How the External Knowledge Stock Affects the Co-authorship Formation?

Mateus Lee・Ming-Hung Weng・Show-Ling Jang・Koichi Miyazaki*

With a strategic game, the current study explores the factors relevant to author collaboration when they face the expanding external knowledge stock. In addition to better coordination, we found authors with more complementary research efforts more likely to form co-authorship. Moreover, as the expanding external knowledge stock accumulates, only the authors with positive combined absorptive capabilities to assimilate it will exhibit a rising co-authorship propensity.

Keywords: Tolerable coordination costs, co-authorship propensity, absorptive capacity, external knowledge stock

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* The authors are Assistant Professor of Department of Diplomacy and International Relations at Tamkang University, Assistant Professor of Department of Economics at National Cheng Kung University, Adjunct Professor, Department of Economics at National Taiwan University and Professor of School of Economics at Hiroshima University, Correspondence: Ming-Hung Weng, email: mhweng@mail.ncku.edu.tw. We appreciate the comments and suggestions provided by two anonymous referees, Jong-Rong Chen, Ming-Fang Tsai and Teyu Chou. We are also grateful for the support provided by the Ministry of Science and Technology of Taiwan (research grant number: MOST 101-2410-H-002-013-MY2). All remaining errors are our own.

1 Introduction

Collaborative research has become increasingly common within academia. Several explanations have been proposed and examined for the prevalence of collaborative research, including increased gains from specialization and the division of labor (McDowell and Melvin, 1983; Barnett, Ault and Kaserman, 1988; Jones, 2009), escalating uncertainty in the editorial review process (Barnett, Ault and Kaserman, 1988), greatly reduced communication costs (Hudson, 1996), technology advancement (Teodoridis, 2018) and improved productivity through collaboration (Laband and Tollison, 2000). While the external knowledge stock has been expanding along this tendency, its possible impact on co-authorship formation remains elusive.

Broadening knowledge may help foster fresh ideas for the researchers by allowing them to "stand on the shoulders of giants." A bulk of empirical literature has documented a positive relationship between co-authorship activities and external knowledge stock (McDowell and Melvin, 1983; Barnett, Ault, and Kaserman, 1988; Wuchty et al., 2007). Nevertheless, the expanding external knowledge stock may also present the "fishing-out effect" (Jones, 2009) to the researchers by increasing the effort they need to learn and digest the new information, squeezing their time for research, or incentivizing them to further focalize on specialized fields. Team working, as proposed by Jones (2009), is likely the cure the researchers rely on to cope with such a "*burden of knowledge*," causing an increasing co-authorship propensity along the expanding external knowledge stock.

While studying the factors that motivate collaborative research, Egbetokun and Savin (2014) implicitly assume firms have positive absorptive capacities of the external knowledge stock. As a result, the higher the latter, the greater the firms' potential benefit and incentive from collaborating with other firms. Laband and Tollison (2000) and Wuchty et al. (2007) also documented similar connections. However, little literature addresses the possible knowledge burden (Jones, 2009) or the potential negative relation between co-authorship and the researchers' productivity except for Jackson and Wolinsky (1996), Hollis (2001), and Ductor (2015).

On the other hand, in contrast to extensive investigations for ascending propensity in collaborative research, little effort has been placed into exploring its coexistence with single authorships or the latter's persistence. As shown in Figure 1, while the percentage of co-authorship papers has constantly been rising in both the Economics and the Physics field, that in the former is significantly lower than the latter. After investigating nearly 20 million academic papers published between 1995 to 2000, Wuchty et al. (2007) even confirm the decreasing propensity of co-authorship in some disciplines, including Veterinary Sciences, Marine Engineering, Philosophy, Film, Radio and TV, Dance, Poetry, and American Literature. A similar finding appears in the work of Henriksen (2016), who analyzes about 4.5 million publications across 56 fields under the Web of Science between 1980-2013. She found that while areas such as Women's Studies, Demography, and Psychoanalysis Psychology have witnessed a trend of collaboration that is first increasing and then decreasing, single authorship has persisted in being the primary form of publication in the areas of History, Cultural Studies, Area Studies, and History of Social Sciences. Specifically, within the field of Economics, Nowell and Grijalva (2011) have identified a relatively low percentage of co-authorship in areas such as General Economics, Economic Thought, Economic History, Quantitative Analysis, Macroeconomics, International Economics, Economics Development, and Economic Systems.

<Figure 1 is inserted about here>

To better understand how the expanding external knowledge will shape co-authorship formation and the coexistence of single and co-authorship, we examine the possible interactions of authors with a strategic game. Moreover, to highlight this potential fishing-out effect while keeping better track of the influences from the expanding external knowledge stock, our framework sets out to permit the expanding external knowledge stock to possibly bring either positive or negative impacts to the researchers to depict the consequences for their choices more generally.

