Instabilities of intense laser beams in air*

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Various aspects of the propagation of an intense laser beam through the atmosphere are considered. The basic laser-fluid equations are presented and a linearized analysis of these equations is given which predicts a very low power threshold for Brueckner–Jorna-type convective instabilities. Another class of instabilities is predicted to be of more practical importance than the convective instabilities and an effective Reynolds number is derived which may help to characterize these turbulent instabilities. In the following paper by the same authors a computer solution of the full set of nonlinear equations is described and the initial development of the laser-fluid interaction is investigated.

I. INTRODUCTION

The present paper and the following paper[1] are concerned with the distortions of a laser beam produced by density and thermal variations in a fluid medium. These distortions have been the subject of many investigations which can be classified into two groups, depending upon whether time dependence is considered. Most available experiments are conveniently understood by reference to theoretical studies of the gross effects of thermal deposition and fluid motion which assume that a steady state will be achieved for the deflection and distortion of the laser beam. On the other hand Brueckner and Jorna[2] have discovered that some of the solutions for beam propagation are unstable so that under certain conditions a steady state may not develop. The Brueckner-Jorna instabilities were discovered in a linearized analysis, but the threshold for such instabilities has not been discussed previously. The Brueckner-Jorna instabilities are not observed in practice because they are convective instabilities and cannot develop within typical distances allowed for the propagation of beams in the laboratory.

In the present paper the linearized analysis is presented, keeping the four-photon coupling induced by periodic fluctuations in the dielectric constant, so that among other things the threshold behavior will be exposed. The basic equations are given in Sec. II and the linearized analysis is presented in Sec. III. In Sec. IV another class of instabilities is predicted to be of more practical importance than the convective instabilities and an effective Reynolds number is derived which may help to characterize these turbulent instabilities.

The companion paper[1] describes a computer solution of the full set of nonlinear laser-fluid equations. Enormous power was presumed for the laser beam in order to drive the laser-fluid interaction as fast as possible in an attempt to watch the onset of distortions of the beam.

II. BASIC EQUATIONS

When an intense laser beam propagates through a fluid, many intersting phenomena take place. This laser-fluid system can be described by a macroscopic model which involves Maxwell's equations, the Navier-Stokes equation, an energy conservation equation, and the continuity equation for fluid motion. These equations, which describe the behavior of intense electromagnetic beams and the associated sound and thermal fluctuations, are coupled by stimulated Raman scattering, electrostriction, the high frequency Kerr effect, absorption heating, and the density and temperature dependence of the dielectric constant. In this paper a systematic discussion is presented for an intense laser beam propagating through air, which has a negligible Kerr constant. If the frequencies are outside the Raman scattering range, the instabilities are primarily caused by optical-acoustic coupling of the laser beam and the gases. These effects are of long duration compared to those of self-focusing. As the beam passes through air, the intensity profile induces a nonuniform temperature gradient transverse to the propagating direction of the beam, due to the energy absorption from the beam. This thermal nonequilibrium and electrostriction together cause the generation of a density gradient and hence a sound wave. These density changes react back on the incident beam through changes in the dielectric constant.

The equations describing propagation of electromagnetic radiation and the equations describing fluid behavior are widely known.[3–5] Dropping unimportant terms from the full equations, we take the following set of nonlinear coupled partial differential equations for description of the macroscopic representation of the laser-fluid system:

Wave equation and Clausius-Mosotti relation:

$$\nabla^2\mathbf{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\epsilon\mathbf{E}) + \frac{\alpha}{c}\frac{\partial}{\partial t}(\sqrt{\epsilon}\,\mathbf{E}), \tag{1}$$

$$\epsilon = \epsilon_L + \epsilon_2\langle E^2\rangle_{\text{av}},$$

$$\rho\left(\frac{\partial\epsilon}{\partial\rho}\right)_T \approx \frac{(\epsilon-1)(\epsilon+2)}{3}, \quad \left(\frac{\partial\epsilon}{\partial T}\right) \approx 0; \tag{2}$$

Navier-Stokes equation:

$$\rho\frac{D\mathbf{v}}{Dt} = \rho\mathbf{g} - \nabla P + \mathbf{f}_{\text{es}} + \mathbf{f}_{\text{visc}}, \tag{3}$$

$$\mathbf{f}_{\text{visc}} = \eta\nabla^2\mathbf{v} + (\eta+\eta')\nabla(\nabla\circ\mathbf{v}), \tag{4}$$

$$\mathbf{f}_{\text{es}} = \tfrac{1}{2}\nabla\left\langle\left[E^2\rho\left(\frac{\partial\epsilon}{\partial\rho}\right)_T\right]\right\rangle_{\text{av}} - \tfrac{1}{2}\langle E^2\nabla\epsilon\rangle_{\text{av}}; \tag{5}$$

heat transfer equation:

