MULTIOBJECTIVE FUZZY OPTIMIZATION WITH RANDOM VARIABLES IN A MIX OF FUZZY AND PROBABILISTIC ENVIRONMENT

C. J. SHIH and R. A. S. WANGSAWIDJAJA
Department of Mechanical Engineering
Tamkang University
Tamsui, Taiwan 25137, R.O.C.

ABSTRACT—The fuzzy 3-formulation is used to solve multiobjective optimization problems in which the design variables are random, the constraints as a whole have a fuzzy probability or the constraints are mixed of deterministic, stochastic and fuzzy constraints. The fuzzy optimization strategy helps one to get the highest degree of satisfaction in reaching an optimum solution. The idea of expected value and the chance constrained programming technique allows one to convert a stochastic optimization problem into its equal deterministic form. The design of a three-bar truss illustrates the proposed design optimization in detail and the design of a machine-tool spindle express the comprehensive work of this technique. Results show that the proposed formulation and method can generate a natural, well-behaved, and reliable design.

1. INTRODUCTION

A lot of efforts have been made to approach engineering design problems as naturally as possible. In nature a lot of aspects are related to the principles of probability. The situation where one often worries about the probability of failure (or reliability) happens to the optimization process [1-4]. It is reasonable that in formulation we consider the probability point of view. The conditions for applying probability concerns is that we must have enough historical data to form the distribution. In case of a known probability density function (pdf), one can construct the associated membership function in accordance with the possibility-probability consistency principle [5].

Usually it is a high cost to obtain these pdf. However, it is relatively easy to obtain the mean and standard deviation. We introduce an approach of applying the fuzzy theory [6], and can still solve this optimum problem under probabilistic base. The fuzzy set theory which was originally developed by professor Zadeh and is a beautiful way of describing a natural optimization condition mathematically. Since the appearance of the fuzzy theory, some researches and applications have also been done such as the works of [7-11].

To deal with optimization problem which is generally recognized to be nondeterministic as well as fuzzy in nature, we present a fuzzy stochastic mathematical formulation to solve such design problems. This nondeterministic condition is not only in the design variables, but it can also be in the allowable limits as well. Using the ideas of expected value and the chance constrained programming technique [2] one can transform the stochastic problem into its deterministic form. By then one can substitute this form into the fuzzy mathematical formulation.

Here we present a design optimization method under a combination of deterministic, probabilistic, and fuzzy environment. The basic idea is to solve the problem under two different expected probabilities of occurrence and/or two extreme values of fuzzy tolerance separately. Having these two results, we can then continue the construction of membership function to fit the fuzzy optimization. An engineering optimization design often involves more than one objectives and sometimes they can be contradicting too. Accordingly, we are interested in concentrating on multiobjective problems.

The following paragraphs we will present and solve the separate pattern of fuzzy, probabilistic and fuzzy probability optimization problem. Each of them is illustrated by a well-known three-bar truss design with minimizing two objective functions. Comparison and observation are given afterwards. Eventually, we made a design problem of four objectives where a machine-tool spindle is optimized. The design process and results under a mix of fuzzy and probabilistic environment will be given subsequently. From the analysis, we conclude this study and present the characteristics between the satisfying degree and factor of safety designs.

2. PROBABILISTIC OPTIMIZATION

In a probabilistic optimization, a general stochastic optimization problems is to find $X=[x_1, x_2, ..., x_n]^T$ by the following formulation:

Minimizes $F(X)$

subjected to:

$P[g_j(X) - g_{j, ad}(X) \leq 0] \geq P_{j, exp}, \quad j=1,2,...,k$  

where $X$ represents a mix of random and deterministic variables, $F(X)$ is the objective function, $P[g_j(X) - g_{j, ad}(X) \leq 0]$ is the occurrence probability of constraint, $g_j(X) - g_{j, ad}(X) \leq 0$, and $P_{j, exp}$ is the expectation value associated to the $j$th constraint. When $P[g_j(X) - g_{j, ad}(X) \leq 0] \geq P_{j, exp}$, the optimization design is based on a reliability point of view. Two important variations according to the kind of allowable limits are described below.
2.1. Optimization With Deterministic Allowable Limit

