Domain wall in ultrathin magnetic film: Internal structure and dynamics

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Detailed micromagnetic study of internal structure and dynamics of domain wall in ultrathin magnetic film with thickness $t_f \ll l_{ex}$ (l_{ex} is the exchange length) is carried out. It is revealed that deviations of stationary magnetization distribution inside the wall from the one of the Bloch domain wall are small and proportional to $(t_f/l_{ex})Q^{-1}$. The limiting velocity of uniform domain wall motion coincides with the same for the Bloch wall (Walker's critical velocity) with an accuracy of terms proportional to $(t_f/l_{ex})^2$. It is also found that the same small parameter describes deviation of stationary distribution of magnetization in a vertical Bloch line and deviation of Bloch line velocity from the expressions found for films with $t_f > l_{ex}$. © *2000 American Institute of Physics.* [S0021-8979(00)08820-4]

I. INTRODUCTION

Physics of ultrathin magnetic films (films with thickness from several atomic layers to several dozens of atomic layers) have been an object of intensive studies during the last ten years. Such unremitting interest of researchers is caused by the significance of these new objects for development of physics of magnetism on the one hand and by their practical importance on the other hand. Ultrathin films of magnetic metals and alloys are the constituting elements for magnetic multilayered structures. These artificially created structures known as magnetic multilayers or magnetic superlattices are of a great interest for a wide range of applications based on the phenomenon of magnetoresistance. Results of studies of magnetic multilayers and ultrathin magnetic films can be found in numerous publications (see, for example, Refs. 1–7). It has to be noted, however, that in spite of the intent attention paid by researchers to investigations of ultrathin magnetic films, the domain walls in these films were not studied in detail.

From the standpoint of a rigorous micromagnetic theory the ultrathin magnetic film is the film whose thickness satisfies the condition

$$t_f \ll l_{ex} = \sqrt{A/(2\pi M_s^2)},$$

where t_f is the film thickness, l_{ex} is the characteristic parameter called the exchange length of magnetic material, A is the constant of the inhomogeneous exchange interaction, $4\pi M_s$ is the saturation magnetization. A comprehensive collection of results of theoretical as well as experimental investigations of domain walls in thin magnetic films with thickness satisfying the opposite condition, namely, $t_f \gg l_{ex}$ can be found in Refs. 8 and 9.

There exist a substantial amount of publications devoted to investigations of domain structures, remagnetization process, and domain wall dynamics in ultrathin magnetic films.[10–22] Studies of remagnetization in ultrathin film[16] showed that the remagnetization occurs via nucleation of small domains with reverse orientation of magnetization. The nucleation of these domains is then followed by the process of their expansion by means of domain wall propagation. As follows from the results of Ref. 16 magnetization reversal in ultrathin magnetic films with perpendicular anisotropy occurs in two stages. The first initial stage of the remagnetization is determined to a large extent by the nanostructure of the film (the defects localized on the crystallite boundaries, the size of atomically flat terraces, etc). The initial stage of remagnetization corresponds to low enough values of bias magnetic field. The domain wall dynamic becomes dominant at higher values of the bias field. The case of the film with inhomogeneous coercivity was numerically analyzed in Ref. 21.

Contemporary technology of ultrathin film preparation made considerable progress and the quality of the samples available at present enables the obtaining of reliable data on dynamic behavior of magnetization in these films. The last circumstance makes us believe that detailed micromagnetic study of domain wall in ultrathin magnetic field is topical. Preliminary results of analysis of static distribution of magnetization in a domain wall in ultrathin film with the thickness satisfying the condition (1) were reported in Ref. 22.

II. INTERNAL STRUCTURE OF THE STATIONARY DOMAIN WALL

The following geometry of the problem is considered in this article. The plane of film is parallel to the $x0y$ plane of the coordinate system. The midplane of the film correspond to the coordinate $z=0$. The axis of magnetic anisotropy is

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oriented along the z axis of the coordinate frame. The distribution of the magnetization inside the transition region between two domains magnetized to the saturation along the directions $+z$ and $-z$ is to be studied. The energy density of magnetic subsystem may be represented as a sum of the following contributions:[8,9]

$$w=w_{\mathrm{ex}}+w_K+w_m. \tag{1}$$

Here w_{ex} is the energy density of inhomogeneous exchange interaction, w_K is the anisotropy energy density, and w_m is the energy density of magnetic dipole interaction.

