Direct Center Adaptive Fuzzy Controller Design

Ching-Chang Wong*, Bing-Chyi Huang*, and Jen-Yang Chen**

* Department of Electrical Engineering, Tamkang University,
Tamsui, Taipei, Taiwan, ROC
** Department of Electronic Engineering, China Institute of Technology and Commerce
No. 245, Sec.3, Yen-Chiu-Yuan Road, Teipei, Taiwan, ROC

Abstract

In this paper, a direct adaptive fuzzy controller with an adaptive law of center regulation is proposed. The adaptive law can modify the membership functions of fuzzy system on line. In this approach, we only use a parameter to design the center adaptive law so that it can reduce the complex of mathematical deduction and increase the speed of computing. Finally, a first-order nonlinear system and a second-order inverse pendulum system are used to prove the efficiency of proposed method.

1. Introduction

The main difference between adaptive fuzzy control systems and non-adaptive fuzzy control systems is an adaptive law is introduced to the adaptive fuzzy control systems to adjust the structure or/and parameters of the fuzzy controller. The fuzzy controller in the non-adaptive fuzzy control system is fixed before real-time operation, whereas the fuzzy controller in the adaptive fuzzy control system changes during real-time operation. The main advantages of the adaptive fuzzy control system over the non-adaptive fuzzy control system are: (a) less information about the process model is required because the adaptive law can help to learn the dynamic of the process during real-time operation, and (b) better performance is usually achieved because the adaptive fuzzy controller can adjust itself to the changing environment. A number of adaptive fuzzy controllers were proposed in [1,2] to discuss unknown or partially unknown nonlinear system. However, several turning parameters are applied in these approaches. In this paper, we develop a direct adaptive fuzzy controller to regulate the parameter of the fuzzy controller. The structure is described in Figure 1. The best merit of our approach is that only a single value regulation would be used to tune the membership functions of the consequent part. Therefore, the process of the proposed approach is more efficiency than that of those approaches employing several tuning parameters. Furthermore, in our arrangement of fuzzy controller the distribution of membership functions in the state space is also reserved after tuning.

2. Direct center adaptive fuzzy controller

2.1 The fuzzy control structure

Consider the following m-th order system with m input variables $(x_1, x_2, \cdots, x_m)$ and a single output variable y:

$$x^{(m)} = f(X) + gu \quad (1)$$

$$y = x \quad (2)$$

where $X = (x_1, x_2, \cdots, x_m)^T = (x, \dot{x}, \cdots, x^{(m-1)})^T$ is the state vector of the system and is assumed to be available for measurement, f is an unknown function and g is an unknown positive constant. We can structure the rule base as follows [3,4]:

$(j_1, j_2, \cdots, j_m)-th$ *rule:*
IF x_1 *is* $A_{(1,j_1)}$ *and* x_2 *is* $A_{(2,j_2)}$ *and…and* x_m *is* $A_{(m,j_m)}$
THEN u *is* $u_{h(j_1,j_2,\cdots,j_m)}$ (3)

where

$$j_i \in \{-n_i, \cdots, -1, 0, 1, \cdots, n_i\},\ i \in \{1, 2, \cdots, m\},$$

$$h(j_1, \cdots, j_m) \in \{-(n_1 + n_2 + \cdots + n_m), \cdots, -1, 0, 1, \cdots, (n_1 + n_2 + \cdots + n_m)\} \quad (4)$$

$$A_{(i,j_i)} \in T(x_i) = \{A_{(i,-n_i)}, \cdots, A_{(i,-1)}, A_{(i,0)}, A_{(i,1)}, \cdots, A_{(i,n_i)}\} \quad (5)$$

$$u_{h(j_1,\cdots,j_m)} \in T(u) = \{u_{-(n_1+n_2+\cdots+n_m)}, \cdots, u_{(0)}, \cdots, u_{(n_1+n_2+\cdots+n_m)}\} \quad (6)$$

From equations (5) and (6), we know the term sets $T(x_i)$ and $T(u)$ have $(2n_i + 1)$ and $2(n_1 + n_2 + \cdots + n_m) + 1$ fuzzy sets in the description of the input variable x_i and the output variable u, respectively. The j_i is considered as index, so $A_{(i,j_i)}$ is the j_i-th term of the term set $T(x_i)$. $h(j_1, \cdots, j_m)$ is a constant index function and defined by

$$h(j_1, \cdots, j_m) = j_1 + j_2 + \cdots + j_m, \quad (7)$$

so that $h(j_1, \cdots, j_m) \in \{(n_1 + n_2 + \cdots + n_m), \quad \cdots, 0, \cdots, (n_1 + n_2 + \cdots + n_m)\}$,