Stochastic problem of this kind has the constraint as Eq. (1), where \( g(X) \) has a nondeterministic value and \( g_{\alpha}(X) \) has a deterministic value. In order to solve this probabilistic optimization problem, we transform the stochastic problem by Taylor expansion into the approximately equal deterministic form [3]:

Minimize \( \Psi(X) = C_{f}(X) + C_{p} \sigma_{p00} \) subject to

\[
g_{j}(X) + \frac{1}{2} \left( \sum_{i=1}^{n} \frac{\partial g_{j}(X)}{\partial x_{i}} \right) \sigma_{j}^{2} \leq 0
\]

(3)

where \( \sigma_{g_{j}}(X) \) is the corresponding value of standard normal variate to the probability, \( \sigma_{p00} \); \( \sigma_{j} \) is the standard deviation of the \( j \)th random design variable; \( \mu_{i} \geq 0, \mu_{o} \geq 0 \) is the degree of importance for minimization. The stochastic optimization problem will be equal to the deterministic form only if \( \sigma_{p00} = 0 \).

2.2. Optimization With Nondeterministic Allowable Limit

For a stochastic optimization problem with a nondeterministic allowable limit, as the similar approach above, Eq. (2) becomes:

\[
g_{j}(X) + \frac{1}{2} \left( \sum_{i=1}^{n} \frac{\partial g_{j}(X)}{\partial x_{i}} \right) \sigma_{j}^{2} \leq 0
\]

(3)

where \( \sigma_{g_{j}}(X) \) is a statistical data with a mean and standard deviation only. Substituting these constraints into the equal deterministic formulation, one can now search for an optimal solution.

3. FUZZY PROBABILISTIC OPTIMIZATION

3.1. The \( \lambda \)-Formulation Method

Designers are often caught in an optimization design situation that minimizes or maximizes more than one objective function under some fuzzy design constraints. In multiobjective fuzzy design, usually there are certain tolerance in which our final design falls in this certain acceptable range. To define this tolerance for several objective functions we must first execute the crisp optimization of each single objective and at the same time calculate the corresponding values of the other objectives. From these values and bounds, we can construct the membership functions for the objective functions.

The mathematical \( \lambda \)-formulation of a multiobjective fuzzy optimization is as following [9]:

Maximize \( \lambda \) Subject to \( \lambda - \mu_{i} \leq 0, i=1,k \)

(8)

\( \lambda - \mu_{j} \leq 0, j=1,m \)

(9)

By neglecting the transition values in the constraints described by the membership function, we are constructing a formulation for the crisp multiobjective optimization.

3.2. \( P_{\exp} \) is a Fuzzy Number

If \( P_{\exp} \) is fuzzy, then \( P_{\exp} \) is restricted between an upper bound, \( (P_{\exp})^{+} \), and a lower bound, \( (P_{\exp})^{-} \). When \( P[g(X)-g_{\alpha}(X) \leq 0] \neq (P_{\exp})^{-} \), the membership function is said to have a degree of satisfaction of one, and when \( P[g(X)-g_{\alpha}(X) \leq 0] = (P_{\exp})^{-} \), the degree of satisfaction is said to be zero.

Since the value of standard normal variate is fuzzy, then we can construct a membership function between \( r(P_{\exp})^{+} \) and \( r(P_{\exp})^{-} \) by changing the equal deterministic form into the following two different conditions.

3.2.1 for a deterministic allowable limit, the constraint can transform to the following:

\[
r_{j}(P_{\exp})^{+} \leq \frac{g_{j}(X) - g_{\alpha}(X)}{\sigma_{g_{j}}(X)}
\]

(10)

3.2.2 for a nondeterministic allowable limit, the constraint can transform to the following:

\[
r_{j}(P_{\exp})^{+} \leq \frac{g_{j}(X) - g_{\alpha}(X)}{\sigma_{g_{j}}(X) \sigma_{\alpha}(X)}
\]

(11)

Thus, the linear membership function of standard normal variate is:

\[
\mu_{\gamma}(P_{\exp}) = \begin{cases} 1, & \text{if } r_{\gamma}(P_{\exp})^{+} \geq r_{\gamma}(P_{\exp})^{-} \\ 0, & \text{if } r_{\gamma}(P_{\exp})^{+} \leq r_{\gamma}(P_{\exp})^{-} \\ 1 - \frac{r_{\gamma}(P_{\exp})^{+} - r_{\gamma}(P_{\exp})^{-}}{r_{\gamma}(P_{\exp})^{-} - r_{\gamma}(P_{\exp})^{+}}, & \text{otherwise} \end{cases}
\]

(12)

By modifying \( \mu_{\gamma} \) in Eq. (9) into Eq. (12) and solving the multiobjective optimization problem Eqs. (7-9), one will has the solution to the fuzzy stochastic problem mentioned above.