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$$\rho C_v \frac{DT}{Dt} - \frac{C_v(\gamma-1)}{\beta}\frac{D\rho}{Dt} = \phi_\eta + \nabla\cdot(\kappa\nabla T) + \alpha c\sqrt{\epsilon}\langle E^2\rangle_{av}, \tag{6}$$

$$\phi_\eta = \left[\eta\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \eta'\frac{\partial v_k}{\partial x_k}\delta_{ij}\right]\frac{\partial v_i}{\partial x_j}\ ; \tag{7}$$

fluid continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0\ ; \tag{8}$$

equation of state:

$$P = P(\rho, T) = R\rho T. \tag{9}$$

In the wave equation the term involving α, the linear absorption coefficient, is associated with a model for the absorption of electromagnetic energy by the fluid.[6] The absorption coefficient is taken to be independent of the frequency of the electromagnetic radiation, so the model is not valid near the resonance lines of the molecules in the fluid. Also the model does not include kinetic rate equations, so saturation effects are not considered. In the present model the electric field $\mathbf{E}$ will be damped by a factor $\exp(-\frac{1}{2}\alpha z)$, where z is the direction of propagation and the energy deposited in the medium is taken to be $\alpha c\sqrt{\epsilon}\langle E^2\rangle_{av} \equiv \alpha I_L$, where I_L is the laser intensity in erg/sec cm^{-2}.[7]

The terms ϵ_L and ϵ_2 are the linear and nonlinear permittivity coefficients, respectively, ρ is the fluid mass density, and $\mathbf{v}$ is the velocity of a "material element" of the fluid. The convective derivative D/Dt follows the motion of a material "particle" of the fluid relative to a fixed coordinate system and is expressed in the form

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + \mathbf{v}\cdot\nabla(\ ).$$

The vector $\mathbf{g}$ is the gravitational acceleration vector and η and η' are the shear and compressional viscosity coefficients, respectively.

The electrostrictive force density $\mathbf{f}_{es}$ is given by

$$(\mathbf{f}_{es})_i = \left\langle\frac{\partial}{\partial x_j}\sigma_{ij}\right\rangle_{av},$$

where σ_{ij} is the interaction stress tensor for the electromagnetic field and the fluid and the angular brackets indicate a time average over several optical periods. A derivation of the stress tensor is given on p. 67 of Ref. 4:

$$\sigma_{ij} = -\tfrac{1}{2}E^2\left[\epsilon - \rho\left(\frac{\partial\epsilon}{\partial\rho}\right)_T\right]\delta_{ij} + \epsilon E_i E_j,$$

but this tensor is not strictly correct for optical fields because an isothermal constraint was imposed. A similar derivation with an isentropic constraint gives the same result, except that the partial derivative $(\partial\epsilon/\partial\rho)_T$ at constant temperature is replaced by $(\partial\epsilon/\partial\rho)_s$, the derivative at constant entropy. The difference in these two constraints is contained in the thermodynamic relation

$$\rho\left(\frac{\partial\epsilon}{\partial\rho}\right)_s = \rho\left(\frac{\partial\epsilon}{\partial\rho}\right)_T + T\left(\frac{\partial\epsilon}{\partial T}\right)_\rho\left(\frac{\gamma C_p}{\beta v_s^2} - \beta T\right)^{-1}. \tag{10}$$

The difference term in Eq. (10) is very small because, for gases, $(\partial\epsilon/\partial T)_\rho \approx 0$. The term $(\gamma C_p/\beta v_s^2 - \beta T)^{-1}$ contributes a factor of roughly $\frac{1}{2}$. Actually, neither constraint is strictly valid, but corrections would be small and would necessitate a detailed examination of fluid boundary layers and the explicit mechanisms of heat deposition in the control volume.

The thermodynamic quantities appearing in the above equations are C_v and C_p, the specific heats in erg/g deg at constant volume and pressure, respectively, and γ is the ratio of specific heats, C_p/C_v; β is the thermal expansion coefficient, $-(1/\rho)(\partial\rho/\partial T)_p$; v_s is the isentropic velocity of sound, $[(\partial P/\partial\rho)_s]^{1/2}$; and κ is the thermal conductivity of the fluid.

Due to the complexity of the laser-fluid equations shown above, it is not possible to obtain exact solutions analytically. The linearized solutions have been discussed[2] and a number of computer solutions have recently been given by various groups.[8] In Sec. III a linearized analysis of this set of equations is presented.

III. LINEARIZED ANALYSIS

Linearized analysis is a standard perturbation technique. In this scheme it is assumed that each of the dependent variables in the problem can be expressed as the sum of its slowly varying zeroth order component and a small first-order correction.[9] In this way, a set of linear equations for small disturbances is obtained. This approach to the analysis of the laser-fluid system was first investigated by Brueckner and Jorna.[2] In the present approach, two variables are used to describe the perturbed electromagnetic field, one for the component of the field which is vibrating in phase with the primary beam and one for the component out of phase. In this way, the four-photon coupling induced by periodic fluctuations in the dielectric constant can be included. This coupling was not included in the original formulation given by Brueckner and Jorna and accounts for the absence of a threshold in their analysis. The dispersion relation for these linearized equations has been evaluated and is more complicated in structure than that presented by Brueckner and Jorna. For propagation through air, however, the numerical differences are minor. The wave with the largest growth rate, resulting from resonant interactions between scattered electromagnetic waves and the thermal wave, propagates almost perpendicularly to the laser beam. The direction is such that the change in frequency of the scattered electromagnetic wave and the frequency of the thermal wave (which is zero) are approximately the same.