The following remark has to be made before we give concrete expressions for terms in formula (1). We will use expressions for contributions to the energy density of ultrathin magnetic film in the continuous medium approximation. Justification of use of a given approximation is, generally speaking, a subject of a separate investigation. However, applicability of continuous medium approach was discussed in Ref. 5 (Chapter 3).

The energy density of inhomogeneous exchange interaction has the form

$$w_{\mathrm{ex}}=A\{(\nabla m_x)^2+(\nabla m_y)^2+(\nabla m_z)^2\}. \tag{2}$$

In this expression $\mathbf{m}(\mathbf{r},t)=\mathbf{M}(\mathbf{r},t)\cdot M_s^{-1}$, $\mathbf{M}(\mathbf{r},t)$ is the vector of the magnetization. The anisotropy energy density is given by the formula

$$w_K=K[\mathbf{m}^2-(\mathbf{n}_A\cdot\mathbf{m})^2], \tag{3}$$

where K is the constant of uniaxial magnetic anisotropy, $\mathbf{n}_A$ is the unit vector along the anisotropy axis (the z axis in our case). The energy of magnetic dipole interaction has a standard form

$$w_m=-\tfrac{1}{2}(\mathbf{H}^{(m)}\cdot\mathbf{M}), \tag{4}$$

where $\mathbf{H}^{(m)}$ is the demagnetization field, which is determined by the magnetostatic equations

$$\mathrm{curl}\,\mathbf{H}^{(m)}=0, \quad \mathrm{div}\{\mathbf{H}^{(m)}+4\pi\mathbf{M}\}=0 \tag{5}$$

with proper boundary conditions. External magnetic field can be easily included in the expression (1) by adding the energy density $w_H=-(\mathbf{M}\cdot\mathbf{H}_b)$ ($\mathbf{H}_b$ is the bias magnetic field) describing interaction with external bias field, for example

The Landau–Lifshitz equation is used for analysis

$$\dot{\mathbf{M}}=-\gamma[\mathbf{M}\times\mathbf{M}^{\mathrm{eff}}], \tag{6}$$

where $\dot{\mathbf{M}}\equiv\partial\mathbf{M}/\partial t$, γ is the gyromagnetic ratio, and the effective field $\mathbf{H}^{\mathrm{eff}}$ is determined by the relation

$$\mathbf{H}^{\mathrm{eff}}=-\frac{\delta w}{\delta\mathbf{M}}, \tag{7}$$

which in our case leads to the expression

$$\mathbf{H}^{\mathrm{eff}}=\alpha\nabla^2\mathbf{m}-\beta[\mathbf{m}-\mathbf{n}(\mathbf{n}_A\cdot\mathbf{m})]+\mathbf{h}^{(m)}. \tag{8}$$

Here

$$\alpha=\frac{2A}{M_s^2}, \quad \beta=\frac{2K}{M_s^2}, \quad \mathbf{h}^{(m)}=\frac{\mathbf{H}^{(m)}}{2M_s}. \tag{9}$$

Equation (6) written in terms of Cartesian components for the static case ($\dot{\mathbf{M}}=0$) has the following form:

$$\begin{gathered} m_x\nabla^2 m_y-m_y\nabla^2 m_x=\frac{1}{\alpha}[m_y h_x^{(m)}-m_x h_y^{(m)}], \\ m_z\nabla^2 m_x-m_x\nabla^2 m_z-\left(\frac{\beta}{\alpha}\right)m_z m_x=\frac{1}{\alpha}[m_x h_z^{(m)}-m_z h_x^{(m)}], \\ m_x^2+m_y^2+m_z^2=1. \end{gathered} \tag{10}$$