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4. MULTIOBJECTIVE OPTIMIZATION WITH MIXED OF FUZZY AND PROBABILISTIC CONSTRAINTS

A natural optimization design problem contains random variables and consists of a mix of deterministic, nondeterministic, fuzzy and probabilistic constraints. The general mathematical formulation for such problem can be described as:

Minimizes \( f_i(X), f_2(X), \ldots, f_N(X) \)
subject to five types of constraints

(A) \( g_1(X) - \alpha_{\text{cd}}(X) \leq 0, i = 1, \ldots, p \)

(B) \( P(g(X) - \beta_{\text{cd}}(X) \leq 0) \geq P_{\text{up}}, \)

(C) \( P(g(X) - \beta_{\text{cd}}(X) \leq 0) \geq P_{\text{up}}, \)

(D) \( P(g(X) - \beta_{\text{cd}}(X) \leq 0) \geq P_{\text{up}}, \)

(E) \( P(R(X) - \alpha_{\text{cd}}(X) \leq 0) \geq P_{\text{up}}, \)

The first step is to solve each single objective optimization under the strict environment and calculate the corresponding value of the other objective functions. Then one repeats this procedure again, only this time one relax the environment based on the allowable fuzzy range of design constraints. Choosing the maximum and minimum values of the objective functions among these results, one can construct the appropriate membership function of the objective functions. Applying the method mentioned in section 3, we can get the solution of this multiple objective in a mixed fuzzy and probabilistic constraints.

5. ILLUSTRATIVE EXAMPLES

5.1 Three-bar Truss Design Optimization

The three-bar truss shown in Fig. 1 is frequently used in describing structural optimum design [9]. We assume that the design variables are random variables. The design objectives are to minimize the weight, \( f_1 \), and the vertical deflection of the loaded joint, \( f_2 \). This problem as solved for five different environments as described in section 2 and 3:

(A) which is depicted in section 3.1. By using \( f_1 = 19.142, f_2 = 3.5549, f_3 = 1.110, \) and \( f_4 = 1.6569 \) one can solve this type of problem as in former solution (B), however, the form of \( r_i(h, \alpha_{\text{cd}}) \) is Eq. (10) instead of Eq. (11).

(B) which is depicted in section 3.1. The problem with deterministic allowable limits is defined as:

Minimize \( F(X) = [f_1(X), f_2(X)] \)
subject to \( [P_{\text{up}}(\alpha_{\text{cd}} h_{\text{up}})] \geq 0 \) for \( j=1,2,3 \)

with \( \alpha_{\text{cd}} = \alpha_{\text{cd}} h_{\text{up}} = 3.0, \omega = 0.2 \) and the expectation value, \( P_{\text{up}} \) is 0.99997, \( j=1,2,3 \). This problem can be transformed into a deterministic form described in section 2.1.

From the single objective optimization, we can get \( f_1 = 19.142, f_2 = 3.5549, f_3 = 1.110, \) and \( f_4 = 1.6569 \). Therefore one can solve this probabilistic constraint problem. The constraints of multiobjective formulation are:

where \( h_{\text{up}} \) has a degree of satisfaction equals one if \( h_{\text{up}} \geq 3.99 \), and zero degree of satisfaction otherwise. Thus one can solve this probabilistic constraint problem.

(C) which is depicted in section 2.1. The probabilistic optimization problem is defined as in Eqs. (13-1, 13-2). Allowable limit, \( \alpha_{\text{cd}} h_{\text{up}} (j=1,2,3) \), is a non-deterministic value with a mean value of 20 and standard deviation of 20%. The mean value of \( \omega_{\text{cd}} h_{\text{up}} \) is -15 and the standard deviation is 20%. One can solve this type of problem as in former solution (B), however, the form of \( h_{\text{up}} \) is Eq. (11) instead of Eq. (10).

