Generating Fuzzy Rules by a GA-Based Method from Input-Output Data

Ching-Chang Wong, Chia-Chong Chen
Department of Electrical Engineering, Tamkang University,
Tamsui, Taipei Hsien, Taiwan, Republic of China
E-mail: wong@ee.tku.edu.tw

ABSTRACT

A method based on the concepts of genetic algorithm (GA) and recursive least- squares method is proposed to construct a fuzzy system directly from some gathered input-output data of the discussed problem. The proposed method can find an appropriate fuzzy system with fewer rules to approach an identified system under the condition that the constructed fuzzy system must satisfy a predetermined acceptable performance. In this method, each individual in the GA is constructed to determine the number of fuzzy rules and the premise part of the fuzzy system, and the recursive least-squares method is used to determine the consequent part of the constructed fuzzy system described by this individual. Finally, two identification problems of nonlinear systems are utilized to illustrate the efficiency of the proposed method.

1. INTRODUCTION

The fuzzy modeling is a new branch of system identification. It concerns with the construction of the fuzzy inference system that can explain the behavior of an unknown system described by a set of sample data [1,3,5-18]. In general, the fuzzy system design is divided into two steps: the structure identification and the parameter identification. In the structure identification step, the input space can be partitioned to describe the inherent structure of an identified system so that the number of fuzzy rules and the shapes of the fuzzy sets in the premise part are determined. On the other hand, in the parameter identification step, a parameter estimation method can be applied to fine tune the parameters of the obtained fuzzy system in the structure identification step. In this paper, each individual in the genetic algorithm (GA) [2,4,15-17] is considered to partition the input space to determine a rough fuzzy system structure and its premise part, then the recursive least-squares method [5,7,14] is applied to determine the consequent part of the constructed fuzzy system. Therefore, a fuzzy system is constructed by the hybrid GA and recursive least-squares method.

One disadvantage of conventional approaches for the fuzzy system design is that the number of fuzzy sets of each input variable must be defined in advance. Therefore, they may have redundant fuzzy sets such that a large number rules are generated for the conventional approaches. Furthermore, when we only have the input-output data of the considered system, it is difficult to extract appropriate fuzzy rules for a predefined system performance directly from numerical data. Consequently, in order to avoid these drawbacks, a method based on the concepts of GA and recursive least-squares method is proposed to construct a fuzzy system directly from some gathered input-output data of the discussed problem. The proposed method can find an appropriate fuzzy system with fewer rules to approach an identified system under the condition that the constructed fuzzy system must satisfy a predetermined acceptable performance. In this method, each individual in the GA is constructed to determine the number of fuzzy rules and fuzzy sets in the premise part of the fuzzy system. Then, the recursive least-squares method is used to determine parameters in the consequent part of the constructed fuzzy system described by this individual. A fitness function is proposed such that it can guide the search procedure to select an appropriate fuzzy system that satisfies the predetermined acceptable performance and has fewer fuzzy rules.

This paper is organized as the following: The structure of the considered fuzzy system is described in Section 2. In Section 3, a GA-based method is proposed to select an appropriate fuzzy system with fewer rules under the condition of satisfying the predetermined performance. In Section 4, two nonlinear systems are utilized to illustrate the efficiency of the proposed approach. Finally, Section 5 concludes the paper.

2. FUZZY SYSTEM STRUCTURE

When m input variables $(x_1, x_2, \cdots, x_m)$ and a single output variable y are considered, a rule base of a fuzzy system can be expressed as follows:

$$\begin{aligned}
&(j_1, j_2, \cdots, j_m)-\text{th rule:}\\
&\text{If } x_1 \text{ is } A_{(1,j_1)} \text{ and } x_2 \text{ is } A_{(2,j_2)} \text{ and} \cdots \text{and } x_m \text{ is } A_{(m,j_m)},\\
&\text{Then } y \text{ is } y_{(j_1,j_2,\cdots,j_m)} = a_{(j_1,j_2,\cdots,j_m;0)} + a_{(j_1,j_2,\cdots,j_m;1)}x_1 + \cdots + a_{(j_1,j_2,\cdots,j_m;m)}x_m\\
&\qquad j_i \in \{1,2,\cdots,d_i\}, i \in \{1,2,\cdots,m\}
\end{aligned} \tag{1}$$