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which decides that $u_{h(j_1,\cdots,j_m)}$ is the $h(j_1,\cdots,j_m)$-th term of the term set $T(u)$. In the study, we apply triangular-shaped fuzzy numbers (Figure 2) and singletons (Figure 3) to define antecedent and consequent membership functions, respectively. The fuzzy numbers of each variable are arranged in symmetrical and equally distributed in individual universe of discourse, so each singleton value of the output variable can be described by

$$u_{h(j_1,j_2,\cdots,j_m)} = b\frac{j_1+j_2+\cdots+j_m}{n_1+n_2+\cdots+n_m}u_s \tag{8}$$

where u_s is the center regulating factor of membership function. If we use the product inference engine and the average defuzzifier, the crisp output of fuzzy controller can be described by

$$\begin{aligned}\hat{u}(X|u_s) &= \frac{\sum_{j_1=-n_1}^{n_1}\cdots\sum_{j_m=-n_m}^{n_m} u_{h(j_1,\cdots,j_m)}\left(\prod_{i=1}^{m}\mu_{A_{(i,j_i)}}(x_i)\right)}{\sum_{j_1=-n_1}^{n_1}\cdots\sum_{j_m=-n_m}^{n_m}\left(\prod_{i=1}^{m}\mu_{A_{(i,j_i)}}(x_i)\right)} \\ &= u_s\frac{\sum_{j_1=-n_1}^{n_1}\cdots\sum_{j_m=-n_m}^{n_m} b\frac{j_1+j_2+\cdots+j_m}{n_1+n_2+\cdots+n_m}\left(\prod_{i=1}^{m}\mu_{A_{(i,j_i)}}(x_i)\right)}{\sum_{j_1=-n_1}^{n_1}\cdots\sum_{j_m=-n_m}^{n_m}\left(\prod_{i=1}^{m}\mu_{A_{(i,j_i)}}(x_i)\right)} \\ &= u_s\eta(X)\end{aligned} \tag{9}$$

where

$$\eta(X) = \frac{\sum_{j_1=-n_1}^{n_1}\cdots\sum_{j_m=-n_m}^{n_m} b\frac{j_1+j_2+\cdots+j_m}{n_1+n_2+\cdots+n_m}\left(\prod_{i=1}^{m}\mu_{A_{(i,j_i)}}(x_i)\right)}{\sum_{j_1=-n_1}^{n_1}\cdots\sum_{j_m=-n_m}^{n_m}\left(\prod_{i=1}^{m}\mu_{A_{(i,j_i)}}(x_i)\right)} \tag{10}$$

2.2 The center adaptive law

Let y_d be the desired output and define the error signal $e = y_d - y = y_d - x_1$. According to (1) and (2), it is easily to find out that the optimal control is

$$u^* = \frac{1}{g}[-f(X) + y_d^{(m)} + c^T e] \tag{11}$$

where $e = (e_1, e_2, \cdots, e_m)^T = (e, \dot{e}, \cdots, e^{(m-1)})^T$ is the error state vector of system and $c = (c_1, c_2, \cdots, c_m)^T$ is the positive constant. Substituting (11) into (1), we have

$$x^{(m)} = f(X) + [-f(X) + y_d^{(m)} + c^T e]$$

$$e^{(m)} + c_m e^{(m-1)} + \cdots + c_1 e = 0 \tag{12}$$

Therefore, we can choose the value of c appropriately to make all the roots of (12) in the open left-half complex plane then the system will be stable. Substituting $\hat{u}(X|u_s)$ into (1), and subtracting gu^* on the both sides of the equation, we can get the error equation as

$$x^{(m)} - gu^* = f(X) + g\hat{u}(X|u_s) - gu^*$$

$$x^{(m)} + f(X) - y_d^{(m)} - c^T e = f(X) + g\hat{u}(X|u_s) - gu^*$$

$$e^{(m)} = -c^T e + g[u^* - \hat{u}(X|u_s)] \tag{13}$$

Let

$$\Xi = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -c_1 & -c_2 & \cdots & \cdots & \cdots & -c_m \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ g \end{bmatrix} \tag{14}$$

then the error equation (13) can be rewritten as the following vector form,

$$\dot{e} = \Xi e + \Omega[u^* - \hat{u}(X|u_s)] \tag{15}$$

For the error equation are related to u_s, we must do some change in (15). In here, we define the optimal parameter value of u_s as

$$u_s^* = \min_{u_s \in R^l}\left[\sup_{X \in R^m}\left|\hat{u}(X|u_s) - u^*\right|\right] \tag{16}$$