(D) which is depicted in section 3.1. The problem with deterministic allowable limits is defined as in Eqs. (13-1, 13-2), but here the expected value has a fuzziness between 0.98030 and 0.99997. For \( P_{\text{up}} = 0.99997 \), the normal variate,
The constraint with fuzzy sense enables the result to have a higher degree of satisfaction than nonfuzzy (crisp or probabilistic) constraints. Therefore, the trended value of $\lambda$ is opposite to the trend of factor of safety. The maximum $\lambda$ in this example is the same as a minimized structural weight and increased deflection. Thus, one can clearly see the characteristics of fuzzy, nondeterministic, and probabilistic design optimization.

6. ENGINEERING DESIGN EXAMPLE

Machine-tool Spindle Design

The design of a machine-tool spindle adopted from [12] with a few modifications is used to illustrate an engineering optimum fuzzy design under mixed constraints environment. The design variable, $\tau_m$ and $\sigma_a$ are assumed to be random variables with deviations of $\sigma_{\tau_m}=\sigma_{\sigma_a}=0.2$, $\sigma_{\omega}=0.2\%$ (EQ), and $\sigma_{\omega}=0.2\%$ (EQ). The model of a machine-tool spindle is shown in Fig. 2. The design objectives are minimizing the total weight of the spindle, $W_t$ (kg), the maximum twist per unit applied torque, $\phi$ (deg), the maximum deflection per unit applied load, $\delta$ (m), and the fundamental natural frequency, $\omega$ (Hz). Therefore, the optimization problem can be mathematically described as to find $X=[D_1,D_2,L_1,L_2,L_3]^T$ by the formulation as:

Minimizes $F(X)=[W_t(X),\phi(X),\delta(X),\omega(X)]$

where

$$W_t = \frac{180M_t}{\pi} \frac{32(L_1+L_2+L_3)}{n(G[D_1^4-D_2^4])} \left( \frac{L_1}{L_2} \right) \left( \frac{L_2}{L_3} \right) \text{(Kg)}$$

$$\phi = \frac{180M_t}{n\pi E(D_1^4-D_2^4)} \frac{32(L_1+L_2+L_3)}{n(G[D_1^4-D_2^4])} \left( \frac{L_1}{L_2} \right) \left( \frac{L_2}{L_3} \right) \text{(Degree)}$$

$$\delta = \frac{64L_2^2}{3nE(D_1^4-D_2^4)} \left( \frac{L_1}{L_2} \right) \left( \frac{L_2}{L_3} \right) \left( \frac{L_1+L_2+L_3}{L_1} \right) \text{(m)}$$

$$\omega = \frac{64(L_1^2+L_2^2)}{64(L_1^2+L_2^2)} \left( \frac{1}{L_1} \right) \left( \frac{1}{L_2} \right) \left( \frac{1}{L_3} \right) \text{(Hz)}$$

subject to two deterministic constraints are:

$$2L_{wa}(D_1-D_2) \leq 0$$

$$\frac{\phi}{180(L_1+L_2+L_3)} - 4 \cdot 6.6120^C \leq 0$$

The two fuzzy constraints are of the form:
where $L_0$ is a fuzzy value between 0.745m and 0.755m in which 0.75m has the highest degree of satisfaction. Therefore membership function for this fuzzy constraint has a degree of satisfaction equals to one when $L_0 = 0.75$ and when $L_0 < 0.745$ or greater than 0.755 the degree of satisfaction is zero, this can be seen in Fig. 3 below.

The three fuzzy probabilistic constraints are expressed as:

$$P\left( \frac{1.047D_1}{n (D_1^2 - D_0^2)} - \alpha_{j1j1} \leq 0 \right) > P_{exp} \quad (24)$$