where d_i denotes the number of fuzzy sets for the input variable x_i, $A_{(i,j_i)}$ is the fuzzy set of the i-th input variable x_i in the premise part, and $y_{(j_1,j_2,\cdots,j_m)}$ is a real number in the consequent part, which is a linear combination of input variables. In this paper, the membership function of the fuzzy set $A_{(i,j_i)}$ is described by

$$A_{(i,j_i)}(c_{(i,j_i)}, w^l_{(i,j_i)}, w^r_{(i,j_i)}; x_i) = \begin{cases} \exp\left(-(\frac{x_i - c_{(i,j_i)}}{w^l_{(i,j_i)}})^2\right) & \text{if } x_i \le c_{(i,j_i)}, \\ \exp\left(-(\frac{x_i - c_{(i,j_i)}}{w^r_{(i,j_i)}})^2\right) & \text{if } x_i > c_{(i,j_i)}. \end{cases} \tag{2}$$

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where $c_{(i,j_i)}$ is the center of the membership function, $w^l_{(i,j_i)}$ is the left width value of the membership function, and $w^r_{(i,j_i)}$ is the right width value of the membership function. Therefore, the shape of the membership function corresponding to the fuzzy set $A_{(i,j_i)}$ in the premise part is determined by the parameters $(c_{(i,j_i)}, w^l_{(i,j_i)}, w^r_{(i,j_i)})$. On the other hand, the real value $y_{(j_1,j_2,\cdots,j_m)}$ in the consequent part is determined by the parameters $(a_{(j_1,j_2,\cdots,j_m;0)}, a_{(j_1,j_2,\cdots,j_m;1)}, \cdots, a_{(j_1,j_2,\cdots,j_m;m)})$.

When the j-th input $x^j = (x^j_1, x^j_2, \cdots, x^j_m)$ is given, the firing strength of the premise of the $(j_1, j_2, \cdots, j_m)$-th rule is calculated by

$$\mu^j_{(j_1,j_2,\cdots,j_m)} = \prod_{i=1}^{m} A_{(i,j_i)}(x^j_i). \tag{3}$$

If there are d_i fuzzy sets corresponding to the input variable x_i, the complete fuzzy system will have $\prod_{i=1}^{m} d_i$ rules. By taking the weighted average of $y_{(j_1,j_2,\cdots,j_m)}$, the output of the fuzzy system with respect to the input x^j can be determined by

$$y = \frac{\sum_{j_1=1}^{d_1} \cdots \sum_{j_m=1}^{d_m} \mu^j_{(j_1,j_2,\cdots,j_m)} \cdot y_{(j_1,j_2,\cdots,j_m)}}{\sum_{j_1=1}^{d_1} \cdots \sum_{j_m=1}^{d_m} \mu^j_{(j_1,j_2,\cdots,j_m)}}$$
$$= \frac{\sum_{j_1=1}^{d_1} \cdots \sum_{j_m=1}^{d_m} \mu^j_{(j_1,j_2,\cdots,j_m)} \cdot (a_{(j_1,j_2,\cdots,j_m;0)} + a_{(j_1,j_2,\cdots,j_m;1)} x^j_1 + \cdots + a_{(j_1,j_2,\cdots,j_m;m)} x^j_m)}{\sum_{j_1=1}^{d_1} \cdots \sum_{j_m=1}^{d_m} \mu^j_{(j_1,j_2,\cdots,j_m)}} \tag{4}$$

According to the above description, each parameter set containing of the premise and consequent parameters determines a fuzzy system. Thus, different parameter sets determine different fuzzy systems resulting in different performances. The goal of this paper is to select an appropriate fuzzy system to approach an identified system where only the input-output data are available. Therefore, the difference between the selected fuzzy system and the identified system can be viewed as a performance index. The objective of the paper is to find a fuzzy system with fewer fuzzy rules to satisfy a predetermined acceptable performance. In the next section, the GA and the recursive least-squares method are applied to find an appropriate fuzzy system to approach the identified system.

