Design of Fuzzy Control Systems with a Switching Grey Prediction

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Abstract

A switching grey prediction fuzzy control system structure is proposed in this paper. In the design of the switching grey predictor, we divide the system response into some regions and propose a switching mechanism so that different forecasting step-size is given in the different region to simultaneously improve the transient and steady state responses of the controlled system. Moreover, we will apply simulated annealing (SA) to find the appropriate parameter values so that the controlled system has a high control performance. Finally, simulation results of the inverted pendulum system will be provided to prove the efficiency of the proposed methods.

1. Introduction

Fuzzy control has been widely applied in many practical applications to industrial process [8,12] for several decades. Much work has been done on the analysis of the parameter structures and control rules of fuzzy controllers so that the fuzzy controller can be constructed efficiently [8,9,11,13]. In this paper, we adopt a systematic and efficient fuzzy controller, called rule mapping fuzzy controller [13], to achieve a desired control performance.

In general, the rise time and the overshoot are the important performance measures in the system response. However, most of them can not reduce the overshoot and the rise time at the same time. In this paper, we propose a control scheme which integrates the grey predictor and fuzzy controller to simultaneously reduce the rise time and the overshoot of the controlled system. In order to get a good performance, we divide the system response into several control regions and propose an algorithm to switch and adjust the forecasting step-size of the grey predictor. The essential concept is that the forecasting step-size in the grey predictor can be switched according to the error of the system during different periods of the system response. Furthermore, in order to prevent the time-consuming trial-and-error method in finding the proper parameters of the control scheme, a simulated annealing approach is proposed to select the control regions and the forecasting steps based on some performance measures of the system response. In this way, we can expect the control parameters selected by the proposed method will provide the controlled system with a good overall performance.

This paper is organized as follows. In Section 2, the concept of the grey predictor is described. In Section 3, the structure of the rule mapping fuzzy controller is constructed. In Section 4, we construct the switching type of grey prediction fuzzy controller. In Section 5, the simulated annealing is applied to select the parameters of the proposed structure. In Section 5, simulations of the inverted pendulum system conclude the paper.

2. Grey Predictor

After the grey system theory was initiated by Deng in 1982 [2], Cheng proposed a grey prediction controller to control an industrial process without knowing the system model in 1986 [1]. From that moment, more and more applications and researches of the grey prediction control were presented [3-6,10]. From the control point of the view, in order to control a system, we usually have to model the system by means of system identification methods in the conventional control schemes. Owing to a large of input and output data of the desired system, this way requires complex computations to model the desired system. However, by means of the grey system theory, we only need few output sample data (four data are enough) for modeling a system. Therefore, it is easy to model a system without complex computations. In the grey theory, the grey model, denoted by GM(n,h), is a dynamic model which contains a group of grey differential equations, where n is the order of the differential equation and h is the number of variables. In this paper, we adopt the most commonly used grey model GM(1,1) for grey prediction. Thus, the grey modeling procedure of GM(1,1) and the generation of the grey predictor are briefly described as follows:

Let $y^{(0)}$ be a non-negative original data sequence,

$$y^{(0)}=\{y^{(0)}(1),y^{(0)}(2),\ldots y^{(0)}(n)\},\ n\geq 4 \quad (1)$$

and take the accumulated generating operation (AGO) on $y^{(0)}$, we can obtain the first order AGO sequence $y^{(1)}$

by

$$y^{(1)}(k) = AGO \circ y^{(0)} \equiv \sum_{m=1}^{k} y^{(0)}(m), \ k = 1,2,\ldots,n. \qquad (2)$$

Then we define $z^{(1)}$ as the sequence obtained by applying the MEAN operation to $y^{(1)}$,

$$z^{(1)}(k) = MEAN \circ y^{(1)} \equiv \frac{1}{2}\left[y^{(1)}(k) + y^{(1)}(k-1)\right], k = 2,3,\ldots,n \qquad (3)$$

The data generating operations, AGO and MEAN operations, are the first step in building grey model. By the way, AGO can weak the randomness of the row data to generate a regular sequence $y^{(1)}$.