$$P\left( \frac{M_2}{2I} \alpha_{j1j2} \leq 0 \right) > P_{exp} \quad (25)$$

$$P\left( \frac{M_2}{2I} \alpha_{j1j3} \leq 0 \right) > P_{exp} \quad (26)$$

where $L_0$ is a fuzzy value between 1.0m and 1.02m in which 1.0m has the highest degree of satisfaction. Thus if $L_0 + L_1 + L_2$ equals 1.0 the degree of satisfaction is equals to one where as a value of $L_0 + L_1 + L_2$ greater/equal to 1.02 has a zero degree of satisfaction. Fig. 4 below depicted this situation.
where \( P_{\text{exp}} \) is a fuzzy value between 0.97 and 0.9999 in which 0.9999 has the greatest satisfaction. Some parameters in the above equations are represented as \( M_i = F_i L_i, M_0 = |F + F_0|, F_1 L_1 \) and \( L = n(D_1^4 - D_2^4) / 64 \), in which \( F \) is the applied force acting on the grinder or cutter; \( F_1 \) and \( F_2 \) are the reaction force at the left bearing and right bearing, respectively. The five side constraints are:

\[
\begin{align*}
0.05(m) &\leq D_1 < 0.1 \ (m), \ i=1,2 \quad (27-1) \\
0.2(m) &\leq L_1 < 0.3 \ (m) \quad (27-2) \\
0.4(m) &\leq L_2 < 0.6 \ (m) \quad (27-3) \\
0.1(m) &\leq L_3 < 1.5 \ (m) \quad (27-4)
\end{align*}
\]

Some numerical data used here are \( F=250N, M_i=1500N.m, \sigma_{\text{mp}}=200Mpa, \sigma_{\text{mp}}=350Mpa, \rho=7.85\times 10^3 \text{kg/m}^3, \ G=8.04\times 10^{10}N/m^2, \ E=2.06\times 10^{11}N/m^2, k_1=k_2=9.8\times 10^5N/m, \ l_{\text{mp}}=8.0\text{kg.m}^2, \) and \( l_{\text{mp}}=2.25\text{kg.m}^2. \) To solve this mixed fuzzy probabilistic optimization problem, we first solve those four optimization objectives in a strict environment one by one. The strict environment will cause several modifications to the above constraints. Here Eqs. (24-26) has only one deterministic \( P_{\text{exp}} \) value which is 0.9999, and the fuzzy constraints in Eqs. (22-23) become \( L_1 + L_2 = 0.75 \) and \( L_1 + L_2 + L_3 = 1.0. \) Therefore we will have four optimized objective values with the corresponding values of the other unoptimized objectives.

The next step is to solve the relaxed single objective optimization. This is done by repeating the above procedure, only this time Eqs. (22-23) become \( L_1 + L_2 \leq 0.745, \ L_1 + L_2 \leq 0.755 \) and \( L_1 + L_2 + L_3 \leq 1.02, \ L_1 + L_2 + L_3 \geq 1.0 \), and \( P_{\text{exp}} \) in Eqs. (24-26) has a value of 0.97. From these two procedures we now have the total of 32 values of the objective functions shown in Table 2 and Table 3.

We choose the best and worst values among these objective functions for the construction of membership function of these objective as described in section 3.1. Completing the membership function constructions for the design constraints and substituting these membership function into the formulations in section 3.1, we can obtain the solutions to this problem in a mixed fuzzy and probabilistic environment. The results are presented in Table 4.

Because of relaxing the constraints, one can see that the optimized value of individual objective function in Table 3 is better than the value in Table 2. The final designs of Table 4 as shown in Fig. 5 which logically falls between the lower bound and the upper bound of the individual optimum result. This optimum design illustrates a natural approach of an optimization problem. Therefore a fuzzy probabilistic optimization does produce a better and reasonable natural solution.

### Table 2: Single objective optimum design for machine-tool spindle under strict environment.

<table>
<thead>
<tr>
<th>Minimize ( W_i )</th>
<th>Minimize ( \phi )</th>
<th>Minimize ( \delta )</th>
<th>Minimize ( \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_i (kg) )</td>
<td>18.030650</td>
<td>38.717230</td>
<td>32.811630</td>
</tr>
<tr>
<td>( \phi (deg) )</td>
<td>0.3227200</td>
<td>0.1263700</td>
<td>0.1635300</td>
</tr>
<tr>
<td>( \delta (m) )</td>
<td>0.084159E-05</td>
<td>0.371222E-05</td>
<td>0.43502E-05</td>
</tr>
<tr>
<td>( \omega (Hz) )</td>
<td>155.35717</td>
<td>248.26556</td>
<td>218.24642</td>
</tr>
</tbody>
</table>