3. FUZZY SYSTEM SELECTED BY A GA-BASED METHOD

The genetic algorithm (GA) is a method to obtain an optimal solution by applying a theory of biological evolution [2,4]. The GA employs the Darwinian survival-of-the-fittest theory to yield the best of the characters among the population and perform a random information exchange to produce superior individual. The GA requires only the information about the fitness value determined by each parameter set. This differs from many optimization methods requiring the derivation information or the complete knowledge of the problem structure and parameter. In this paper, the GA is applied to choose an appropriate fuzzy system to approach an identified system. In the GA approach, each individual is represented to determine a fuzzy system. The individual is used to partition the input space so that a rough fuzzy system and the fuzzy sets in the premise part are determined, and then the number of rules is determined from the complete concept. Furthermore, the recursive least-squares algorithm is applied to determine the parameters in the consequent part corresponding to the individual. Here, a set of L individuals, P, called population, is expressed in the following:

$$P = \{p^1, p^2, \cdots, p^L\}. \tag{5}$$

Each individual is viewed as a solution candidate and expressed by the following string expression:

$$p^k = \{p^k_{11}, p^k_{12}, \cdots, p^k_{1b_1}, p^k_{21}, p^k_{22}, \cdots, p^k_{2b_2}, \cdots, p^k_{m1}, p^k_{m2}, \cdots, p^k_{mb_m}\},\ k \in \{1,2,\cdots,L\} \tag{6}$$

where $p^k_{ij}, i \in \{1,2,\cdots,m\}, j \in \{1,2,\cdots,b_i\}$, is a bit variable taking of either "1" or "0". We use the substring, $\{p^k_{i1}, p^k_{i2}, \cdots, p^k_{ib_i}\}$, to partition the i-th input space and determine the shapes of membership functions for the i-th input variable. The substring is expressed in terms of strings consisting of "0" and "1". The center position of each membership function is denoted by "1". If the number of bits with "1" is d_i, d_i is viewed as the number of membership functions for the i-th input variable. Therefore, the rule number of the fuzzy system described by this individual p^k is $m(p^k) = \prod_{i=1}^{n} d_i$ in the sense of the complete concept. The notation, $I_{(i,j_i)} \in \{1,2,\cdots,b_i\}$, is defined to represent the index of the substring, $\{p^k_{i1}, p^k_{i2}, \cdots, p^k_{ib_i}\}$, taking a value "1" with respect to the i-th input space.

Assume that n input-output data $(x^j_1, x^j_2, \cdots, x^j_m, y^j)$, $j \in \{1,2,\cdots,n\}$, are gathered from the observation of the identified system, where $(x^j_1, x^j_2, \cdots, x^j_m)$ is the input vector of the j-th input-output pair and y^j is the corresponding output. By means of the input-output data of the identified system, the range $[x^{min}_i, x^{max}_i]$ of the i-th input variable is determined, where $x^{min}_i = \min_{j \in \{1,2,\cdots,n\}} x^j_i$ and $x^{max}_i = \max_{j \in \{1,2,\cdots,n\}} x^j_i$. Therefore, the center position of each membership function with respect to the i-th variable can be determined by:

$$c_{(i,j_i)} = x^{min}_i + (I_{(i,j_i)} - 1) \cdot s_i,\ j_i \in \{1,2,\cdots,d_i\}, \tag{7}$$

where $s_i = (x^{max}_i - x^{min}_i)/(b_i - 1)$. On the other hand, from (2), the left and right width values of each membership function with respect to the i-th input variable can be respectively calculated by

$$w^l_{(i,j_i)} = \sqrt{\frac{(c_{(i,j_i-1)} - c_{(i,j_i)})^2}{-\ln(\alpha)}} \tag{8}$$

and

$$w^r_{(i,j_i)} = \sqrt{\frac{(c_{(i,j_i+1)} - c_{(i,j_i)})^2}{-\ln(\alpha)}}\,, \tag{9}$$

where $c_{(i,0)} = x_i^{min}$, $c_{(i,d_i)} = x_i^{max}$ and $\alpha \in [0,1]$ is a constant to control the overlapping of two adjacent membership functions. That is, the membership function values of $A_{(i,j_i)}(c_{(i,j_i+1)})$ and $A_{(i,j_i)}(c_{(i,j_i-1)})$ are both equal to the value of α. Consequently, each individual can be used to determine a rough fuzzy system. By means of the process, the number and shapes of membership functions in the premise part are constructed to partition the input space, and the number of fuzzy rules is determined.