The equation

$$y^{(0)}(k) + az^{(1)}(k) = u_g \qquad (4)$$

is called a grey differential equation of GM(1,1), where the parameters a and u_g are called the development coefficient and the grey input, respectively. The equation

$$\frac{dy^{(1)}}{dt} + ay^{(1)} = u_g \qquad (5)$$

is called the whitening equation corresponding to the grey differential equation of Equation (4). In order to find out the solution of the above differential equation, the parameters a and u_g must be decided. They can be solved by means of the least-square method as follows:

$$\hat{\theta} = \begin{bmatrix} a \\ u_g \end{bmatrix} = (B^T B)^{-1} B^T y^N, \qquad (6)$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & 1 \\ -z^{(1)}(n) & 1 \end{bmatrix} \qquad (7)$$

and

$$y^N = [y^{(0)}(2)\ y^{(0)}(3) \cdots y^{(0)}(n)]^T. \qquad (8)$$

Since the solution of the whitening equation (5) is

$$y^{(1)}(t) = (y^{(0)}(1) - \frac{u_g}{a}) \cdot e^{-a(t-1)} + \frac{u_g}{a}, \qquad (9)$$

the solution of the corresponding grey differential equation can be expressed by

$$\hat{y}^{(1)}(n+p) = (y^{(0)}(1) - \frac{u_g}{a}) \cdot e^{-a(n+p-1)} + \frac{u_g}{a}, n \geq 4 \qquad (10)$$

where the parameter "p" is the forecasting step-size and the upscript "^" means the value $\hat{y}$ is a forecasting value of y. Take the inverse AGO (IAGO) on $\hat{y}^{(1)}$, the corresponding IAGO sequence $\hat{y}^{(0)}$ is defined by

$$\hat{y}^{(0)}(k) = IAGO \circ y^{(1)} = \hat{y}^{(1)}(k) - \hat{y}^{(1)}(k-1), \ k = 2,3,\ldots,n \qquad (11)$$

and we can obtain the forecasting value of $y^{(0)}(n+p)$ expressed as follows:

$$\hat{y}^{(0)}(n+p) = (y^{(0)}(1) - \frac{u_g}{a}) \cdot e^{-a(n+p-1)} \cdot (1 - e^{a}), \ n \geq 4. \qquad (12)$$

Based on the above description, the grey predictor composed of AGO, IAGO, and GM(1,1) can be constructed by

$$\hat{y}^{(0)} = IAGO \circ GM(1,1) \circ AGO \circ y^{(0)}. \qquad (13)$$

According to the grey prediction procedure, the input data of the grey predictor must be a non-negative sequence. However, the response sequence of the system may be positive or negative. Therefore, we have to map the negative sequence to the relative positive sequence by some methods of data mapping. In other words, the conventional grey predictor must be modified. In this paper, we adopt two data mapping operations proposed by Huang and Huang [6]. One is referred as the mapping generating operation (MGO), and the other is the inverse MGO (IMGO). They are defined as follows:

Let $y^{(0)}$ be an original sequence and $y_m^{(0)}$ be the MGO image sequence of $y^{(0)}$, then

$$y_m^{(0)} = MGO \circ y^{(0)} \equiv b^{r \cdot y^{(0)}}, \ b, r > 0, \qquad (14)$$

where b and r are positive constants selected by designer. Similarly, the IMGO can also got as follows:

$$y^{(0)} = IMGO \circ y_m^{(0)} \equiv \frac{1}{r} \log_b y_m^{(0)} \qquad (15)$$

Therefore, the modified grey predictor can be constructed by

$$\hat{y}^{(0)} = IMGO \circ IAGO \circ GM(1,1) \circ AGO \circ MGO \circ y^{(0)}. \qquad (16)$$