Our significant contributions are as follows. First, we demonstrate that single and co-authorship will likely coexist in equilibrium. As the authors engage in the interaction like the coordination game, some may adhere to single authorship as it appears as the focal point to the researchers in specific fields. Second, based on the authors' constant elasticity of substitution (CES) production function under co-authorship, we find that authors with more complementary research efforts are more likely to form co-authorship. Third, by examining how the expanding external knowledge stock may impact co-authorship formation, we identify the crucial role of the combined absorptive capacity in assimilating it. To address the impacts of growing external knowledge stock on co-authorship formation, we first derive two novel measures to quantify the authors' incentive and propensity to collaborate. The first measure, the *tolerable coordination costs,* helps assess the authors' incentive to collaborate given their absorptive capacities, the volume of the external knowledge stock, and the stochastic coordination costs they incur. The appraisal of such incentive is a critical factor in the authors' decisions on whether to engage in co-authorships since only those authors with tolerable coordination costs that are higher than their actual coordination costs are likely to collaborate. We then apply authors' tolerable coordination costs to derive the *co-authorship propensity* (*COP*), the second measure to evaluate how likely the authors will form a co-authorship.

Furthermore, based on the link between the coordination costs and the volume of the external knowledge stock, we can further examine the potential impact that the latter has on the authors' incentives to engage in collaborations. Specifically, by decomposing its impact into *individual* and *collaborative effects,* our framework helps to understand better how the expanding external knowledge stock may cause diverse transitional patterns in the tolerable coordination costs and the co-authorship propensity for authors with distinct absorptive capacities.

In pace with the growing external knowledge, we found co-authorships more likely to emerge only if the benefit from collaboration can outweigh the possible knowledge burden the individual author experiences, i.e., when the authors have positive *combined absorptive capacities* under co-authorship. This is in line with the findings of McDowell and Melvin (1983) and Barnett, Ault, and Kaserman (1988).[2] In contrast, probabilities of forming co-authorship are eventually declining among other authors with negative combined absorptive capacity amid the increasing external knowledge stock. These contrasting scenarios identified in our model may shed some light on why, in an era of knowledge profusion, both single and co-authorships can coexist within academia since varying impacts on the research output under co-authorship are likely to be observed alongside the increasing external knowledge stock. Moreover, the critical role of the combined absorptive capacities signifies the importance of both individual and additional absorptive, instead of anyone alone, in guiding the evolvement of co-authorship amid the expanding external knowledge stock.

The remainder of this paper is organized as follows. The model and its equilibrium,

[2] As noted by McDowell and Melvin (1983) and Barnett, Ault, and Kaserman (1988), collaborations are more likely to be formed if there is a greater likelihood of the productivity levels of the authors being increased through such co-authorships.

as well as the derivation of an author's tolerable coordination costs and the co-authorship propensity, are introduced in Section 2. In Section 3, we present two comparative static analyses to investigate further the impacts of the varying external knowledge stock on the authors' incentives to collaborate and the co-authorship propensity. Finally, the conclusions and discussion on potential future research are offered in Section 4.

2 Model

We construct a multi-stage game to model the interactions between two authors who can use different formats to produce knowledge output, such as published articles, conference papers, technical reports, research manuscripts, and patents. Let $N = \{1,2\}$ be the set of authors. We assume they will first determine whether to collaborate simultaneously and separately before choosing their corresponding levels of research effort in the second stage. We describe these potential formats of knowledge production below.

2.1 Knowledge Production by the Authors

When working under single authorship, we assume that $\forall i \in N$, Author i's knowledge output level, $z_{s_i}(e_{s_i})$, is generated by:

$$z_{s_i}(e_{s_i}) = e_{s_i}K^{r_i}, \tag{1}$$

where $e_{s_i} \geq 0$ denotes the level of research effort comprised of the amount of time, brainpower, and other resources exerted by Author i when working individually.[3] The variable $K > 0$ refers to the external knowledge stock to which the researchers are exposed, including that in the same research field as Author i, or indeed, in a completely different field. The individual absorptive capacity, r_i, represents Author i's ability to assimilate the external knowledge stock and turn it into the transformed knowledge base, K^{r_i}, when working under single authorship (Cohen and Levinthal, 1989; Egbetokun and Savin, 2014; Savin and Egbetokun, 2016).