Equations (10) have to be accompanied by the boundary conditions for magnetic moment on the film surfaces. For the above-discussed geometry these boundary conditions may be chosen as follows:[8,23,24]

$$\begin{gathered} m_y\left[\alpha'\frac{\partial m_z}{\partial\mathbf{n}}-\beta'\mathbf{n}(\mathbf{n}\cdot\mathbf{m})\right]-m_z\alpha'\frac{\partial m_y}{\partial\mathbf{n}}=0, \\ m_x\left[\alpha'\frac{\partial m_z}{\partial\mathbf{n}}-\beta'\mathbf{n}(\mathbf{n}\cdot\mathbf{m})\right]-m_z\alpha'\frac{\partial m_x}{\partial\mathbf{n}}=0, \\ m_x\frac{\partial m_y}{\partial\mathbf{n}}-m_y\frac{\partial m_x}{\partial\mathbf{n}}=0, \ \text{at} \ z=\pm t_f/2. \end{gathered} \tag{11}$$

Parameters α' and β' characterize the exchange interaction and magnetic anisotropy at the surface of the film correspondingly, $\mathbf{n}$ is the direction of the external normal to the film surface. The anisotropy of the easy axis type at the surface of the film is considered (see Refs. 4 and 5 for details about surface anisotropy of ultrathin magnetic films). The components of the magnetization vector at $z=\pm t_f/2$ have to satisfy the following conditions m_x, $m_y\to 0$, $m_z\to\pm 1$ at $y\to\pm\infty$. The system of Eqs. (10) with boundary conditions (11) has to be analyzed to obtain the distribution of the magnetization inside the domain wall separating two domains with opposite directions of magnetization $\mathbf{M}\|\mathbf{n}$ and $-\mathbf{M}\|\mathbf{n}$. Solution of magnetostatic Eqs. (5) for the case when the domain wall plane coincides with the $x0z$ plane of the coordinate system are given by expressions

$$\begin{gathered} h_y^{(m)}=-4\pi\int_0^\infty du\, e^{-(u\tau/2)}\cos(u\eta)\frac{\sinh(u\varsigma)}{\sinh(\pi u/2)}, \\ h_z^{(m)}=-4\pi\int_0^\infty du\, e^{-(u\tau/2)}\sin(u\eta)\frac{\cosh(u\varsigma)}{\sinh(\pi u/2)}. \end{gathered} \tag{12}$$

In these expressions $\varsigma=z/\Delta_B$, $\eta=y/\Delta_B$, $\Delta_B=(A/K)^{1/2}$ is the width of the Bloch domain wall, and $\tau=t_f/\Delta_B$.

It is convenient to introduce the spherical coordinates with the polar axis oriented along the normal to the domain wall. The vector of magnetization $\mathbf{m}$ has the following components in this system

$$m_x=\sin\vartheta\sin\varphi, \quad m_y=\cos\vartheta, \quad m_z=\sin\vartheta\cos\varphi, \tag{13}$$

where polar ϑ and azimuthal φ angles of magnetization are the functions of ς and η. The system of Eqs. (10) has to be rewritten in terms of angles ϑ and φ. Solutions of the system (10) will be represented in the form

$$\vartheta=\vartheta_0+\theta(\eta,\varsigma), \quad \varphi=\phi_0(\eta)+\psi(\eta,\varsigma), \tag{14}$$

where ϑ_0 and $\phi_0(\eta)$ describe the distribution of magnetization inside the stationary Bloch domain wall

$$\vartheta_0=\frac{\pi}{2}, \text{ and } \sin\phi_0=\frac{1}{\cosh\eta}. \tag{15}$$

Equations (10) now have the form

$$\left[\hat{O}-\frac{1}{Q}\right]\theta(\eta,\varsigma)=\frac{1}{2\pi Q}h_y^{(m)}(\eta,\varsigma),$$
$$\hat{O}\psi(\eta,\varsigma)=\frac{1}{2\pi Q}\sin\phi_0(\eta)h_z^{(m)}(\eta,\varsigma), \tag{16}$$

where the operator $\hat{O}$ is determined by the expression

$$\hat{O}=\left(\frac{\partial^2}{\partial\eta^2}+\frac{\partial^2}{\partial\varsigma^2}\right)-\cos 2\phi_0(\eta) \tag{17}$$

and $Q=(l_{\mathrm{ex}}/\Delta_B)^2$. Parameter Q is called the quality factor of a magnetic material. This parameter is greater than unity $Q>1$ for materials under consideration. The main small parameter of the problem is $\tau=t_f/\Delta_B=t_fQ^{1/2}/l_{\mathrm{ex}}$. Boundary conditions (11) are transformed into

$$(\partial\theta(\eta,\varsigma)/\partial\varsigma)_{\varsigma=\pm\tau/2}=0, \quad (\partial\psi(\eta,\varsigma)/\partial\varsigma)_{\varsigma=\pm\tau/2}=0. \tag{18}$$