and optimal minimum approximation error as

$$\varepsilon = \hat{u}(X|u_s^*) - u^* \tag{17}$$

Using (9) and (17), we can rewrite (15) as

$$\dot{e} = \Xi e + \Omega(u_s^* - u_s)\eta(X) - \Omega\varepsilon \tag{18}$$

The following work is to determine an adjust mechanism for u_s such that the tracking error e and $u_s^* - u_s$ are minimized. So we choose the Lyapunov function candidate V as

$$V = \frac{1}{2}e^T P e + \frac{g}{2\gamma}(u_s - u_s^*)^2 \tag{19}$$

where P is a $m \times m$ positive definite matrix satisfying the Lyapunov function $\Xi^T P + P\Xi = -Q$, Q is also a $m \times m$ positive definite matrix and γ is a positive constant. Obviously V is positive. According the Lyapunov theorem, if the $\dot{V} < 0$, then V will decrease gradually. Therefore, our control goals of $e = 0$ and $u_s = u_s^*$ will be achieve. If we differentiate (19) with respect to time, and substitute (18), we have

$$\begin{aligned}\dot{V} &= \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} + \frac{g}{\gamma}(u_s - u_s^*)\dot{u}_s \\ &= -\frac{1}{2}e^T Qe - e^T P\Omega\varepsilon + \frac{g}{\gamma}(u_s - u_s^*)[\dot{u}_s - \gamma e^T P\Omega\eta(X)]\end{aligned} \tag{20}$$

If we choose the center adaptive law as

$$\dot{u}_s = \gamma e^T P\Omega\eta(X) \tag{21}$$

and substitute (21) into (20), then

$$\dot{V} = -\frac{1}{2}e^T Qe - e^T P\Omega\varepsilon \tag{22}$$

As indicated in [2], we only need enough rules to describe the control action, then the ε will be very small such that $|e^T P\Omega\varepsilon| < \frac{1}{2}e^T Qe$. In other words, $\dot{V}$ will be smaller than 0. Figure 4 is the flowchart of direct center adaptive fuzzy controller.

3. Result of simulation

To prove the efficiency of the proposed method, we apply it in the controlling of first-order nonlinear system and second-order inverse pendulum system, and compare with the Wang's study [2].

3.1 Application of first-order nonlinear system

Consider the first-order nonlinear system, where the state equation is described by:

$$\dot{x}(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + u(t) \tag{23}$$

$$y(t) = x(t) \tag{24}$$

Use the method of Section 2.1 to build rule base, where $i = 1$ and $n_1 = 3$. Because of the fuzzy system is one dimension, fuzzy rule can simply express as follows:

$$IF\ x\ is\ A_{(1,j)}\ THEN\ u\ is\ u_{h(j)} \tag{25}$$

where

$$j = \{-3,\cdots,-1,0,1,\cdots,3\},\quad h(j) = \{-3,\cdots,-1,0,1,\cdots,3\} \tag{26}$$

In other words, the antecedent part and consequent part use 7 fuzzy numbers respectively to define membership functions. The desire control objective is $y_d = 0$. In the simulative process, set the universe of discourse of input variable is [-a,a]=[-3,3], the universe of discourse of output variable is [-b,b]=[-9,9]. Furthermore, we choose $\gamma = 1$, $u_y(0) = 1$ and sample time is 0.01. Figure 5 is the simulation result with initial condition $x(0) = 2$, where the trajectory with the solid line is the result or the proposed direct center adaptive fuzzy controller, and the trajectory with dashed line is the result of Wang's approach. According to the simulation result, we can find that the proposed approach in the performance of the rise time and overshoot is better than that of Wang's approach. The reason is that Wang's method about the design of adaptive law that is able to adjust the center value of each consequent membership function. Furthermore, the initial center value is obtained by random. This method not only causes the slow speed of computing, but also obtains the greater difference rule with real control rule. So, the speed of convergence becomes slowly.

4.2 Application of second-order inverse pendulum system

In this example, the control objective is to produce an appropriate actuator force u to control the motion of the cart such that the pole can be balanced in the vertical position. Assume $x_1 = \theta$ is the angle of pole with respect to the vertical axis, and $x_2 = \dot{\theta}$ is the angular velocity of the pole. The state equation can be expressed by

$$\dot{x}_1 = x_2 \tag{27}$$

$$\dot{x}_2 = f + bu \tag{28}$$

where

$$f = \frac{g \sin x_1 - \frac{mLx_2^2 \cos x_1 \sin x_1}{m+M}}{L\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m+M}\right)} \tag{29}$$