For example, an individual with two substrings ($b_1 = 5$ and $b_2 = 7$) is expressed in the string {110101010111}. That is, the substrings $\{p^k_{11}, p^k_{12}, \cdots, p^k_{15}\}$ and $\{p^k_{21}, p^k_{22}, \cdots, p^k_{27}\}$ for x_1 and x_2 are {11010} and {1010111}, respectively. Assume the ranges of the two input variables, x_1 and x_2, are $[-2,2]$ and $[-3,3]$, respectively. Following the above description, some results are determined as follows:

$$\begin{aligned}
&s_1 = 1, s_2 = 1;\\
&d_1 = 3, d_2 = 5;\\
&(I_{(1,1)}, I_{(1,2)}, I_{(1,3)}) = (1,2,4); (I_{(2,1)}, I_{(2,2)}, I_{(2,3)}, I_{(2,4)}, I_{(2,5)}) = (1,3,5,6,7),\\
&(c_{(1,1)}, c_{(1,2)}, c_{(1,3)}) = (-2,-1,1), (c_{(2,1)}, c_{(2,2)}, c_{(2,3)}, c_{(2,4)}, c_{(2,5)}) = (-3,-1,1,2,3),
\end{aligned} \tag{10}$$

Thus, there are three and five membership functions for input variables x_1 and x_2, respectively. Therefore, a rule base of a two-input-one-output fuzzy system can be described as follows:

$$\begin{aligned}
&(j_1, j_2) - \text{th rule}:\\
&\qquad \text{If } x_1 \text{ is } A_{(1,j_1)} \text{ and } x_2 \text{ is } A_{(2,j_2)},\\
&\qquad \text{Then } y \text{ is } y_{(j_1,j_2)} = a_{(j_1,j_2;0)} + a_{(j_1,j_2;1)} x_1 + a_{(j_1,j_2;2)} x_2\\
&\qquad\qquad j_1 \in \{1,2,3\},\ j_2 \in \{1,2,3,4,5\}
\end{aligned} \tag{11}$$

The number of rules is $3 \times 5 = 15$ and the membership functions of each input variable for this example can be described as Fig. 1, where the left and right width values of each membership function are calculated according to (8) and (9), and the value of α is set to be 0.15.

In the following, we should introduce how to determine the parameters in the consequent part of the fuzzy system so that a fine fuzzy system can be determined. The goal of the paper is to select a fuzzy system to approach the identified system. Therefore, the summation of squared error corresponding to the individual p^k is

$$E(p^k) = \sum_{j=1}^{n} [y^j - y_f^{(k,j)}]^2\,, \tag{12}$$

where y^j is the desired output corresponding to the j-th input $(x^j_1, x^j_2, \cdots, x^j_m)$ and $y_f^{(k,j)}$ is the output of the fuzzy system determined by the individual p^k corresponding to the same input. As E is as small as possible, the constructed fuzzy system will approximate the identified system as well as possible. In order to minimize E, the recursive least-squares method is applied [5,7,14]. The idea case is that the outputs of the constructed fuzzy system are all equal to these of the identified system. This can be described from (4) as follows:

$$y^j = \sum_{j_1=1}^{d_1} \cdots \sum_{j_m=1}^{d_m} [g^j_{(j_1,j_2,\cdots,j_m)} a_{(j_1,j_2,\cdots,j_m;0)} + \sum_{i=1}^{m} g^j_{(j_1,j_2,\cdots,j_m)} a_{(j_1,j_2,\cdots,j_m;i)} x^j_i],\ j \in \{1,2,\cdots,n\} \tag{13}$$

where $g^j_{(j_1,j_2,\cdots,j_m)}$ is the normalized firing strength of the $(j_1, j_2, \cdots, j_m)$-th rule corresponding to the j-th input $x^j = (x^j_1, x^j_2, \cdots, x^j_m)$ and is described by

$$g^j_{(j_1,j_2,\cdots,j_m)} = \frac{\mu^j_{(j_1,j_2,\cdots,j_m)}}{\sum_{j'_1=1}^{d_1} \cdots \sum_{j'_m=1}^{d_m} \mu^j_{(j'_1,j'_2,\cdots,j'_m)}} \tag{14}$$