Operator $\hat{O}$ is a modified Winter's operator.[25,26] The spectrum of the original Winter's operator

$$\hat{O}_w=-\frac{\partial^2}{\partial\eta^2}+\cos 2\phi_0(\eta) \tag{19}$$

contains two modes. One of these modes $\psi_0^{(\mathrm{tr})}=\sin\phi_0(\eta)$ corresponds to translational motion of the domain wall and another one $\psi_0^{(\mathrm{pr})}\approx[i\kappa+\cos\phi_0(\eta)]e^{i\kappa\eta}$ (κ is the dimensionless wave vector $\kappa=k\Delta_B$) corresponds to precession of magnetization inside domains. We represent solutions of Eqs. (16) as expansion with respect to these modes

$$\sigma(\eta,\varsigma)=\sigma^{(\mathrm{tr})}(\varsigma)\psi_0^{(\mathrm{tr})}(\eta)+\int_{-\infty}^{\infty}du\,\sigma^{(\mathrm{pr})}(\varsigma,u)\psi_0^{(\mathrm{tr})}(\eta,u). \tag{20}$$

Here we denoted values $\theta(\eta,\varsigma)$ or $\psi(\eta,\varsigma)$ by $\sigma(\eta,\varsigma)$. The following result for the contribution of the translational mode localized in the vicinity of the domain wall to the $\theta(\eta,\varsigma)$ angle can be obtained

$$\theta^{(\mathrm{tr})}(\eta,\varsigma)\approx-\frac{\tau^2}{8Q}\Xi(\tilde{\varsigma})\sin\phi_0(\eta), \tag{21}$$

where $\tilde{\varsigma}=2\varsigma/\tau$ and

$$\Xi(\tilde{\varsigma})=\tfrac{1}{2}[(1+\tilde{\varsigma})^2\ln(1+\tilde{\varsigma})-(1-\tilde{\varsigma})^2\ln(1-\tilde{\varsigma})$$
$$-2\tilde{\varsigma}(1+2\ln 2)]. \tag{22}$$

The value of $\theta^{(\mathrm{tr})}(\eta,\varsigma)$ at $\eta=0$ (the center of the domain wall) and $\varsigma=\pm\tau/2$ describes the domain wall twisting. As one can easily see this contribution is really very small $\theta^{(\mathrm{tr})}(y=0,z=\pm t_f/2)\approx(t_f/l_{\mathrm{ex}})^2$. The contribution of the precession mode obtained in a similar manner is described by the formula

$$\theta^{(\mathrm{pr})}(\varsigma,k)$$
$$=\frac{1}{2\pi Qp(k)}\left\{\int_0^{\varsigma}d\varsigma'\sinh[p(k)(\varsigma-\varsigma')]\right.$$
$$\times h_y^{(m)}(k,\varsigma')-\frac{\sinh[p(k)\varsigma]}{\cosh[\tau p(k)]}$$
$$\left.\times\int_0^{\tau/2}d\varsigma'\cosh\left[p(k)\left(\frac{\tau}{2}-\varsigma'\right)h_y^{(m)}(k,\varsigma')\right]\right\}, \tag{23}$$

where $p(k)=\sqrt{1+(k\Delta_B)^2}$ and

$$h_y^{(m)}(k,\varsigma)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\eta\,h_y^{(m)}(\eta,\varsigma)\psi_0^{(\mathrm{pr})*}(k,\eta). \tag{24}$$

Expression (23) has the following form at the film surface that is at $z=\pm t_f/2$

$$\theta^{(\mathrm{pr})}\left(\frac{\tau}{2},k\right)=-\frac{1}{Qp(k)}\frac{1}{\cosh[\tau p(k)/2]}$$
$$\times\int_0^{\tau/2}d\varsigma\sinh[\varsigma p(k)]h_y^{(m)}(k,\varsigma). \tag{25}$$