Several features in our model set it apart from the existing literature. First, to better

[3] The number of published articles is a common index used to evaluate academic performance within the existing literature, such as McDowell and Melvin (1983), Jackson and Wolinsky (1996) and Goel and Rich (2005). Furthermore, as reported in Goel and Rich (2005), 2% of the patents granted in 1998 were assigned to academia or academic institutions.

reflect the complexity, cumulativeness, and uncertainty of the external knowledge stock, we assume the transformed knowledge base takes an exponential form instead of a linear one as in the prior studies (Cohen and Levinthal, 1989; Egbetokun and Savin, 2014; Savin and Egbetokun, 2016). Second, as the accumulation in K may bring not only a source of which the researchers can take advantage but also obstacles and challenges, as suggested by Jones (2009), we assume $r_i \geq 0$ for authors being exposed to this "standing on the shoulders of giants" effect or $r_i < 0$ when they work in a field exhibiting a "fishing-out" effect. Moreover, it is noteworthy that r_i is related to the marginal productivity of Author i's research efforts. Hence, its variation also features potential differences between authors in terms of productivity. Thus, in contrast to a positive and limited absorptive capacity ($0 < r_i < 1$) in the prior studies, our model is more capable of reflecting the potential realities where an author's efficiency in assimilating the external knowledge stock may either rise or fall as K expands. This setting may help to capture the fishing-out effect on knowledge production (Jones, 2009). At last, we assume Author i's knowledge output under single authorship to be the product of e_{s_i} and K^{r_i}. As knowledge production's effectiveness depends on the author's research effort and the amount of transformed knowledge, this assumption is a reasonable step to highlight the essential roles played by both elements in the knowledge production process as shown in Equation (1).

Alternatively, as opposed to working independently, Author i could opt to work cooperatively with Author $j \in N$ and $j \neq i$. We assume each of them, under co-authorship, would share equally their joint knowledge output characterized by the constant elasticity of substitution (CES) production function. Specifically, Author i's knowledge output under co-authorship, $z_{c_i}(e_{c_i}, e_{c_j})$, when collaborating with Author $j \neq i$, is generated as follows:

$$z_{c_i}(e_{c_i}, e_{c_j}) = \frac{1}{2}\left[\left(e_{c_i} K^{r_i+b}\right)^{\rho} + \left(e_{c_j} K^{r_j+b}\right)^{\rho}\right]^{\frac{1}{\rho}}, \quad (2)$$

where $e_{c_i}, e_{c_j} \geq 0$ denote Author i and j's research effort under co-authorship. In addition to r_i and r_j, the individual absorptive capacity of each author, b, refers to the *'additional absorptive capacity'* as the extra impact on efficiency in absorbing, assimilating, and exploring the external knowledge stock due to co-authorship. Although the value of b may depend on shared knowledge, research interests, skill complementarity, or proximity between authors (Jones, 2009; Egbetokun and Savin, 2014), Jackson and Wolinsky (1996) highlighted the synergy effect as the benefit of co-authorship. This claim was supported by the empirical finding of Ductor (2015), who showed that co-authorship led to higher academic output. However, Jackson and Wolinsky (1996) also pointed out possible impaired productivity due to co-authorship, echoed by Hollis (2001), who found reduced publications in economics papers when researchers engaged in co-authorship. To summarize, a researcher who collaborates with a partner may face either a rise or fall in his ability to transform external knowledge stock into valuable knowledge output compared to working individually. Therefore, the additional absorptive capacity can be either positive to highlight the benefits the co-authorship adds to the knowledge production or negative when it, in turn, creates more obstacles.

In addition, the parameter ρ indicates the degree of complementarity between the authors' efforts in the CES knowledge production function, where smaller ρ refers to more complementary efforts[4]. It should be addressed that the increase in the number of collaborating firms in the CES production function will only lower their combined output level for $\rho \in (-\infty, 0)$ (Adam 2006). Moreover, although ρ is usually interpreted as the complementarity of efforts between distinct academic fields, its variation may be related to the age difference between collaborating authors. By modeling the CES complementarity parameter, $\rho \in (0,1]$, as the function of the age difference between collaborating authors, Krapf (2015) found their complementarity to be maximized at an age difference of 10 years. As the current work explores the co-authorship formation, we consider only the range of the parameter, $\rho \in (0,1]$, where collaboration brings increased output to the researchers.

In summary, our model is distinctive in considering the joint knowledge output arising from the cooperative agreement between the authors in contrast to the prior studies that were based on the authors' separate knowledge outputs.[5] Assuming a CES production function also separates us from the related literature whose analysis was based on the linear addition of knowledge output, as shown by Cohen and Levinthal (1989) because it ensures a more flexible and general examination of how the expanding external knowledge may impact the co-authorship propensity.

[4] The function earns its name as its elasticity of substitution between inputs (effort levels from different authors in the current study), $1/(1-\rho)$, is a constant.

[5] See for example, Hollis (2001), Egbetokun and Savin (2014), Ductor (2015), and Savin and Egbetokun (2016).