Writing

$$\begin{aligned}
&\mathbf{E} \approx \mathbf{E}_{(0)} + \mathbf{E}_{(1)},\\
&\rho \approx \rho_0 + \rho_1,\\
&T \approx T_0 + T_1,\\
&\epsilon_{0e} \equiv \epsilon_{L(0)} + \epsilon_{2(0)}\langle E_{(0)}^2\rangle_{av}
\end{aligned} \tag{11}$$

and taking

$$\mathbf{E}_{(0)} = \tfrac{1}{2}\hat{e}_y E_0 \exp[i(\omega_L t - k_L z)]\exp(-\tfrac{1}{2}\alpha z) + \text{c.c.}, \tag{12}$$

$$\mathbf{E}_{(1)} = \tfrac{1}{2}\hat{e}_y\{f\exp[i(\omega_+ t - \mathbf{k}_+\cdot\mathbf{x})] + g\exp[-i(\omega_- t - \mathbf{k}_-\cdot\mathbf{x})]\}\exp(-\tfrac{1}{2}\alpha z) + \text{c.c.}, \tag{13}$$

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$$\begin{pmatrix}\rho_1\\ T_1\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\rho'\\ T'\end{pmatrix}\exp[i(\omega t-\mathbf{k}\cdot\mathbf{x})]+\text{c. c.},\tag{14}$$

where

$$\omega_L \equiv \text{laser frequency},$$
$$\omega_\pm \equiv \omega_L \pm \omega,\tag{15}$$
$$\mathbf{k}_\pm \equiv k_L \hat{e}_z \pm \mathbf{k},$$

and treating the velocity $\mathbf{v}$ to be a first-order quantity, we obtain from Eqs. (1)–(9) the following four equations relating f, g, ρ', and T':

$$[(\epsilon_{0e}+\tfrac{1}{2}\epsilon_{2(0)}E_0^2)\omega_+^2-c^2k_+^2]f+(\tfrac{1}{2}\epsilon_{2(0)}E_0^2\omega_+^2)g$$
$$+(\tfrac{1}{2}AE_0\omega_+^2)\rho'+(\tfrac{1}{2}BE_0\omega_+^2)T'=0;\tag{16}$$

$$(\tfrac{1}{2}\epsilon_{2(0)}E_0^2\omega_-^2)f+[(\epsilon_{0e}+\tfrac{1}{2}\epsilon_{2(0)}E_0^2)\omega_-^2-c^2k_-^2]g$$
$$+(\tfrac{1}{2}AE_0\omega_-^2)\rho'+(\tfrac{1}{2}BE_0\omega_-^2)T'=0;\tag{17}$$

$$[\alpha c(\epsilon_{0e})^{1/2}E_0]f+[\alpha c(\epsilon_{0e})^{1/2}E_0]g+\{i[C_v(\gamma-1)/\beta]\omega\}\rho'$$
$$+(-\kappa k^2-i\rho_0C_v\omega)T'=0;\tag{18}$$

$$[\tfrac{1}{6}(\epsilon_{0e}-1)(\epsilon_{0e}+2)E_0k^2]f+[\tfrac{1}{6}(\epsilon_{0e}-1)(\epsilon_{0e}+2)E_0k^2]g$$
$$+\{\omega^2-iN\omega k^2-[u^2-\tfrac{1}{6}(\epsilon_{0e}-1)]AE_0^2k^2\}\rho'$$
$$+\{[-u^2\beta\rho_0+\tfrac{1}{6}(\epsilon_{0e}-1)BE_0^2]k^2\}T'=0.\tag{19}$$

Throughout these equations, E_0 has been written in place of $|E_0|$. Since only the magnitude of the complex amplitude appears, E_0 may be considered to be real and positive without loss of generality. For convenience in writing these equations, the following notation has been introduced:

$$u^2\equiv v_s^2/\gamma,\tag{20}$$

so that u is the isothermal speed of sound in the unperturbed medium, and

$$A\equiv\left(\frac{\partial\epsilon_L}{\partial\rho}\right)_T+\left(\frac{\partial\epsilon_2}{\partial\rho}\right)_T\langle E_{(0)}^2\rangle_{\text{av}}\approx\frac{1}{\rho_0}\frac{(\epsilon_{0e}-1)(\epsilon_{0e}+2)}{3},$$
$$B\equiv\left(\frac{\partial\epsilon_L}{\partial T}\right)_\rho+\left(\frac{\partial\epsilon_2}{\partial T}\right)_\rho\langle E_{(0)}^2\rangle_{\text{av}},\tag{21}$$
$$N\equiv(2\eta+\eta')/\rho_0.$$