$$b = \frac{\frac{\cos x_1}{m+M}}{L\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m+M}\right)} \tag{30}$$

and g (acceleration due to the gravity) is 9.8 meter/sec^2, L (half length of the pole) is 0.5 meter, M (mass of the cart) is 1.0 kg, and m (mass of the pole) is 0.1 kg. If we consider the $i = 2$, $n_1 = 2$, and $n_2 = 2$, then the fuzzy rule can be expressed by:

$$IF\ x_1\ is\ A_{(1,j_1)}\ and\ x_2\ is\ A_{(2,j_2)}\ THEN\ u\ is\ u_{h(j_1,j_2)} \tag{31}$$

where

$$j_1, j_2 \in \{-2,-1,0,1,2\},\ h(j_1,j_2) \in \{-4,\cdots,-1,0,1,\cdots,4\} \tag{32}$$

In other words, the input variables x_1 and x_2 of antecedent part apply 5 fuzzy numbers respectively to define membership functions. Furthermore, the output variable of consequent part applies 9 fuzzy numbers to define membership functions. In the simulative process, we set the universe of discourse of input variables x_1 and x_2 both over the same interval $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$, and the universe of discourse of output variable u over the interval [-b,b]=[-60,60]. We choose $\gamma = 0.01$, $u_y(0) = 1$ sample time is 0.01 and let $Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, so we can get the $P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$. Figure 6 is the simulation result of initial condition $x_1(0) = \frac{-\pi}{4}$, $x_2(0) = 0$, where the solid trajectory is the result of the proposed approach, and dashed trajectory is the result of Wang's approach. Similarly, we know our method is better in control performance than Wang's approach that usually causes divergence of the inverse pendulum system. The reason still is the initial center value of membership function by random so that it produces an inappropriate rule base.

4. Conclusions

In this paper, we proposed a direct center adaptive fuzzy controller. It improved the defect that we must spend more time to adjust the membership functions in the design of traditional fuzzy controllers. In here, we only decide the fuzzy numbers and initial values of each input and output variables, then the center adaptive law will adjust membership functions of fuzzy system by the system characteristic in that time. This is simpler and more effective than the existing approaches using genetic algorithms and several parameter adaptation approach. Furthermore, only a regulating parameter factor is needed in the center adaptive law. This way of design, not only reduce the complex of mathematical deduction and, increase the speed of computing.

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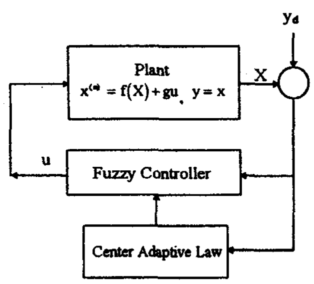

Figure 1. The structure of direct center adaptive fuzzy controller.

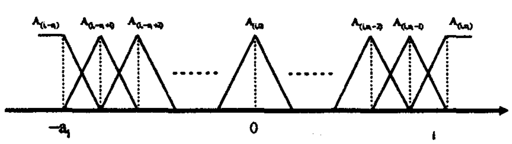

Figure 2. The membership functions of input variables.

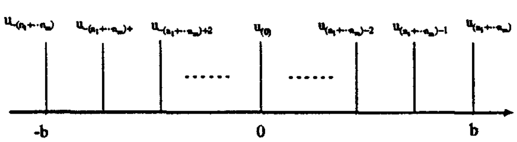

Figure 3 The membership function of output variable

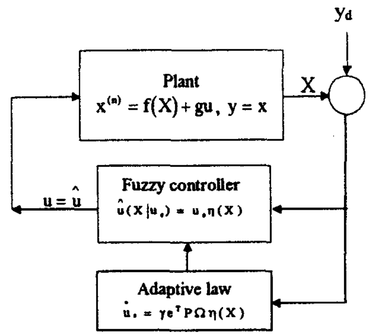

Figure 4. The structure of direct center adaptive fuzzy control system.

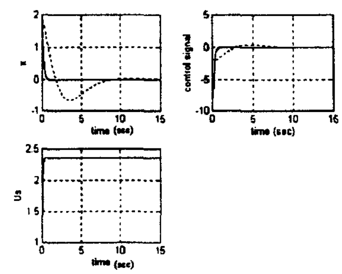

Figure 5. The response of first order nonlinear system.

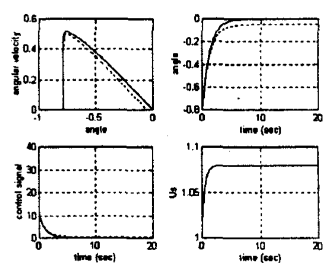

Figure 6. The response of second order inverse pendulum system.