We can represent equation (12) as the following vector equation:

$$Y = WB\,, \tag{15}$$

where

$$Y = [y^1\ \ y^2\ \ \cdots\ \ y^n]^T, \tag{16}$$

$$B = [A_{(1,1,\cdots,1)}\ \cdots\ A_{(1,1,\cdots,d_m)}\ \cdots\ A_{(j_1,j_2,\cdots,j_m)}\ \cdots\ A_{(d_1,d_2,\cdots,1)}\ \cdots\ A_{(d_1,d_2,\cdots,d_m)}]^T, \tag{17}$$

$$\begin{aligned}
&A_{(j_1,j_2,\cdots,j_m)} = [a_{(j_1,j_2,\cdots,j_m;0)}\ \ a_{(j_1,j_2,\cdots,j_m;1)}\ \cdots\ a_{(j_1,j_2,\cdots,j_m;m)}],\\
&j_i \in \{1,2,\cdots,d_i\}, i \in \{1,2,\cdots,m\}
\end{aligned} \tag{18}$$

and

$$W = \begin{bmatrix}
g^1_{(1,1,\cdots,1)} & g^1_{(1,1,\cdots,1)}x^1_1 & \cdots & g^1_{(1,1,\cdots,1)}x^1_m & \cdots & g^1_{(d_1,d_2,\cdots,d_m)} & g^1_{(d_1,d_2,\cdots,d_m)}x^1_1 & \cdots & g^1_{(d_1,d_2,\cdots,d_m)}x^1_m \\
g^2_{(1,1,\cdots,1)} & g^2_{(1,1,\cdots,1)}x^2_1 & \cdots & g^2_{(1,1,\cdots,1)}x^2_m & \cdots & g^2_{(d_1,d_2,\cdots,d_m)} & g^2_{(d_1,d_2,\cdots,d_m)}x^2_1 & \cdots & g^2_{(d_1,d_2,\cdots,d_m)}x^2_m \\
\vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
g^n_{(1,1,\cdots,1)} & g^n_{(1,1,\cdots,1)}x^n_1 & \cdots & g^n_{(1,1,\cdots,1)}x^n_m & \cdots & g^n_{(d_1,d_2,\cdots,d_m)} & g^n_{(d_1,d_2,\cdots,d_m)}x^n_1 & \cdots & g^n_{(d_1,d_2,\cdots,d_m)}x^n_m
\end{bmatrix}. \tag{19}$$

Using pseudo inverse of W, we can obtain the consequent parameter values in the form of the matrix B by

$$B = (W^T W)^{-1} W^T Y\,. \tag{20}$$

When the dimension is large, it is computationally expensive to directly calculate the pseudo inverse $(W^T W)^{-1} W^T$. Therefore, we apply the following recursive least-squares method to calculate the consequent parameter values in the form of the matrix B. Let w_j be the j-th row vector of the matrix W, k=1,2,…,n, then B is recursively calculated as follows:

$$B_{j+1} = B_j + S_{j+1} \cdot w^T_{j+1} \cdot (y^j - w_{j+1} \cdot B_j),\quad j = 0,1,\cdots,n-1, \tag{21}$$

$$S_{j+1} = S_j - \frac{S_j \cdot w_{j+1}^T \cdot w_{j+1} \cdot S_j}{1 + w_{j+1} \cdot S_j \cdot w_{j+1}^T}, \; j = 0,1,\cdots,n-1, \qquad (22)$$

The initial values of the algorithm are defined as $B_0=0$ and $S_0 = \beta I$, where β is a positive large number and I is the identity matrix of dimensions $[(\prod_{i=1}^{m} d_i)\cdot(m+1)]\times[(\prod_{i=1}^{m} d_i)\cdot(m+1)]$. Thus, the parameter values of the consequent part are determined by the recursive least-squares estimate $B = B_n$ of the algorithm and a fine fuzzy system is constructed for the representation of individual.

According to the above description, an individual determines a fuzzy system. In order to construct a fuzzy system which satisfies a predetermined acceptable performance and has fewer rules, the fitness function is defined as follows:

$$F(p^k) = g_1(E(p^k)) \cdot g_2(m(p^k)) \qquad (23)$$

where $g_1(E(p^k))$ and $g_2(m(p^k))$ are respectively defines as

$$g_1(E(p^k)) = \begin{cases} \exp(-(\frac{E(p^k)}{\delta_e})), & \text{if } E(p^k) \le \delta_{AP} \\ 0, & \text{otherwise.} \end{cases} \qquad (24)$$

and

$$g_2(m(p^k)) = \exp(-(\frac{m(p^k)}{\delta_r})), \qquad (25)$$

where δ_{AP} denotes an acceptable performance determined by the designer and σ_e and σ_r is a constant determined by the designer. Consequently, under the condition of satisfying the acceptable performance, the fitness function will guide the individual to find a fuzzy system with fewer rules. In this way, as the fitness function value increases as greatly as possible based on the guidance of the proposed fitness function, the fuzzy system corresponding to the individual will satisfy the desired objective as well as possible. That is, the selected fuzzy system can achieve the acceptable performance and has fewer rules simultaneously.