3. Rule Mapping Fuzzy Controller

The fuzzy controller is defined by a rule base of individual control rules which are conditional linguistic statements of the relationship between inputs and outputs. In a rule mapping method applied in this paper, a complete fuzzy control rule base with two input variables and one output variable can be represented as

$$R(j_1, j_2): \text{If } x_1 \text{ is } A_{(1,j_1)} \text{ and } x_2 \text{ is } A_{(2,j_2)} \text{ Then } u \text{ is } O_{K=f(j_1,j_2)} \qquad (17)$$

where x_1 and x_2 stand for input linguistic variables and u stands for output linguistic variables. $A_{(1,j_1)}$, $A_{(2,j_2)}$ and $O_{K=f(j_1,j_2)}$ are fuzzy sets characterized by membership functions. $f(j_1, j_2)$ is a constant index function which decides a linguistic value of u. The set of control rules described above can by considered as a mapping

$$f:\{-m_1,\cdots,0,\cdots m_1\}\times\{-m_2,\cdots,0,\cdots m_2\}\rightarrow\{-m,\cdots,0,\cdots m\}. \quad (18)$$

The rule mapping method is an approach to approximate the mapping of the rules by an index function f. It is clear that any fuzzy controller with a separable rules set has an index-represented rule mapping.

In this paper, we consider

$$f(j_1,j_2)=<a_f(j_1+j_2)>=<b>, \quad (19)$$

where

$$<b>=\begin{cases}-m & \text{if } b\le -m+\frac{1}{2}\\ \text{int}(b-\frac{1}{2}) & \text{if } -m+\frac{1}{2}<b\le 0\\ \text{int}(b-\frac{1}{2})+\text{sign}(b-\frac{1}{2}) & \text{if } 0<b\le m-\frac{1}{2}\\ m & \text{if } b>m-\frac{1}{2}\end{cases} \quad (20)$$

There are 2m+1 fuzzy sets O_K, $-m\le K\le m$, for the output variable. Since there are 2m+1 fuzzy sets O_K, $-m\le K\le m$, we sum up all the output fuzzy sets to form the resultant output set and convert it into a scalar by

$$u=\frac{\sum_{j_1=-m_1}^{m_1}\sum_{j_2=m_2}^{m_2} w(i,j_i)\cdot O_K}{\sum_{j_1=-m_1}^{m_1}\sum_{j_2=m_2}^{m_2} w(i,j_i)} \quad (21)$$

where $w(i,j_i)=\prod_{i=1}^{2}A_{(i,j_i)}(x_i)$.

In this paper, the following bell-shape fuzzy numbers and singletons are used for defining the antecedent and consequent membership function, respectively. The antecedent membership, $A_{(i,j_i)}$, is defined as

$$A_{(i,j_i)}(x_i)=\begin{cases}e^{-\left(\frac{x_i-j_iD_i}{s_iD_i}\right)^2} & \text{if } |x_i|\le m_iD_i,\ j_i\in\{-m_i,\cdots,m_i\}\\ 0 & \text{if } |x_i|>m_iD_i\end{cases} \quad (22)$$

$$O_K=\begin{cases}1 & \text{if } u=KD\\ 0 & \text{if } u\ne KD\end{cases},\ K\in\{-m,\cdots,m\} \quad (23)$$

where the central values of $A_{(i,j_i)}$ and O_K are j_iD_i and KD, respectively. D_i and D denote the length of the subdivision of each universe of discourse. s_i regulates the width of the bell-shape membership function.

According to the observation of Equations (19), (20), (21), (22) and (23), we find that the defuzzified output involves directly the choice of the rule base and the membership functions. The control rule base is decided by the parameters a_f; and the membership functions are decided by the parameters m_i, m, D_i, s_i and D. Therefore, the performance of the fuzzy controller depends on the parameters a_f, m_i, m, D_i, s_i and D.