The consistency condition for the linearized equations is the vanishing of the determinant of the coefficients of f, g, ρ', and T'. Thus we obtain the following relation between the frequency ω and the wave vector $\mathbf{k}$, the dispersion relation for the system:

$$\{(\omega-i\kappa'k^2)[\omega^2-iN\omega k^2-(u^2-A')k^2]-(\gamma-1)(u^2-B')\omega k^2\}$$
$$\times[\xi_+\xi_--\tfrac{1}{2}\epsilon_{2(0)}E_0^2(\xi_+\omega_-^2+\xi_-\omega_+^2)/2\omega_L]$$
$$=-\big(Ak^2[\tfrac{1}{2}\rho_0A(\omega-i\kappa'k^2)+i(\alpha cn_{0e}\beta/C_v)(u^2-B')]$$
$$+B\{[(\gamma-1)/\beta]\tfrac{1}{2}A\omega k^2+i(\alpha cn_{0e}/\rho_0C_v)[\omega^2-iN\omega k^2$$
$$-(u^2-A')k^2]\}\big)\tfrac{1}{2}E_0^2(\xi_+\omega_-^2+\xi_-\omega_+^2)/2\omega_L.\tag{22}$$

In order to put Eq. (22) in the slightly more compact form shown above, the following additional abbreviations have been introduced:

$$\kappa'\equiv\kappa/\rho_0C_v,$$
$$\eta_{0e}\equiv(\epsilon_{0e})^{1/2}=\text{index of refraction of unperturbed medium},$$
$$I_L\equiv\tfrac{1}{2}\eta_{0e}cE_0^2=\text{power in incident laser beam/unit area},$$
$$A'\equiv\tfrac{1}{6}(\epsilon_{0e}-1)AE_0^2,\tag{23}$$
$$B'\equiv\tfrac{1}{6}(\epsilon_{0e}-1)BE_0^2/\rho_0\beta,$$
$$2\omega_L\xi_+\equiv c^2k_+^2-\epsilon_{0e}\omega_+^2,$$
$$2\omega_L\xi_-\equiv c^2k_-^2-\epsilon_{0e}\omega_-^2.$$

For comparison, the dispersion relation obtained by Brueckner and Jorna[2] for frequencies outside the Raman scattering range is

$$\omega(\omega^2-v_s^2k^2-iN\omega k^2)\xi_L+I_L(\nu\omega+i\delta)k^2=0,\tag{24}$$

where

$$\xi_L\equiv[k^2-(2\epsilon_{2(0)}/n_{0e}^3c)I_Lk_L^2]-4(n_{0e}k_L/ck)^2[\omega-(c/n_{0e})k_z]^2,$$
$$\nu\equiv\rho_0A^2k_L^2/n_{0e}^3c,\tag{25}$$
$$\delta\equiv2A\beta v_s^2\alpha k_L^2/n_{0e}^2C_\rho.$$

Although Eq. (22) is considerably more complicated in structure than Eq. (24), the general features of the two equations are the same. As a first approach to the analysis of (22), one should realize that the power in the primary laser beam is proportional to E_0^2. Thus, the free modes of the system can be obtained by letting $E_0^2\to0$. With no power in the incident beam, therefore, (22) reduces to

$$\{[\omega-i(\kappa/\rho_0C_v)k^2](\omega^2-iN\omega k^2-u^2k^2)-(\gamma-1)u^2\omega k^2\}$$
$$\times[c^2(k_L^2+k^2+2k_Lk_3)-\epsilon_{L(0)}(\omega_L+\omega)^2]$$
$$\times[c^2(k_L^2+k^2-2k_Lk_3)-\epsilon_{L(0)}(\omega_L-\omega)^2]=0.\tag{26}$$

The first factor in (26) contains a nonpropagating thermal wave and two damped sound waves coupled by the term $(\gamma-1)u^2\omega k^2$. The last two factors correspond to the four free modes for scattered undamped electromagnetic waves:

$$\omega/\omega_L=\sigma\pm[1+k^2/k_L^2+2\sigma k_3/|k_L|]^{1/2},\tag{27}$$

where $\sigma=\pm1$, specified by given values of k^2 and k_3. Two of the roots are low frequency ($\omega\ll\omega_L$), whereas the other two have frequencies of the same order as the laser frequency. It is clear that the roots at the high frequency should be eliminated, because it has been assumed previously, in evaluating time averages, that the perturbed solutions vary much more slowly than the optical waves. Therefore, factors like

$$c^2k^2-\epsilon_{L(0)}\omega_+^2\tag{28}$$

will be replaced by

$$\epsilon_{L(0)}(2\omega_L)(c_Lk_+-\omega_+),\tag{29}$$

where

$$c_L\equiv c/[\epsilon_{L(0)}]^{1/2}=\text{velocity of light in the medium}.$$

Thus the free modes will include three thermal-sound waves and two electromagnetic waves.