2.2 Payoffs for the Two Authors

Let $p_i \in \{0,1\}$ $\forall i \in N$ represent the decision taken by Author *i* regarding the format of authorship, where $p_i = 1$ when Author *i* chooses to collaborate with Author $j \in N$ and $j \neq i$; otherwise, 0. As any unilateral attempt to collaborate with the other author may not necessarily lead to a successful co-authorship, the formation of a co-authorship by the two authors is defined as follows:

Definition 1: *A co-authorship will be established if and only if each author agrees to collaborate, such that* $p_i = 1$ $\forall i \in N$*; otherwise, each maintains single authorship.*

To formally examine their collaborative choices, we assume that Author *i* tries to maximize the payoff function as:

$$\pi_i(p_i, p_j, e_i, e_j) = \left(1 - p_i p_j\right) e_{s_i} K^{r_i} + p_i p_j \cdot \frac{1}{2}\left[\left(e_{c_i} K^{r_i+b}\right)^{\rho} + (e_{c_j} K^{r_j+b})^{\rho}\right]^{\frac{1}{\rho}}$$

$$-\tfrac{1}{2}\left[\left(1 - p_i p_j\right) e_{s_i} + p_i p_j e_{c_i}\right]^2 - p_i c_i \,. \qquad (3)$$

The first term in Equation (3), $\left(1 - p_i p_j\right) e_{s_i} K^{r_i}$, is the knowledge output of Author *i* under single authorship (i.e., $p_i = 0$ or $p_j = 0$), while the second term, $p_i p_j \cdot 1/2\left[\left(e_{c_i} K^{r_i+b}\right)^{\rho} + (e_{c_i} K^{r_j+b})^{\rho}\right]^{1/\rho}$, is that under a successful co-authorship ($p_i = 1$ $\forall i \in N$).

Since effort is required to produce knowledge output, each author must bear costs relating to the physical and mental resources exhausted during the research process. We assume that these costs, similar to most convex production costs, will be quadratic in the magnitude of the author's total effort[6], as represented by the third component in Equation (3), $1/2\left[\left(1 - p_i p_j\right) e_{s_i} + p_i p_j e_{c_i}\right]^2$. Regardless of whether the effort is exerted under single or co-authorship, we assume the personalized unit price of an author's effort is a constant of 1/2.

Furthermore, when an author decides to collaborate with the other author, (s)he will inevitably incur additional "coordination costs." Referring to Lee (2014), we assume $c_i \geq 0$ $\forall i \in N$ to represent these exogenous, randomized, and individualized

[6] It is straightforward to reason that the efforts will only be exerted for collaboration once both authors agree to collaborate or for single authorship otherwise. Hence the analysis is simplified by omitting the choices of effort on single or co-authorship after possible four combinations of 1st-stage decisions.

coordination costs faced by Author i when reaching out to the other author, regardless of whether such an attempt is successful or not. These costs are comprised of: (i) communication costs; (ii) the opportunity costs related to the time and effort spent in building a shared understanding of a particular research issue; and (iii) the opportunity costs related to any delay in, or termination of, the research project.[7] Specifically, we assume c_i to be drawn from the probability distribution, F_i, over $[0, \underline{d}_i]$, where $\underline{d}_i > 0$ refers to the maximum value of the actual coordination costs. Although these coordination costs are likely random and fluctuating across distinct fields, the information regarding their values could be easily acquired during the internet era. Hence, we assume them to be common knowledge when authors are contemplating whether to collaborate.

Moreover, it should be noted that the authors will also encounter the possible free-rider problem found in Adam (2006) and McGinty (2014). Since each author only gets a fraction of their combined knowledge output, (s)he tends to work less diligently compared to the socially efficient level. We will address this issue in the subsequent subsection.

To summarize, we investigate the interaction between two authors with a multi-stage two-player game as described below:

Let $N = \{1,2\}$ be the set of authors. At the beginning of the day, the actual coordination cost of Author $i \in N$ is drawn and becomes common knowledge to both authors before they contemplate whether to collaborate.

In stage 1, the authors simultaneously and separately make their choices regarding how to conduct the scientific research. Author $i, \forall i \in N$, can choose "to collaborate" ($p_i = 1$) with the other author $j \in N$ and $j \neq i$, by unilaterally making a co-author propose while incurring the coordination costs or "not to collaborate" ($p_i = 0$).