4. Switching Grey Prediction Fuzzy Controllers

The structure configuration of the grey prediction fuzzy controller with a fixed positive forecasting step-size is shown in Figure 1. The control strategy is based on the forecasting value $\hat{y}$ of the output y. Then, the forecasting value of the output is transmitted to the fuzzy controller and a control signal $\hat{u}$ is generated to control the plant. However, the forecasting step-size decides the forecasting value. If the forecasting step-size is positive, and the decreasing response of the system is considered, the forecasting value of the output will get smaller so that the fuzzy controller generate a smaller forecasting control signal to prevent the system over the set point. Therefore, it leads to a slow response of the system so that the overshoot of the fuzzy control system with grey predictor is smaller than that of the fuzzy control system without grey predictor in the sense of "prevention is better than cure". Although this reduces the overshoot, it will cause a long rise time. To overcome the drawback, we have to speed the response of the system when the system starts work. Therefore, if the predictor have the ability of predicting the "past" behavior of the system, the forecasting value of the output will get larger for a decreasing system response. Therefore, the fuzzy controller will transmit a larger control signal to speed the response of the system and result in a shorter rise time. In order to predict the "past" behavior of the system that is a monotonous decreasing function, we must make the forecasting step-size negative. From these above points of view, we propose a switching mechanism to switch the forecasting step-size to integrate the advantages and remove the disadvantages. In addition, to avoid the long rise time, we find a proper forecasting step-size between the given positive and negative one. The configuration of the proposed control structure is shown in Figure 2, where the switching mechanism is defined by

$$p=\begin{cases}pys & \text{if } y>\theta_l\\ pym & \text{if } \theta_s<y\le\theta_l\\ pyl & \text{if } \theta_s\ge y\end{cases} \quad (24)$$

where p is the forecasting step-size of the system; pys, pym and pyl are the forecasting step-sizes for the large output, the middle output and the small output, respectively; θ_l and θ_s are the switching time of the large output and the small output, respectively. By the way, the proposed controller not only can drastically reduce the overshoot, but also can cause a shorter rise time than the conventional design methods.

5. Parameters Selected by Simulated Annealing

According to the description in Section 4, the proposed control structure still includes the switching times (θ_l, θ_s) and three forecasting step-size (pyl, pym, pys). In order to decrease the number of searching parameters, we let that θ_s is the half of θ_l and pym is the mean of pyl and pys. Therefore, a set of parameters $R_i = (\theta_l, pyl, pys)$ is the solution that we will apply simulated annealing to obtain for a desired control task. The simulated annealing (SA) originated in statistical mechanics and is based on a Monte Carlo model that was used by Metropolis. In 1983, Kirkpatrick was the first who applied simulated annealing to solve combinatorial optimization problem [7]. In this paper, a desired control system with good performance is decided by the rise time (rt), the overshoot (os) and the integral absolute error (IAE). Therefore, we define the cost function as follows:

$$E(R_i) = \{1 - [g_1(rt(R_i)) \cdot g_2(IAE(R_i))]\} \cdot \{1 - g_3(os(R_i))\} \quad (25)$$

where $E(R_i)$ is the cost value of the selected parameter R_i; $rt(R_i)$, $IAE(R_i)$ and $os(R_i)$ are the values of the rise time, integral absolute error and overshoot of the selected parameter R_i, respectively. $g_1(.)$, $g_2(.)$, and $g_3(.)$ are any proper membership functions for rt, os and IAE of the system, respectively. In this paper, we define the functions as follows:

$$g_1(rt(R_i)) = \exp(-(\frac{rt(R_i)}{\sigma_{rt}})^2), \quad (26)$$

$$g_2(IAE(R_i)) = \exp(-(\frac{IAE(R_i)}{\sigma_{IAE}})^2), \quad (27)$$

$$g_3(os(R_i)) = \exp(-(\frac{os(R_i)}{\sigma_{os}})^2), \quad (28)$$

where σ_{rt}, σ_{IAE} and σ_{os} are 1, 3, and 1, respectively. If the final solution is obtained, it can provide the controlled system with a high overall performance.