Using the above expression one can get an approximate formula for $\theta^{(\mathrm{pr})}(z=t_f/2,y)$ in the region $|y|<t_f$ corresponding to the vicinity of the domain wall

$$\theta^{(\mathrm{pr})}(z=t_f/2,y)\cong\frac{1}{2\pi\sqrt{Q}}\left(\frac{t_f}{l_{\mathrm{ex}}}\right)\left[\ln 4-\pi\frac{|y|}{t_f}\right]. \tag{26}$$

It has to be noted that the contribution of the precession mode in the interval $|y|<\Delta_B$ is even smaller because the expression (26) has to be multiplied by the factor of the order of Q^{-1}. It is clearly seen that both contributions of the translational and precession modes are small and can not influence the initial value of the angle ϑ and thus $\vartheta\cong\vartheta_0=\pi/2$.

Let us now consider the contribution of translational and precession modes into the angle $\psi(\eta,\varsigma)$. Taking into account that $H_z^{(m)}(y)=-H_z^{(m)}(-y)$ and using the second equation from Eq. (16) one can see that the translational mode does not give any contribution to $\psi(\eta,\varsigma)$. The contribution of the precession mode is given by the expression

$$\psi^{(\mathrm{pr})}(z=t_f/2,y)\cong\frac{1}{\pi\sqrt{Q}}\left(\frac{y}{l_{\mathrm{ex}}}\right)\left[1+\ln\left|\frac{t_f}{y}\right|\right] \quad \text{for } |y<t_f|, \tag{27}$$

which was obtained in the frames of the same approximations as formula (26).

Thus the consideration of the internal structure of the domain wall in ultrathin magnetic film carried above allows us to conclude that the structure of the stationary domain wall is close to the one of the Bloch wall and contributions of twisting caused by the translational and precession modes are negligibly small being proportional to $Q^{-1}(t_f/l_{ex})$ or even smaller.

III. DOMAIN WALL LIMITING VELOCITY AND BLOCH LINE MOTION

We have to make several remarks before starting the analysis of the domain wall motion. First, the analysis of the dynamic behavior of domain wall will be done using different orientation of the polar axis of a spherical coordinate frame for magnetization. Now the z axis will be the polar axis and thus the following expressions for the components of the magnetization will be used:

$$m_x=\sin\theta\cos\varphi,\quad m_y=\sin\theta\sin\varphi,\quad m_z=\cos\theta. \tag{28}$$

Second, the static structure of the domain wall is now described by the formulas

$$\sin\theta_0=\frac{1}{\cosh(y/\Delta_B)},\quad \varphi_0=0. \tag{29}$$

As one can see we have neglected all static features of the internal structure of the stationary domain wall analyzed in the previous section because they are negligibly small.

Third, we will consider the translational motion of the domain wall along the y axis, which remains normal to the domain wall plane. The motion of the domain wall is caused by the presence of an external driving magnetic field $\mathbf{H}_b$ parallel to the anisotropy axis $\mathbf{H}_b \| 0z$. Thus, the problem is similar to the one treated by Walker[27] (see also Refs. 8 and 9). As was shown in Ref. 27 the motion of the domain wall was accompanied by the additional dynamic deviation of magnetization from the domain wall plane. This deviation is described by the azimuthal angle of magnetization ψ in the midpoint of the domain wall. The appearance of a magnetization component in the direction of normal to the domain wall plane leads to the change of components of demagnetization field. Thus the first step of our analysis is to obtain new expressions for components of the demagnetizing field, which are now determined as follows:

$$\begin{aligned} h_y^{(m)}&=-4\pi\int_0^\infty du\cos(u\eta)\left\{e^{-(u\tau/2)}\frac{\sinh(u\varsigma)}{\sinh(\pi u/2)}\right.\\ &\qquad\left.+\sin\psi\frac{1-e^{-(u\tau/2)}\cosh(u\varsigma)}{\cosh(\pi u/2)}\right\};\\ h_z^{(m)}&=-4\pi\int_0^\infty du\sin(u\eta)e^{-(u\tau/2)}\left\{\frac{\cosh(u\varsigma)}{\sinh(\pi u/2)}\right.\\ &\qquad\left.-\sin\psi\frac{\sinh(u\varsigma)}{\cosh(\pi u/2)}\right\}. \end{aligned} \tag{30}$$