In stage 2, both choices of whether to collaborate have been revealed. If a decision 'not to collaborate' has been made by any author in the previous stage, no co-authorship is formed. Author $i, \forall i \in N$, will then simultaneously and independently choose $e_{si} \geq 0$, the effort level under single authorship, by maximizing the payoff as:

$$\pi_{si}(e_{si}) = e_{si}K^{r_i} - \frac{1}{2}{e_{si}}^2 - p_i c_i. \tag{4}$$

Alternatively, suppose a decision 'to collaborate' has been made by both authors

[7] See Jackson and Wolinsky (1996), Jackson and Watts (2002), Goyal and Vega-Redondo (2005) and Cuijpers, Guenter, and Hussinger (2011).

in the previous stage so that a co-authorship is formed. In that case, they will simultaneously and independently choose their level of effort, $e_{ci} \geq 0$, by maximizing their own payoff as:

$$\pi_{ci}(e_i, e_j) = \frac{1}{2}\left[\left(e_{c_i} K^{r_i+b}\right)^{\rho} + (e_{c_j} K^{r_j+b})^{\rho}\right]^{\frac{1}{\rho}} - \frac{1}{2} e_{ci}{}^2 - c_i,\ j \neq i. \qquad (5)$$

Finally, the authors obtain their respective payoff based on whether a co-authorship is formed and their efforts under the corresponding authorship.

2.3 Sub-game Perfect Equilibrium and Tolerable Coordination Costs

The equilibrium concept in our model adopts the 'Sub-game Perfect Equilibrium' (SPE), which is similar to the work by Lee (2014). By solving the game backward, the equilibrium effort under single authorship in the second stage[8] of Author $i, \forall i \in N$, can be derived by maximizing the payoff function in Equation (4). Alternatively, if a co-authorship is established, the equilibrium efforts of the two authors will be the Nash Equilibrium that solves the simultaneous first-order conditions maximizing the respective individual payoffs in Equation (5). Given *K*, Author *i*'s research efforts exerted under single authorship (e_s^*) and co-authorship (e_c^*) are respectively:

$$e_{si}^* = K^{r_i},\ \ \forall i \in N, \qquad (6)$$

and

$$e_{ci}^* = \frac{1}{2} K^{b+\frac{\rho r_i}{2-\rho}} [K^{\frac{2\rho r_i}{2-\rho}} + K^{\frac{2\rho r_j}{2-\rho}}]^{\frac{1-\rho}{\rho}},\ \forall i, j \in N, j \neq i. \qquad (7)$$

Rolling back to stage 1, each author must simultaneously choose whether to collaborate with the other based upon the above decision rules. Given the optimal level of effort, Author *i's* maximized payoff under single authorship is:

$$\pi_{si}^* = \frac{K^{2r_i}}{2} - p_i c_i\ . \qquad (8)$$

Note that in Equation (8), Author *i* still must pay for the coordination costs in the case where (s)he reaches out for collaborating with Author $j \neq i$ who turns down the offer.

Meanwhile, Author *i*'s maximized payoff under co-authorship is:

$$\pi_{ci}^* = \frac{1}{8} K^{2b} \left[K^{\frac{2\rho r_i}{2-\rho}} + K^{\frac{2\rho r_j}{2-\rho}}\right]^{\frac{2-2\rho}{\rho}} \left(K^{\frac{2\rho r_i}{2-\rho}} + 2K^{\frac{2\rho r_j}{2-\rho}}\right) - c_i, \qquad (9)$$

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[8] It's noteworthy that single authorship will be outcome of the sub-games where either of the two authors chooses not to collaborate in the first stage.

after both authors agree to collaborate in the first stage. As the authors engage in a stag hunt type coordination game, both will likely choose to collaborate, and a co-authorship will be successfully built only if

$$\pi_{ci}^* - \pi_{si}^* > 0, \forall i \in N. \tag{10}$$

Substituting Equations (8) and (9) into Equation (10), we obtain:

$$\left[\frac{1}{8}K^{2b}\left[K^{\frac{2\rho r_i}{2-\rho}} + K^{\frac{2\rho r_j}{2-\rho}}\right]^{\frac{2-2\rho}{\rho}}\left(K^{\frac{2\rho r_i}{2-\rho}} + 2K^{\frac{2\rho r_j}{2-\rho}}\right) - c_i\right] - \frac{K^{2r_i}}{2} > 0. \tag{11}$$

By rearranging Equation (11), we can derive the threshold value of the actual coordination costs, below which Author *i* will yield a higher payoff under co-authorship as below:

$$c_i < \underline{c_i} \equiv \frac{1}{8}K^{2b}\left[K^{\frac{2\rho r_i}{2-\rho}} + K^{\frac{2\rho r_j}{2-\rho}}\right]^{\frac{2-2\rho}{\rho}}\left(K^{\frac{2\rho r_i}{2-\rho}} + 2K^{\frac{2\rho r_j}{2-\rho}}\right) - \frac{K^{2r_i}}{2}. \tag{12}$$

Denoted as $\underline{c_i}$ in Equation (12), we refer to this threshold value as the '*tolerable coordination cost*' as it represents the maximal level of the actual coordination costs Author *i* is willing to bear when seeing a higher knowledge output from co-authorship than from single authorship. On the other hand, as $\underline{c_i}$ is the difference of payoffs between co- and single authorship, it can also be regarded as the quantitative measure of the author's incentive to engage in collaboration in equilibrium as compared to its counterpart.