| \( D_1 (m) \)   | 0.0854280        | 0.1000000        | 0.0994270        | 0.0834130 |
| \( D_2 (m) \)   | 0.06622770       | 0.0609940        | 0.0599550        | 0.0622940 |
| \( L_1 (m) \)   | 0.2831400        | 0.2801600        | 0.2000000        | 0.2524900 |
| \( L_2 (m) \)   | 0.4668600        | 0.4694800        | 0.5500000        | 0.4911900 |
| \( L_3 (m) \)   | 0.2566400        | 0.2500000        | 0.2500000        | 0.2587600 |
Table 3  Single objective optimum design for machine-tool spindle under relaxed environment.

<table>
<thead>
<tr>
<th>Minimize $W_1$ (kg)</th>
<th>Minimize $\phi$ (deg)</th>
<th>Minimize $\delta$ (m)</th>
<th>Minimize $\omega$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.871940</td>
<td>0.3978700</td>
<td>0.10044E-04</td>
<td>139.91751</td>
</tr>
<tr>
<td>46.193810</td>
<td>0.1160200</td>
<td>0.61092E-05</td>
<td>259.10027</td>
</tr>
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<td>47.165140</td>
<td>0.1184600</td>
<td>0.41188E-05</td>
<td>255.41843</td>
</tr>
<tr>
<td>15.054530</td>
<td>0.4318500</td>
<td>0.67976E-05</td>
<td>134.29929</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_1$ (m)</th>
<th>$D_2$ (m)</th>
<th>$L_1$ (m)</th>
<th>$L_2$ (m)</th>
<th>$L_3$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0823620</td>
<td>0.1000000</td>
<td>0.2742100</td>
<td>0.4805800</td>
<td>0.3257700</td>
</tr>
<tr>
<td>0.0659420</td>
<td>0.0500000</td>
<td>0.2619400</td>
<td>0.4928100</td>
<td>0.2442500</td>
</tr>
<tr>
<td>0.0812770</td>
<td>0.0500000</td>
<td>0.2000000</td>
<td>0.5550000</td>
<td>0.2650000</td>
</tr>
<tr>
<td>0.0748290</td>
<td>0.0500000</td>
<td>0.0200000</td>
<td>0.5214500</td>
<td>0.2773900</td>
</tr>
</tbody>
</table>

Table 4  Multi-objective optimum design for machine-tool spindle under mixed fuzzy and probabilistic environment.

<table>
<thead>
<tr>
<th>$W_1$ (kg)</th>
<th>$D_1$ (m)</th>
<th>$D_2$ (m)</th>
<th>$L_1$ (m)</th>
<th>$L_2$ (m)</th>
<th>$L_3$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.895910</td>
<td>0.0947950</td>
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<td>0.2742100</td>
<td>0.4928100</td>
<td>0.3257700</td>
</tr>
<tr>
<td>0.0659420</td>
<td>0.0500000</td>
<td>0.2619400</td>
<td>0.4928100</td>
<td>0.3257700</td>
<td></td>
</tr>
<tr>
<td>0.0812770</td>
<td>0.0500000</td>
<td>0.2000000</td>
<td>0.5550000</td>
<td>0.3573900</td>
<td></td>
</tr>
<tr>
<td>0.0748290</td>
<td>0.0500000</td>
<td>0.0200000</td>
<td>0.5214500</td>
<td>0.2773900</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5  Optimum points of single and multiobjective optimization in machine-too spindle design.
7. CLOSING

The degree of satisfaction and the factor of safety designs under uncertainties are presented and compared in this study. Both of them can neatly be a design index or design criterion. The higher degree of satisfaction means the design reaches a level of compromising and optimizing all objective functions in a common fuzzy design space. A kier design space can generate a higher satisfying degree, but a lower factor of safety. However, the traditional factor safety design is too conservative to reach a reasonable design. Probabilistic-based optimization does provide a much reliable design and a stronger structure, nonetheless, it demands higher cost and is not usually used in all kind of design. To formulate a design problem with mixed fuzzy, probabilistic, and deterministic information can contribute a natural, well-behaved and reliable design, yet it is not the only way to achieve the design target. Other researches on this subject can be widely done.

8. ACKNOWLEDGEMENT

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9. REFERENCES