Here $\eta=y/\Delta$ and $\varsigma=z/\Delta$, where $\Delta=\Delta(\psi)$ is the width of the moving domain wall. It was shown by Walker[27] (see also Refs. 8 and 9) that the translational motion of the domain wall leads to the dependence of the domain wall width on the azimuthal angle characterizing the deviation of magnetization from the domain wall plane.

It is convenient to use a variation principle[8,9] for analysis of the limiting velocity of domain wall motion. The structure of the moving domain wall is given by the following formulas:

$$\sin\theta=\frac{1}{\cosh\left(\dfrac{y-q(t)}{\Delta}\right)},\quad \psi=\psi(t). \tag{31}$$

Here $q(t)$ is the coordinate of the midpoint of the domain wall and $\psi(t)$ is the azimuthal angle of the magnetization in the center of the domain wall. Generally speaking these quantities are the functions of the coordinates in the domain wall plane

$$q=q(x,z;t),\quad \psi=\psi(x,z;t). \tag{32}$$

Dependence of parameters q and ψ on the z coordinate is due to the account of demagnetizing field and dependence of q and ψ on the x coordinate appears if one takes into account the presence of vertical Bloch lines in the domain wall. The statics and dynamics of vertical Bloch lines will be studied further.

Equations describing the domain wall motion can be obtained by variation of the following density of the Lagrangian function:[8,9]

$$L=\int_{-\infty}^{\infty}dy\int_{-t_f/2}^{t_f/2}dz\{M_s\gamma^{-1}\dot\theta\psi\sin\theta+w_{\mathrm{DW}}\}, \tag{33}$$

where

$$\begin{aligned} w_{\mathrm{DW}}=M_s^2\{&\tfrac{1}{2}[(\nabla\theta)^2+(\nabla\psi)^2\sin^2\theta]\\ &+\tfrac{1}{2}\beta\sin^2\theta-(\mathbf{m}\cdot\mathbf{h}^{(m)})-h_b\cos\theta\}. \end{aligned} \tag{34}$$

In these expressions $\dot\theta\equiv\partial\theta/\partial t$, $h_b=H_b/M_s$ other parameters are determined by formulas (9).

The variation procedure is standard and described in Refs. 8 and 9 in detail. It has to be noted, however, that in the case under consideration, namely, $t_f \ll l_{ex}=\Delta_B\sqrt{Q}$, all calculations can be carried out analytically. The equation for the azimuthal angle of magnetization in the domain wall's midpoint has the form

$$\psi''=v[\sin 2\psi-\dot{\tilde q}-2\tilde h_y^{(m)}(\tilde z)\cos\psi]. \tag{35}$$

Here $\dot\psi\equiv\partial\psi/\partial t$, $v=t_f^2/(8l_{ex}{}^2)$, $\dot{\tilde q}=\dot q/V_W$, $V_W=2\pi\gamma M_s\Delta_B$ and

$$\tilde h_y^{(m)}(\tilde z)=\int_{-\infty}^{\infty}d\eta h_y^{(m)}(\eta,\zeta),\quad \tilde z=2z/t_f. \tag{36}$$

It has to be noted that in order to simplify further analysis we consider material quality factor Q to be $Q\gg 1$. This assumption permits us to neglect the difference between Walker's critical velocity[27] and Schlömann's limiting velocity,[28] which is always greater than Walker's critical one in materials with $Q>1$ (see also Ref. 29). The solution of Eq. (35) will be represented in the form

$$\psi=\psi_0+v\psi_1+v^2\psi_2+\cdots \tag{37}$$

and similar expansion has to be used for the domain wall velocity $\dot q$

$$\dot{\bar{q}}=\dot{\bar{q}}_0+v\dot{\bar{q}}_1+v^2\dot{\bar{q}}+\cdots. \tag{38}$$