Accordingly, Author *i's* best response on whether to engage in co-authorship can be summarized by the following equation:

$$p_i^*(p_j) = \begin{cases} 1, & if\ p_j = 1\ and\ c_i \le \underline{c_i} \\ 0, & otherwise \end{cases}, \text{for } j \neq i, \tag{13}$$

where $\underline{c_i} = \frac{1}{8}K^{2b}\left[K^{\frac{2\rho r_i}{2-\rho}} + K^{\frac{2\rho r_j}{2-\rho}}\right]^{\frac{2-2\rho}{\rho}}\left(K^{\frac{2\rho r_i}{2-\rho}} + 2K^{\frac{2\rho r_j}{2-\rho}}\right) - \frac{K^{2r_i}}{2}$.

Let $Z = \{[x_i^*, (y_{ci}^*, y_{si}^*)], \cdots\}, i \in N$ be the SPE profile of the game where x_i^* is Author *i's* first-stage equilibrium choice and y_{ci}^* is Author *i's* optimal effort level in the subgame under successful co-authorship. For simplicity, let y_{si}^* represent Author *i's* optimal effort level in all the subgames under single authorship. Based on the above, we can establish the SPE of the interaction between the two authors as follows [9]:

[9] For simplicity, our attention here is restricted to pure strategy SPE.

Proposition 1. *Given the level of the external knowledge stock (K), the authors' individual absorptive capacity* (r_i , r_j), $i, j \in N$ and $j \neq i$, *and the 'additional absorptive capacity'* (b),

$$[x_i^*, (y_{ci}^*, y_{si}^*)] = [0, (e_{ci}^*, e_{si}^*)]\ \forall\, i \in N, \tag{14-1}$$

is the SPE for all values of c_i*; in addition, when* $c_i \leq \underline{c_i}\ \forall\, i \in N$*, there is another SPE such that*

$$[x_i^*, (y_{ci}^*, y_{si}^*)] = [1, (e_{ci}^*, e_{si}^*)]\ \forall\, i \in N, \tag{14-2}$$

where $e_{ci}^* = \frac{1}{2} K^{b+\frac{\rho r_i}{2-\rho}} [K^{\frac{2\rho r_i}{2-\rho}} + K^{\frac{2\rho r_j}{2-\rho}}]^{\frac{1-\rho}{\rho}}$, $\quad e_{si}^* = K^{r_i}$,

and $\underline{c_i} \equiv \frac{1}{8} K^{2b} \left[K^{\frac{2\rho r_i}{2-\rho}} + K^{\frac{2\rho r_j}{2-\rho}}\right]^{\frac{2-2\rho}{\rho}} \left(K^{\frac{2\rho r_i}{2-\rho}} + 2K^{\frac{2\rho r_j}{2-\rho}}\right) - \frac{K^{2r_i}}{2}$.

Proposition 1 specifies the conditions under which a co-authorship will likely prevail as the outcome in the sub-game perfect equilibrium. We find that in equilibrium, both authors will choose to collaborate, and thus a co-authorship will be established only if their actual coordination costs are both lower than their respective tolerable coordination costs (that is, $c_i \leq \underline{c_i}\ \forall\, i \in N$). In words, only when the benefits from co-authorship outweigh costs for each of the two authors will they likely agree to collaborate. Therefore, these tolerable coordination costs are critical for establishing co-authorship and can also serve as a quantitative measure to evaluate each author's incentive to engage in co-authorship. More importantly, as $\underline{c_i}$ depends on the level of the existing stock of knowledge, the authors' 'individual' and the 'additional absorptive capacity' of the external knowledge stock, Proposition 1 provides a theoretical foundation for us to examine further their impacts on the authors' incentive to collaborate.

Moreover, Proposition 1 also confirms the possibility that single authorship may remain the author's choice even when co-authorship appears more attractive than its counterpart (that is, $c_i \leq \underline{c_i}\ \forall\, i \in N$). In this case, the authors are involved in the stag hunt game where either both choosing to collaborate or both choosing not to collaborate may be the equilibrium outcome in the first stage. In other words, the prevalence of either co- or single authorship depends on how the authors coordinate. This finding may explain the coexistence of the two formats of authorship or the persistence of single authorship in specific fields amid the prevalence of co-authorship across most fields.

For instance, the lower percentage of co-authored papers in Economics than in the Physics field as shown in Figure 1 is possibly because researchers in Economics or social sciences are less used to team working. Lack of faith in others' intention to co-author may be one of the reasons deterring the knowledge from being produced efficiently. In a broader perspective, the distinctive history and feature of each academic field in embracing collaboration may also contribute to the variation of co-authorship tendency across disciplines.