Based on the above description, the procedure can be described as follows:

Step 1: Set the number of individuals (L), the length of bits for each input variable (b_i), the crossover probability (p_c), the mutation probability (p_m), the overlapping control constant (α), the number of generations (N), the constants for the fitness function (δ_e and δ_r), and the predetermined acceptable performance (δ_{AP}).

Step 2: Generate the initial individual for G=0.

Step 3: Let k=1.

Step 4: Determine the premise part of fuzzy system from the k-th individual and determine the consequent part by the recursive least-squares method.

Step 5: Calculate the fitness value for the k-th individual by equation (23).

Step 6: If k=L, then go to Step 7; otherwise let k=k+1 and go to Step 4.

Step 7: If G=N, then go to Step 9; otherwise let G=G+1 and go to Step 8.

Step 8: Generate new individuals by the reproduction, crossover and mutation operators, and then go to Step 3.

Step 9: Based on the individual with the best fitness value of the final result, the desired fuzzy system can be determined.

4. ILLUSTRATED EXAMPLES

In order to illustrate the usefulness of the proposed method, three identification problems of nonlinear systems are discussed here.

Example 1: (Approaching a fifth-order polynomial)

In this example, we use the proposed method to approximate a function with a fifth-order polynomial as follows:

$$y = 3x_1(x_1-1)(x_1-1.9)(x_1+0.7)(x_1+1.8) \qquad (26)$$

From the evenly distributed points of the input range [-2,2] of the preceding equation, 100 input-output pairs are obtained. Following the proposed method, some simulation results for different predetermined acceptable performances are shown in Fig.2-3, where the initial conditions for the proposed method in example 1 are given in the following: The number of individuals: L=40, the length of bits for the input variable: b_1=30, the length of individual: b_1=30, the crossover probability: p_c=0.9, the mutation probability: p_m=0.3, the overlapping control constant: $\alpha = 0.15$, the number of generations: N=30, the constant $\delta_e = 20$, and the constant $\delta_r = 5$. The obtained performances for different predetermined acceptable performances are shown in Table 1. From the simulation results, the membership functions are appropriately selected so that the selected fuzzy system approximates the identified system well.

Example 2: (Modeling a two-input sinc function)

In this example, the proposed method is used to model a two-dimensional sinc equation defined by

$$y = \sin c(x_1, x_2) = \frac{\sin(x_1)\sin(x_2)}{x_1 x_2} \qquad (27)$$

From the evenly distributed grid points of the input range $[-10,10]\times[-10,10]$ of the preceding equation, 121 input-output pairs are obtained and shown in Fig. 4. Following the proposed method, some simulation results according to different predetermined acceptable performance are respectively shown in Fig. 5-6, where the initial conditions for the proposed method are given in the following: The number of individuals: L=40, the length of bits for the input variable x_1: b_1=10, the length of bits for the input variable x_2: b_2=10, the length of individual: b_1+b_2=20, the crossover probability: p_c=0.9, the mutation probability: p_m=0.3, the overlapping control constant: $\alpha = 0.15$, the number of generations: N=30, the constant $\delta_e = 1$, and the constant

$\delta_r = 16$. The performance corresponding to each predetermined acceptable performance is shown in Table 2. From the above results, the membership functions for each input variable are appropriately selected so that the selected fuzzy system approaches the identified system well.

5. CONCLUSIONS

When we only have the input-output data of the considered system, the conventional fuzzy system design is difficult to extract the appropriate fuzzy rules such that the constructed fuzzy system simultaneously satisfies a predefined performance and has fewer rules. Consequently, a method based on the GA and the recursive least-squares method is proposed to effectively construct a fuzzy system directly from the gathered input-output data. In this method, each individual in the GA is represented to determine a rough fuzzy system, then the recursive least-squares method is applied to determine the consequent parameters of the fuzzy system so that the constructed fuzzy system has a high performance. Furthermore, a fitness function is proposed such that it can effectively guide the search procedure to choose an appropriate fuzzy system that satisfies the predetermined acceptable performance and has fewer fuzzy rules. Thus, the proposed method can improve the drawbacks of conventional fuzzy system designs that the number of fuzzy sets of input variable (or the number of the fuzzy rules) must be defined in advance. The simulation results can illustrate the efficiency of the proposed method in the fuzzy system design.