Limiting ourselves by the first correction to the azimuthal angle of the magnetization and taking into account that ψ_0'' due to that $\psi_0=0$, we have

$$\psi_1''=-\dot{\bar{q}}_0+\sin 2\psi_0-2\tilde{h}_y^{(m)}(\bar{z})\cos\psi_0 \tag{39}$$

and

$$\psi_2''=-\dot{\bar{q}}_1+2\psi_1\cos 2\psi_0+2\psi_1\tilde{h}_y^{(m)}(\bar{z})\sin\psi_0. \tag{40}$$

The solution of Eq. (39), which satisfies the boundary conditions, is given by

$$\psi_1=-\Xi(\bar{z})\cos\psi_0+B_2, \tag{41}$$

where $\Xi(\bar{z})$ is determined by expression (22). The condition $\psi_1'|_{z=\pm t_f/2}=0$ demands the fulfillment of the relation $\dot{\bar{q}}_0=\sin 2\psi_0$, which is just standard dependence of the domain wall velocity on the angle of deviation of magnetization from the domain wall plane. Substitution of expression for the first order term ψ_1 into Eq. (40) allows us to determine the constant B_2 and $\dot{\bar{q}}_1$. The procedure of integration of Eq. (40) is rather standard but unwieldy because of the cumbersome form of expression (41). Limiting ourselves to the terms linear in a small parameter v and taking into account the boundary condition as well as condition of absence of divergent term (these divergent terms appear at $\psi_0=\pi/4$), we can obtain the following expression for domain wall velocity;

$$\dot{\bar{q}}=\sin 2\psi_0\left[1+\frac{v}{3}(\ln 2+1)\right], \tag{42}$$

which determines the limiting velocity of the domain wall in ultrathin magnetic film. Theoretically speaking, the limiting velocity of the domain wall in ultrathin films

$$V_{\lim}=V_W\left[1+\left(\frac{t_f}{l_{\text{ex}}}\right)^2\frac{\ln 2+1}{24}\right] \tag{43}$$

is higher than in the films with $t_f>l_{\text{ex}}$. At the same time the difference between $V_{\lim}$ and V_W is negligibly small being proportional to $(t_f/l_{\text{ex}})^2$.

Let us consider the stationary dynamics of the Bloch line in domain wall in ultrathin film. The stationary motion of the vertical Bloch line in the domain wall, which itself moves under the action of driving magnetic field, will be considered. Generalization of Eq. (35) has the following form:

$$\frac{\dot{q}}{\Delta}-\lambda_r\dot{\psi}=-\omega_M\left[l_{\text{ex}}{}^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\psi -\sin\psi\cos\psi+\frac{1}{4}h_y^{(m)}(z,0)\cos\psi\right], \tag{44}$$

where $\omega_M=4\pi\gamma M_s$. This equation was obtained with an account of dissipation.[8,9] The standard dissipative function corresponding to the Gilbert's form of the relaxation term in the Landau–Lifshitz equation was added to Eq. (33), λ_r is the Gilbert relaxation parameter. The velocity of the domain wall is now determined by the driving field $\dot{q}=\Delta\lambda_r^{-1}\gamma H_b$. Static distribution of the magnetization inside the vertical Bloch line that is the solution of Eq. (44) at $\dot{q}=\dot{\psi}=0$ will be analyzed considering the last term in the right-hand side of Eq. (44) as perturbation

$$\psi=\psi_0(x)+\psi_1(x,z)+\cdots. \tag{45}$$

The zero order distribution of magnetization in the vertical Bolch line is standard[8,9]

$$\sin\psi_0=\frac{1}{\cosh(x/l_{\text{ex}})}. \tag{46}$$

Using this formula one can obtain

$$\psi_1(x,z)=v\tanh(x/l_{\text{ex}})\Xi(z), \tag{47}$$

where $\Xi(z)$ is given by formula (22). As one can see, the span of the magnetic moment distribution in the vertical Bloch line becomes dependent on the coordinate along the film thickness and it is maximal in the film's midplane and minimal at the film surfaces. However, it has to be noted that the additional dependence of the azimuthal angle of magnetization inside the vertical Bloch line on the z coordinate is proportional to the small parameter $(t_f/l_{\text{ex}})^2$ and thus negligibly small.