2.4 Social Planner's Problem

The free-rider problem that possibly arises when players engage in team works has been confirmed by Adams (2006) and McGinty (2014). They found team members likely exerting an inefficient effort level at the Nash equilibrium. In this sub-section, we will show that a similar free-rider problem exists in our co-authorship model when collaboration emerges as the equilibrium. To do so, we compare the authors' optimal effort levels exerted under co-authorship, e_{ci}^*, to those under the social planner's decision-making.

Consider the social planner who aims to maximize the overall payoff of the two researchers under co-authorship as below:

$$\text{Max}_{\{e_{ci}, e_{cj}\}} \; [(e_{ci}K^{r_i+b})^{\rho} + (e_{cj}K^{r_j+b})^{\rho}]^{\frac{1}{\rho}} - \sum_i \frac{1}{2} e_{ci}^2 - \sum_i c_i, \; i,j \in N \text{ and } j \neq i. \quad (15)$$

Solving simultaneously the first-order conditions that must be satisfied when the social planner chooses e_{ci} and e_{cj} respectively to maximize the overall payoff, one can derive the Pareto efficient effort level under co-authorship as

$$e_{ci}^{PE} \equiv K^{b+\frac{\rho r_i}{2-\rho}} \left[K^{\frac{2\rho r_i}{2-\rho}} + K^{\frac{2\rho r_j}{2-\rho}} \right]^{\frac{1-\rho}{\rho}}, \forall i,j \in N, \text{and } j \neq i. \quad (16)$$

Comparing the above Pareto efficient effort level with e_{ci}^*, Author i's optimal effort level under co-authorship in Equation (7), we get

$$e_{ci}^* = \frac{1}{2} e_{ci}^{PE}. \quad (17)$$

The above result confirms an inefficient equilibrium effort level under co-authorship since each author will exert only half of the socially optimal one. Its intuition is straightforward: the authors are locked into the public good dilemma. First, as each author cannot enjoy the joint knowledge output alone, the latter is non-excludable. Second, as each author is guaranteed a fixed portion of the joint knowledge output, it is non-rival. Therefore, on the one hand, Author i can simply enjoy the contribution from the collaborating partner, constituting a free-rider problem in co-authorship. On the other hand, as each author can obtain half of the social payoff[10] that depends on the contribution of both authors, Author i's incentive to exert effort will hence be halved as compared to the social planner's choice. Thus, the current framework replicates the similar inefficiency that rises from research collaboration as described by Adams (2006) and McGinty (2014).[11]

2.5 Co-authorship propensity

Combining Proposition 1 and Definition 1, the measure, '*co-authorship propensity*' (*COP*), is constructed to quantify the likelihood that a co-authorship will be successfully established at any given level of the external knowledge stock. Since a co-authorship will be formed only when both authors' actual coordination costs are smaller than their tolerable coordination costs, *COP* can be expressed as:

$$COP \equiv Pr(p_i^* = 1, \forall i \in N) = Pr\big(c_i \leq \underline{c_i}, \forall i \in N\big)\,. \tag{18}$$

Though the formation of collaboration has been extensively examined, unfortunately, existing literature (Jackson and Wolinsky, 1996; Egbetokun and Savin, 2014; Savin and Egbetokun, 2016) fails to provide a quantitative measure to assess the probability of its formation among either authors or firms. Equation (18) would not only help evaluate the chances of co-authorship formation but also provide a theoretical foundation to discuss how this propensity will be affected as discussed in the next section.

3 Comparative Statics

We are living in an era where external knowledge is growing at an unprecedented rate. Whether it brings the "standing on the giant's shoulders" effect or the "fishing out"

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[10] It should be addressed that the public good dilemma still exists even when each author obtains the full size of joint research output. As each author only obtains half of the total social payoff, the incentive to exert effort will thus be halved.

[11] In addition, the social planner not only prefers more effort from the authors when they voluntarily choose to co-author, but it may also prefer their collaboration to no collaboration when the authors choose to work individually for c_i slightly higher than $\underline{c_i}$ for at least one of them. This can be reasoned in the following way. When $c_i = \underline{c_i}, \forall i \in N$, both authors are indifferent between co- and single authorship while the social planner strictly prefers co-authorship with the socially optimal effort levels. Then by the continuity of payoffs, co-authorship with the socially optimal effort levels will still be preferred by the social planner to single authorship when c_i is slightly above $\underline{c_i}$ for at least one author. In other words, it is still socially inefficient for the authors to choose not to collaborate.

effect becomes a critical issue for the researchers whose output relies heavily on external knowledge. As the quantitative measure for evaluating the author's incentive, $\underline{c}_i$, is dependent on K, its closed form in Equation (12) would allow us to investigate how the level of the external knowledge stock, as well as other factors, may impact the co-authorship formation and the *co-authorship propensity* (*COP*).

