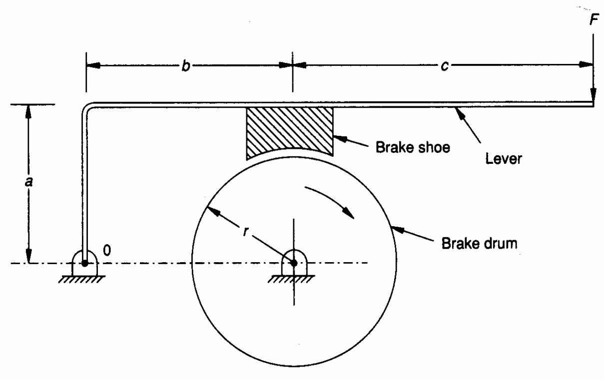

Figure 6. A single-shoe brake

f = N(0.3, 0.03), a = N(10, 1) in, b = $N(\bar{b},\ 0.01\bar{b})$ in, c = N(20, 2) in, and r = $N(\bar{r},\ 0.01\bar{r})$ in. The brake-shoe has a contact area of $\bar{A} \pm \Delta A$ = 4 $\pm$ 0.4 in^2. The optimization problem can be stated as follows:

$$\text{Find } X = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} \bar{b} \\ \bar{r} \end{Bmatrix}$$

$$\text{Maximize } F(X) = z_1 = \frac{\bar{T}_b - \bar{T}_C}{\sqrt{\sigma_{T_b}^2 + \sigma_{T_C}^2}} \quad (31)$$

$$\text{subject to } g_1: \frac{50(\bar{b}+20)}{\bar{b}-0.3\bar{r}} - 120 \le 0 \quad (32)$$

$$10 \le \bar{b} \le 25 \quad (33)$$

$$5 \le \bar{r} \le 15 \quad (34)$$

where $\bar{T}_b$ can be written as the following formulation:

$$\bar{T}_b = \frac{15\bar{r}(\bar{b}+20)}{(\bar{b}-0.3\bar{r})} \quad (35)$$

The parameter σ_{T_b} represents the standard deviation of T_b that can be found in the reference of Rao (1992). The standard deviation of T_C is represented as σ_{Tc} =35/3=11.67 lb/in^2. Table 4 shows the design results of this problem. The last column in the table is the robust performance design that is the same as the result of minimizing the variability of the objective function shown in the first column. This phenomenon can be

Table 4. Design of a mechanical shoe brake

Item	Min $\lvert z_{1U} - z_{1L}\rvert$	Max $\lvert z_{1U} - z_{1L}\rvert$	Robust design
Average length $\bar{b} = x_1$ (in)	22.0004	25.0	22.0004
Average radius $\bar{r} = x_2$ (in)	15.0	5.0	15.0
Performance function z_1	2.0751	3.9998	2.0751
Reliability of z_1 (%)	98.1	0	98.1
Deviation of performance σ_F (kg)	0.05166	0.1258	0.05166

investigated and explained as follows. Let's look at the robust design that requires to maximizing z_1 and minimizing $\sigma_{z_1}^2$, simultaneously. The maximization of z_1 via Eq. (31) yields to the maximization of $\overline{T}_b$ and the minimization of $\sigma_{T_b}^2$ where the parameters $\overline{T}_C$ and σ_{TC}^2 are known constants. When one simply analyze $\sigma_{z_1}^2$ by Eq. (6), one can found out that the minimization of $\sigma_{z_1}^2$ has a tendency of maximizing $\overline{T}_b$ and minimizing $\sigma_{T_b}^2$. From the other hand, one can analyze the minimization of $|z_{1U} - z_{1L}|$ that is equivalent to the minimization of $\sigma_{z_1}^2$. This investigation shows that the result of minimizing $|z_{1U} - z_{1L}|$ is equivalent to the robust design. Thus, it is no doubt to know that the result of the robust design is nothing different from the result of minimizing the variability of design goal in this particular case. However, this is not always true for all kinds of design problems. Moreover, the design process presenting in this paper definitely can be valuable to confirm a question: is a certain design result a truly robust performance design?

7. Conclusions

A robust performance design method and process, which applied the strategy of fuzzy optimization to optimize the performance function and minimize its variation simultaneously, is presented in this paper. The minimization of the largest variation between two extreme performance values is taken as the objective function for the variability optimization. A computational algorithm for this design process is presented that results in the final design with the maximum design level via the fuzzy design concept. Three mechanical design examples further illustrate the proposed method for the problems with fixed expected design target, the maximum structural system reliability, and optimum design performance with the minimum variance. From this study, one generally conclude that the effect of the robust design sometimes is not very obvious, as compared to the problem of minimizing the largest variation of performance function. Particularly, when the strength-base reliability is the design performance, the result of the robust design is equivalent to the result of minimizing the largest performance variability. However, the presenting robust design process resulting in the final design can be used as the confirmation of a truly robust performance design.

Acknowledgements

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