All of the terms in (26) result from the left-hand side of (22) because the right-hand side is proportional to the power in the primary laser beam. Now, as the power in the laser beam is turned on, the right-hand side couples the five free modes described by (26). Additional tiny coupling arises inside the left-hand side itself through the A', B', and $\epsilon_{2(0)}$ terms.

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For detailed consideration, the case of a primary laser beam at 10.6 μ propagating through air at approximately 10 °C and at standard pressure will be discussed. The numerical values for the parameters appearing in the dispersion relation are[10]

$$\begin{aligned}
&\omega_L = 1.773\times10^{14}\ \text{sec}^{-1},\\
&k_L = 5920\ \text{cm}^{-1},\\
&\rho_0 = 1.25\times10^{-3}\ \text{g/cm}^3,\\
&N = 0.284\ \text{cm}^2/\text{sec},\\
&\beta = 3.67\times10^{-3}\ \text{deg}^{-1},\\
&C_v = 7.143\times10^6\ \text{erg/g deg},\\
&\kappa' = 0.28\ \text{cm}^2/\text{sec},\\
&n_0 = 1 + 2.82\times10^{-4},\\
&u^2 = 8.39\times10^8\ (\text{cm/sec})^2,\\
&\gamma = 1.4,\\
&\epsilon_{L(0)} = 1 + 5.65\times10^{-4},\\
&\alpha = (3\times10^{-7}\ \text{cm}^{-1})\alpha_0,\\
&\alpha\eta_{0e}cu^2\beta/C_v = 3.831\times10^3\alpha_0,\\
&\alpha\eta_{0e}/\rho_0C_v = 0.988\alpha_0,\\
&\tfrac{1}{2}\epsilon_{2(0)}E_0^2 = (1.37\times10^{-25}\ \text{sec}^3/\text{g cm}^2)I_L,\\
&A = 0.452\ \text{cm}^3/\text{g} + (1.10\times10^{-22}\ \text{cm sec}^3/\text{g}^2)I_L,\\
&A(\gamma-1)/2\beta = 24.7\ \text{cm}^3\,\text{deg/g}\\
&\qquad + (6.00\times10^{-21}\ \text{cm sec}^3\,\text{deg/g}^3)I_L,\\
&\epsilon_{0e} = 1 + 5.65\times10^{-4} + (1.37\times10^{-25}\ \text{sec}^3/\text{g cm}^2)I_L,\\
&A' = (2.84\times10^{-15}\ \text{cm}^2\,\text{sec/g})I_L + (1.38\times10^{-36}\ \text{sec}^4/\text{g}^2)I_L^2,\\
&B = \epsilon_{2(0)}E_0^2/T_0 = -(4.84\times10^{-28}\ \text{sec}^3/\text{g cm}^2\,\text{deg})I_L,\\
&B' = -(0.662\times10^{-36}\ \text{sec}^4/\text{g})I_L^2.
\end{aligned} \tag{30}$$

In the above list a dimensionless absorption constant α_0 of order unity has been introduced and the power I_L is in units of erg/sec per cm². Now, using these numerical values, one finds that the power-dependent terms are very small for power fluxes less than 10 MW/cm², except for the term which represents energy absorption. In other places in the dispersion relation, the power-dependent terms are connected with the nonlinear index, and will be omitted in the following. (The terms omitted are related to self-focusing in a manner described by Brueckner and Jorna.[2]) This neglect of the nonlinear index and of the weak dependence of the optical coefficients of gases on the temperature for fixed density allows the simplification of the dispersion relation to

$$\begin{aligned}
&[(\omega - i\kappa'k^2)(\omega^2 - iN\omega k^2 - u^2k^2) - (\gamma-1)u^2\omega k^2]\\
&\quad\times(c_Lk_+ - \omega_+)(c_Lk_- - \omega_-)\\
&\qquad = -(Ak^2\omega_LI_L/2c\epsilon_L)[(c_Lk_+ - \omega_+) + (c_Lk_- - \omega_-)]\\
&\qquad\quad\times[\tfrac{1}{2}\rho_0A(\omega - i\kappa'k^2) + i\alpha cn_0\beta u^2/C_v].
\end{aligned} \tag{31}$$

In addition, for air at reasonable powers, the term $\frac{1}{2}\rho_0A(\omega - i\kappa'k^2)$ on the right-hand side is negligible compared to $\alpha cn_0\beta u^2/C_v$. Introducing the variable

$$\nu \equiv k_3/k, \tag{32}$$

instead of k_3, and defining

$$\tau \equiv A\omega_L\beta\alpha I_L/\sqrt{\epsilon_L}C_p, \tag{33}$$

which corresponds to the power parameter used in Ref. 2, the dispersion relation can be written in the form

$$\begin{aligned}
&(\omega - i\kappa'k^2)(\omega^2 - iN\omega k^2 - u^2k^2) - (\gamma-1)u^2\omega k^2\\
&\quad = \tfrac{1}{2}i\tau v_s^2k^2\{[\omega - \tfrac{1}{2}i\alpha c_L - c_L(k\nu + k^2/2k_L)]^{-1}\\
&\qquad - [\omega - \tfrac{1}{2}i\alpha c_L - c_L(k\nu - k^2/2k_L)]^{-1}\}.
\end{aligned} \tag{34}$$