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Table 1 The obtained performances for different predetermined acceptable performances in Example 1.

Predetermined Acceptable Performance (δ_{AP})	Actual Error of the Selected Fuzzy System	Rule Number of the Selected Fuzzy System
40	7.5831	7
7.5	6.5888	8
5	3.7116	9
3	2.1942	10

Table 2 The obtained performances for different predetermined acceptable performances in Example 2.

Predetermined Acceptable Performance (δ_{AP})	Actual Error of the Selected Fuzzy System	Rule Number of the Selected Fuzzy System
0.2	0.1911	15
0.15	0.1315	20
0.1	0.095	32
0.05	0.0458	42

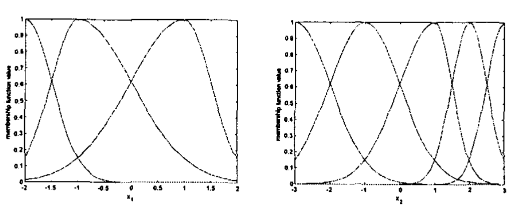
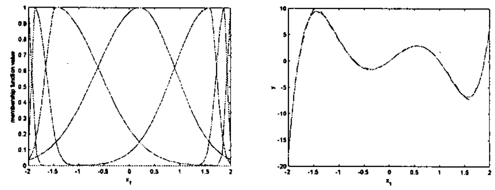
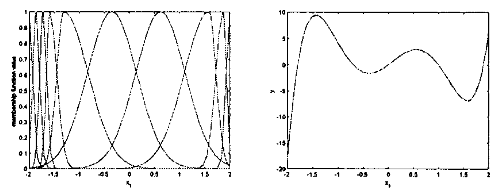


Fig. 1 The membership functions of each input variable for (10).



(a) (b)

Fig. 2 The simulation results for δ_{AP} =40 in Example 1. (a) The selected membership functions; (b) The desired outputs and the outputs obtained by the selected fuzzy system respectively represented by a solid line and a dashed line.



(a) (b)

Fig. 3 The simulation results for δ_{AP} =3 in Example 1. (a) The selected membership functions; (b) The desired outputs and the outputs obtained by the selected fuzzy system respectively represented by a solid line and a dashed line.

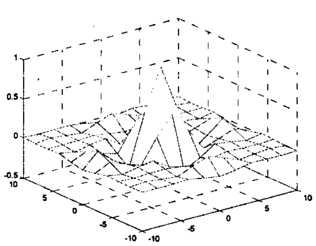
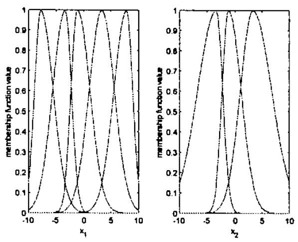
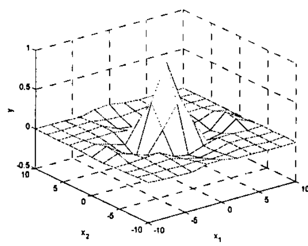


Fig. 4 The input-output pairs of the two-input sinc function in Example 2.

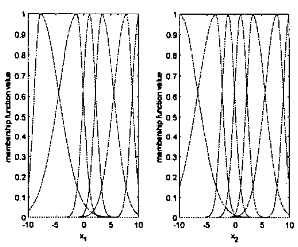


(a)

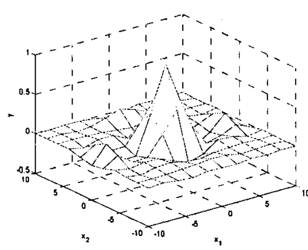


(b)

Fig. 5 The simulation results for δ_{AP} =0.2 in Example 2. (a) The selected membership functions for each input variable; (b) The outputs of the selected fuzzy system.



(a)



(b)

Fig. 6 The simulation results for δ_{AP} =0.05 in Example 2. (a) The selected membership functions for each input variable; (b) The outputs of the selected fuzzy system.