6. Simulations and Conclusions

To demonstrate the effectiveness of the proposed method, an inverted pendulum control problem is used as illustrated examples. The scheme of the inverted pendulum system is shown in Figure 3, where a free-falling pole is mounted on a cart which is controlled by an actuator. The control objective is to produce an appropriate actuator force F to control the motion of the cart so that the pole can be balanced in the vertical position. Let $x_1(t) = \theta(t)$ and $x_2 = \dot{\theta}(t)$, where θ and $\dot{\theta}$ stand for angle and angular velocity of the pole with respect to the vertical axis, respectively, then the state equation can be expressed by

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{g\sin(x_1) + \cos(x_1)(\frac{-F - mlx_2^2\sin(x_1)}{m+M})}{l(\frac{4}{3} - \frac{m\cos^2(x_1)}{m+M})}, \quad (29)$$

where g (acceleration due to the gravity) is 9.8 m/sec^2, M (mass of cart) is 1.0 kg, m (mass of pole) is 0.1 kg, l (half length of pole) and F is the applied force in Newton.

In the rule mapping fuzzy controller, we adopt parameters of the antecedent membership function, (D_1, s_1, D_2, s_2)=(0.0784,0.6843,0.7451,0.9784), selected by [13], and change parameters of the consequent membership function to be $(a, D) = (1,15)$ in order to speed the response of the system. Set $m_1 = m_2 = m = 3$, and then generate 49 rules, where the rules are represented as .

$$R(j_1, j_2): \text{If } \theta \text{ is } A_{(1,j_1)} \text{ and } \dot{\theta} \text{ is } A_{(2,j_2)} \text{ Then F is } O_{K=f(j_1,j_2)} \quad (30)$$

In the parameter selection problem, the searching regions of the parameters are [0,20] for θ_l, [0,10] for pyl, and [-10,0] for pys and the following parameters of SA are considered: T=10, d=0.85, k=0.0005 and ξ =0.01. Following the proposed method described in Section 5, we obtain the parameter set (θ_l, pyl, pys)=(4.5321, 7.7140, -7.3646) which provides the controlled system with a good overall performance. The comparison of our proposed method with the fuzzy control method without the grey predictor [13] for the same plant is shown in Table 1 and Figure 4. We can find that our proposed method not only reduces the rise time strongly but also maintain a small extent of overshoot. From the above comparison, it is clear that the proposed control structure and method are available.

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Table 1. The comparison of our proposed method with other fuzzy control method

		Non-Self-Regulating Fuzzy Controller	Switching Grey Prediction Fuzzy Controller
Parameters	a_f	1	1
	D_1	0.0784	0.0784
	s_1	0.6843	0.6843
	D_2	0.7451	0.7451
	s_2	0.9784	0.9784
	D	10	15
	θ_1	no	4.5321
	pyl	no	7.7140
	pys	no	-7.3646
Performance	rt	0.29(sec)	0.21(sec)
	os	0.5959(deg)	0.08358(deg)
	IAE	2.8528(deg·sec)	2.193(deg·sec)

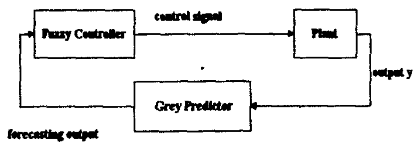

Figure 1. Block diagram of the grey prediction fuzzy control system with a fixed forecasting step-size.

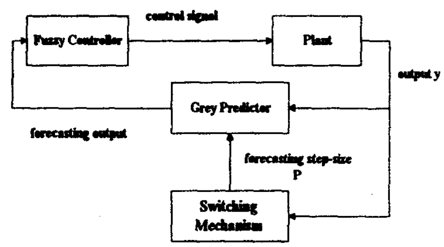

Figure 2. Block diagram of the switching grey prediction fuzzy control system.

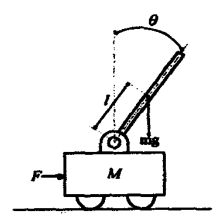

Figure 3. The scheme of inverted pendulum system

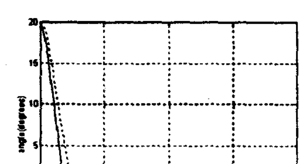

Figure 4. The comparison of our proposed design (solid) with non-self-regulateing fuzzy controller (dotted).