Let us now analyze how this change of the structure of the vertical Bloch line may affect its dynamics. Dynamic solution of Eq. (44) will be sought in a form $\psi[x-X(z,t),z)]$, where $X(z,t)=V_{\text{BL}}t+\tilde{X}(z,x)$ describes the position of the vertical Bloch line and distortion of its shape due to the action of gyrotropic forces [it is clear that $\tilde{X}(z,x)$ is expected to be small].

Equation (44) can be now rewritten in the form

$$\frac{\dot{q}}{\Delta}-\lambda_r\dot{X}\frac{\partial\psi_0}{\partial x}+\omega_M l_{\text{ex}}{}^2\left(\frac{\partial\psi_0}{\partial x}\frac{\partial^2 X}{\partial z^2}+\frac{\partial^2\psi_0}{\partial x\partial z}\frac{\partial X}{\partial z}\right) =-\omega_M\left\{l_{\text{ex}}^2\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial x^2}\right)\chi-\chi\cos 2\psi_0 +\frac{1}{4}\chi h_y^{(m)}(z)\sin\psi_0\right\}, \tag{48}$$

where $\chi(z,x)=\partial\psi/\partial x\tilde{X}$ in approximation of small distortions of the vertical Bloch line shape. The right-hand side of Eq. (48) becomes equal to zero if expression (45) along with Eqs. (46) and (47) is used for ψ in calculation of $\chi(z,x)$. The inhomogeneous Eq. (48) can be solved if the expression for $\chi(z,x)$ is orthogonal to the left-hand side of the equation. One can get from Eq. (45)

$$\chi(z,x)=\frac{1}{l_{\text{ex}}}\left\{-\frac{1}{\cosh(x/l_{\text{ex}})}+v\frac{f(z)}{\cosh^2(x/l_{\text{ex}})}\right\}\tilde{X}(z,x). \tag{49}$$

The condition that the above expression for $\chi(z,x)$ is orthogonal to the left-hand side of Eq. (48) can be written in the form of the equation

$$\lambda_r\dot{X}-\omega_M l_{\text{ex}}^2\frac{\partial^2}{\partial z^2}X=\frac{\pi}{2}\dot{q}\sqrt{Q}\left[1+v\left(\frac{\pi}{2}-\frac{2}{\pi}\right)\Xi(z)\right], \tag{50}$$

which has to be solved along with the following boundary conditions $(\partial X/\partial z)_{z=\pm t_f/2}$. The solution is given by

$$X(z,t)=V_{\mathrm{BL}}t+v^2\left(\frac{\pi^2}{4}-1\right)\dot{q}\frac{\sqrt{Q}}{\omega_M}g(\tilde{z}), \tag{51}$$

where

$$V_{\mathrm{BL}}=\dot{q}\frac{\pi\sqrt{Q}}{2\lambda_r} \tag{52}$$

coincides with the standard expression[9] for the velocity of the vertical Bloch line in the domain wall moving with velocity $\dot{q}$ and

$$g(\tilde{z})=\tilde{z}\left(\ln 2+\frac{1}{2}\right)-\tilde{z}^3\frac{2\ln 2}{3}+\tilde{z}^5\frac{(2\ln 2-1)}{10}. \tag{53}$$

The second term in expression (51) describes the dynamic distortions of the distribution of magnetization inside the vertical Bloch line. Very small (being proportional to v^2) distortion of the vertical Bloch line shape is, however, antisymmetric with respect to the film thickness.

Summarizing the analysis carried out here, we can conclude that our analysis provides theoretical justification of the fact that static and dynamic properties of domain wall in ultrathin magnetic film are notably close to the ones of a standard Bloch wall. The limiting velocity of the uniform motion of the domain wall in ultrathin films has to be greater than that observed in thicker films with $t_f>l_{\mathrm{ex}}$, because it is practically equal to Walker's critical velocity.

Our study has also shown that the distribution of the magnetization inside the stationary Bloch line is very close to the one in films with $t_f>l_{\mathrm{ex}}$. We also showed that the dynamics of vertical Bloch lines in ultrathin magnetic films is similar to that of thicker films with thickness $t_f>l_{\mathrm{ex}}$.

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