The problem at this stage is the determination of the maximum growth rate of any Fourier component of a distortion of the plane wave as a function of the absorbed power from the beam. That is, one must solve the dispersion relation for the frequencies as a function of k, ν, I_L, and the characteristic parameters of the medium, and then find the maximum value of $-\text{Im}\,\omega$ for real ν and k with $|\nu| < 1$. Such a problem cannot be solved analytically without further approximations. One region of interest would be the high power limit, where the driving term would overwhelm the losses resulting from thermal conduction, viscosity, and the absorption of electromagnetic energy. In that case, all the imaginary terms in Eq. (34), with the sole exception of the i immediately preceding τ, can be dropped, reducing the dispersion relation to the form employed by Bruecker and Jorna in Eq. (45) of Ref. 2.

To proceed analytically, Brueckner and Jorna neglected the term in the second set of brackets on the right-hand side of (34), and assumed that the maximum growth rate would occur somewhere on the curve in the ν, k plane determined by the constraint

$$\text{Re}[\omega - c_L(k\nu + k^2/2k_L)] = 0. \tag{35}$$

Along that curve, the maximum growth rate is

$$(-\text{Im}\,\omega)_{\max} = \tfrac{1}{2}\sqrt{\tau}(1.08), \tag{36}$$

which corresponds to

$$v_sk = (\tfrac{1}{2}\tau)^{1/2}\left(\frac{0.97515}{0.56305}\right)\left(\frac{1}{0.93063}\right)^{1/2}. \tag{37}$$

(These results differ from those in Ref. 2, which are erroneous.) There is no assurance that the actual maximum growth rate does lie along the one-dimensional subset of the ν, k plane assumed in Ref. 2. We have conducted a search along the line

$$\nu + k/2k_L = 0.$$

However, the result for the maximum,

$$\tfrac{1}{2}\sqrt{\tau}(1.06), \tag{38}$$

is 2% smaller. No other curve in the ν, k plane has been found which allows an analytical search. Nevertheless, one suspects that these answers are sufficiently close and that further analytical effort is not justified, because of the previous approximations.

An interesting unknown not discussed previously is the power flux required to stimulate these instabilities. This threshold power is clearly a critical function of the losses in the system, which therefore renders it important to treat them carefully. If the second term on the right-hand side of Eq. (34) is dropped, the instabil-

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ity appears to have no threshold, because the conduction loss, which must be overcome, vanishes as $k\to 0$. However, as $k\to 0$, the Stokes and anti-Stokes terms on the right-hand side of Eq. (34) tend to cancel each other, and, therefore, there is a threshold power flux for these stimulated thermal Rayleigh scattering instabilities. Using (34), a computer search for this threshold was performed and led to the result.

$$(I_L)_{\text{threshold}} = 0.329 \text{ mW/cm}^2. \qquad (39)$$

This threshold was located at $k = 0.04$ cm^{-1} with $\nu = \pm 1.1\times 10^{-6}$. The degeneracy in the value of ν occurs because of the symmetry property contained in (34), $\nu \to -\nu$ implies $i\omega \to (i\omega)^*$. The dispersion relation of Brueckner and Jorna contained no threshold for the convective instabilities and, therefore, suggested that instabilities might be present for extremely low beam intensities. The $\frac{1}{3}$-mW per cm^2 threshold obtained in the present analysis is certainly small in relation to intensities available for experiments.

The presence of a wind does not alter the growth rates for distortions. This can be easily seen by considering the problem from a frame of reference moving with the fluid. A uniform beam remains a uniform beam in the moving frame, although its direction of propagation is shifted. This shift in direction has no effect upon the stability discussion. The very low convective instability threshold does not mean that such effects are easy to observe or are of practical importance. An extremely long, and carefully protected, optical path length would be required in order to see these instabilities grow to observable sizes. Indeed, such instabilities have never been observed in the laser-fluid system.

There is, however, another class of instabilities involving the balance between inertial terms and bouyancy forces in the fluid. Such instabilities are discussed in Sec. IV.

IV. BEAM-FED TURBULENCE

Many experiments have been reported for which theories assuming steady-state beam profiles, after initial transients die out, provide rather good explanations of the principal features. However, that is probably true only because these experiments are conducted at relatively low power fluxes. Theoretically, one expects a time-dependent state of the system because of the instabilities discussed earlier. Such instabilities are not observed in practice because they cannot develop within the distances allowed for the propagation of beams. However, alternate considerations for a beam of finite cross section suggest that the beam may drive the fluid into a time-dependent, or turbulent, state at powers which are not completely unreasonable.

It may be impossible to prove analytically that such a turbulent state develops, because the investigation of hydrodynamic stability is very difficult even for the simplest flows. However, an argument can be made from dimensional considerations, an approach that promises to be very useful. Namely, for the problem of a beam of radius a and power flux I passing through air, it is possible to estimate a parameter W, which plays the role of an effective Reynolds number for our problem. It will be shown that the parameter W takes on values of the order of 30 000 for a beam with intensity $I = 1$ kW/cm^2 of radius 1 m. Since it is known that some flows with Reynolds numbers substantially lower than 30 000 are turbulent, the flows for the laser-heated atmospheric path should also be expected to show significant time dependence.

Consider the equations of motion for the air and the equation governing heat transfer, which take the following form if it is assumed that the air can be assumed incompressible (that amounts to dropping terms of order u^2/v_s^2, where u is a typical flow speed, and v_s is the speed of sound; for the problems under consideration, u^2/v_s^2 will be less than 10^{-5}, and the incompressible fluid approximation will be quite good):

$$\begin{aligned}\rho(\mathbf{v}\cdot\nabla)\mathbf{v} &= -\nabla P - \beta T_1\rho\mathbf{g} + (\rho/\rho_0)\eta\nabla^2\mathbf{v} \\ &\quad + (\rho/\rho_0)(\eta+\eta')\nabla(\nabla\circ\mathbf{v}), \\ \rho(\nabla\circ\mathbf{v}) &= -(\mathbf{v}\cdot\nabla)\rho = \beta\rho(\mathbf{v}\cdot\nabla)T_1, \\ \rho C_p(\mathbf{v}\cdot\nabla)T_1 &= \alpha I + \nabla\cdot(\kappa\nabla T_1) \\ &\quad + (\rho/\rho_0)[\eta(v_{i,j}+v_{j,i})^2 + \eta'(\nabla\circ\mathbf{v})^2].\end{aligned} \qquad (40)$$

Assuming the Reynolds number is high, the inertial terms will dominate the viscous terms in the Navier-Stokes equation. Thus, there must be a balance between the inertial terms and the bouyancy forces, which implies that $\rho u^2/a \sim \alpha\beta T_1 g$, where ρ is the density of air, β is the coefficient of thermal expansion, T_1 is a typical value for the temperature rise, and g is the acceleration due to gravity. The pressure variation will be of order ρu^2. In the heat-transfer equation, the convection term will dominate the conduction term, and the beam heating will overwhelm the viscous dissipation, so that there must be a balance between heat deposition from the beam and convective heat transfer, which implies that $\rho C_p u T_1/a \sim \alpha I$. Combining these two relations we find that $u^3 \sim \alpha\beta a^2 I g/\rho C_p$. With this expression for u, we then define a parameter W, which is expected to indicate regimes where steady flow and where time-dependent flow may be anticipated. W is an estimate of the relative importance of inertial terms to viscous terms in controlling the flow:

$$\begin{aligned}W &= a u \rho_0/\eta \\ &= a(\rho_0/\eta)(\alpha\beta a^2 I g/\rho C_p)^{1/3}.\end{aligned} \qquad (41)$$

For a beam with $I = 10^{10}$ erg/cm^2 sec, $a = 100$ cm, $T_0 = 273$ °K, and $\alpha = 3\times 10^{-6}$ cm^{-1}, $W = 30\,000$.

For many experiments described in the literature, the values of W are much smaller, and thus one would not expect any turbulent fluid flow to be observed. For example, in the original experiment of Gordon, Leite, Moore, Porto, and Whinnery[11] the parameter W takes on a value about 10^{-3}, and in the more recent experiments of Smith and Gebhardt,[12] W is of order 10.

The parameter W introduced here is different from the Grasshof number, which is referred to in some discussions of the convective flows set up by the absorption of energy from a laser beam.[13] In fact, the conceptual basis for using the Grasshof number in a discussion attempting to explain the transition between smooth flow and time-dependent flow seems less relevant because

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the Grasshof number appears to be more sensible when the thermal bouyant forces are balanced by viscous forces. In the present discussions, the thermal bouyant forces are balanced by inertial effects. It turns out that the number W is essentially the square root of the Grasshof number.[14]

We are planning experiments to determine the critical value of W, W_{cr}, which determines the onset of turbulent convective flows for the geometry appropriate to laser beam transmission. It is also our aim to attempt a theoretical evaluation of this critical value. At the present time we can only speculate that W_{cr} may be between 10^3 and 10^4.[15] The theoretical approach appears fairly difficult because the question of the stability of flows even without heat sources has only been answered theoretically for very simple geometries.[16,17] The question of stability for fluids which are heated or cooled appears to have been treated mainly for cases in which the fluid would be motionless, and has not been explored for a problem like the present one.[17,18] The first part of that problem would be to determine a steady-state flow pattern for a fluid with a distributed heat source within a right circular cylinder with its axis aligned at some angle to the vertical. For the case of a horizontal cylinder of infinitely small radius, the flow pattern has been calculated by Yih.[19,20]

Unfortunately, however, that solution is not of great value for the present problem because the size of the cylinder radius is a critical parameter. Nevertheless, it is expected that Yih's solution will assist in obtaining the asymptotic form of the steady-state flow at large distances from the laser beam cylinder. Once that time-independent flow pattern has been determined, the linearization of the hydrodynamic equations for perturbations from the flow pattern will lead to an eigenvalue problem, which eventually will yield a critical value for W. Ostrach[21] suggests that the eigenvalue problem can be bypassed as the stability of fully developed natural convection flows can be found by using the appropriate velocity profile in the classical theory of hydrodynamic stability. This assertion rests upon his analysis of the stability of free convection above a flat heated plate, where instability first appears for a Reynolds number of 283.

Above the threshold for beam-induced turbulence, governed by W_{cr}, general arguments[22] lead to a size for the smallest eddies, $a(W_{cr}/W)^{3/4}$. For a 1-m beam, if W_{cr} should be about 10^3, then the eddies might have sizes as small as 7 cm for a power flux of 1 kW/cm^2. The associated density fluctuation would then be expected to result in considerably increased scattering of the beam.

The arguments presented here show that there are substantially more important sources of instability in the laser-fluid system than those discussed in earlier linearized analyses. It is felt that these fluid instabilities will be enhanced by their interaction with the scattering of the laser beam, because of the general result that instabilities in fluids result if the heating of the fluid is greater in those regions where the density of the fluid is greater.[23]

At the present time we can only outline the general nature of the effects to be expected above a critical power level. Much additional work clearly needs to be done, both of an experimental and theoretical nature.

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[1] J. D. Reichert, W. G. Wagner, and W. Y. Chen, following paper, J. Appl. Phys. **44**, 3647 (1973).

[2] K. A. Brueckner and S. Jorna, Phys. Rev. **164**, 182 (1967).

[3] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Addison-Wesley, New York, 1959).

[4] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Addison-Wesley, New York, 1960).

[5] Sir G. C. Stokes, Philos. Mag. **1**, 305 (1851).

[6] Instead of adding this term explicitly, the same effect can be obtained by considering the permittivity, ϵ to be complex and frequency dependent. The present procedure is used to avoid the logical difficulty of putting frequency dependence in the space-time wave equation.

[7] Heavyside-Lorentz units are used for electrical quantities throughout this paper.

[8] Many of the computer calculations have not yet appeared in the journals, but a number of groups of workers have developed sophisticated computer codes and have interesting results. Among these groups are: P. V. Avizonis and C. B. Hogge at Air Force Weapons Lab, Kirtland Air Force Base, Albuquerque, New Mexico; J. N. Hayes, A. H. Aitken, and P. B. Ulrich, Naval Research Laboratory, Washington, D. C.; J. Hermann and and L. C. Bradley, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Mass.; and J. Wallace and M. Camac, Avco Everett Research Lab, Everett, Mass. Other codes are being developed by J. Marburger at USC, J. Fleck at Livermore, K. Gustafson at Berkeley, J. Armstrong at IBM Yorktown Heights, and R. Shimizu at IBM Zürich.

[9] C. Eckart, Phys. Rev. **73**, 68 (1948).

[10] *Handbook of Chemistry and Physics* (Chemical Rubber, Cleveland, Ohio, 1969).

[11] J. P. Gordon, R. C. C. Leite, R. S. Moore, S. P. S. Porto, and J. R. Whinnery, J. Appl. Phys. **36**, 3 (1965).

[12] David C. Smith and Frederick G. Gebhardt, Appl. Phys. Lett. **16**, 275 (1970).

[13] S. A. Akhmanov, D. P. Krindach, A. V. Migulin, A. P. Sukhorukov, and R. V. Khokhlov, IEEE J. Quantum Electron. **4**, 568 (1968).

[14] W. G. Wagner and J. H. Marburger, Opt. Commun. 3, 19 (1971).

[15] A. J. Ede, in *Advances in Heat Transfer*, edited by J. P. Hartnett and T. F. Irvine Jr. (Academic, New York, 1967), Vol. 4.

[16] C. C. Lin, *The Theory of Hydrodynamic Instability* (Cambridge U. P., Cambridge, England, 1955).

[17] J. T. Stuart, in *Laminar Boundary Layers*, edited by L. Rosenhead (Oxford U. P., Oxford, England, 1963).

[18] H. K. Cheng (private communication).

[19] C. S. Yih, in *Symposium on the Use of Models in Geophysical Fluid Mechanics* (U. S. GPO, Washington, D. C., 1956).

[20] C. S. Yih, *Fluid Mechanics* (McGraw-Hill, New York, 1969), pp. 412–416.

[21] Simon Ostrach, in *Theory of Laminar Flows*, edited by F. K. Moore (Princeton U. P., Princeton, New Jersey, 1964), p. 691.

[22] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, London, 1959), p. 122.

[23] O. K. Mawardi, *Aero-thermoacoustics*, Reprints on Progress in Physics, Vol. 19, p. 180, edited by E. C. Strickland (Physical Society, London, England, 1956).

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