As the authors in our model are symmetric, we begin the comparative static analysis with the following assumption to better depict the impact brought about by the accumulation of *K*.

Assumption 3.1. $r_i = r,\ \forall i \in N$.

We assume that the authors have identical individual absorptive capacities. Although this simplification may limit our understanding of the full picture, it is still a reasonable consideration when our primary concern is on the average impact that the growing external knowledge stock may bring to the authors' intentions to engage in collaboration.

3.1 The Impacts of the Expanding External Knowledge Stock on an Author's Tolerable Coordination Cost

Under the assumption of identical r, the author's research efforts under single and co-authorship in Proposition 1 become:

$$e_{Si}^* = e_S^* \equiv K^r, \forall i \in N \tag{19}$$

and

$$e_{Ci}^* = e_C^* \equiv \frac{1}{4} \cdot 2^{\frac{1}{\rho}} K^{r+b}, \forall i \in N\ . \tag{20}$$

In equilibrium, the corresponding payoffs under single and co-authorship are as follows:

$$\pi_{si}^* = \pi_s^* \equiv \frac{K^{2r}}{2} \tag{21}$$

and

$$\pi_{ci}^* = \pi_c^* \equiv \frac{3}{32} \cdot 2^{\frac{2}{\rho}} K^{2(r+b)} - c_i\ , \forall i \in N. \tag{22}$$

Accordingly, the author's tolerable coordination cost, $\underline{c}$, can be derived as:

$$\underline{c}_i = \underline{c} \equiv \frac{K^{2r}}{2} (\frac{3}{16} \cdot 2^{\frac{2}{\rho}} \cdot K^{2b} - 1), \forall i \in N. \tag{23}$$

As the author's tolerable coordination cost depends on K, r, ρ, and b, the ‘additional absorptive capacity’ was confirmed as one of the determinants for co-authorship formation. Because it represents the potential influence that collaboration brings to the knowledge production process, the assumption of a positive b will ensure collaboration is beneficial as K accumulates. Since the subsequent analyses examine how the expanding external knowledge stock and other factors impact the co-authorship formation, the following assumption thus appears necessary.

Assumption 3.2. $b > 0$.

Nevertheless, it should be addressed that a positive additional absorptive capacity does not guarantee the prevalence of co-authorship over its counterpart. From Equation (23), it can be found that the sign of the tolerable coordination cost will depend on the specific value of the external knowledge stock. We summarize the relevant findings in Lemma 1 below:

Lemma 1. *A threshold value of the external knowledge stock ($\underline{K}$) exists in determining the sign of an author's tolerable coordination costs ($\underline{c}$) with*

(i) $\underline{c} < 0$ *if* $K < \underline{K}$

(ii) $\underline{c} = 0$ *if* $K = \underline{K}$*; and*

(iii) $\underline{c} > 0$ *if* $K > \underline{K}$*;*

where $\underline{K} = \left(\frac{16}{3} \cdot 2^{\frac{-2}{\rho}}\right)^{\frac{1}{2b}}$ *is increasing in* ρ.

Specifically, $\underline{K}$ is the critical value of the external knowledge stock below which we have $\underline{c} < 0$. Thus, a co-authorship is never feasible as the authors face some positive actual coordination costs. Alternatively, when $K \geq \underline{K}$ the authors may obtain a greater knowledge output under co-authorship than under single authorship and collaborations thus are likely to be formed. In other words, a certain level of external knowledge stock is required to foster co-authorship.

Moreover, this essential level of the external knowledge stock may depend on the complementarity among authors as $\underline{K}$ is increasing in ρ. For authors with a smaller ρ in their joint production function, their research efforts are more complementary. Their joint knowledge output would also be higher than that of other authors, thanks to a more significant synergy effect. Accordingly, $\underline{K}$ will be lower for these authors, making their collaboration more commonly observed. This finding provides another explanation for why the percentage of co-authored papers in the Physics field is much higher than that of Economics in Figure 1. Because the research efforts among authors in natural sciences like the Physics field are more complementary than that in social sciences like Economics, a relatively lower threshold of the external knowledge stock is required to foster collaboration, thus making co-authorship more commonly seen in the Physics field.

To further understand how the tolerable coordination costs would vary with the expansion of the external knowledge stock, we differentiate $\underline{c}$ with respect to K and rearrange the results as the following:

$$\frac{d\underline{c}}{dK} = \frac{d\pi_C^*}{dK} - \frac{d\pi_S^*}{dK} = K^{2r-1}\left[\underbrace{(-1)\cdot r}_{individual\ effect} + \underbrace{\left(\frac{3}{16}\cdot 2^{\frac{2}{\rho}}\cdot K^{2b}\right)\cdot (r+b)}_{collaborative\ effect}\right]. \quad (24)$$

Equation (24) clearly describes the two potential channels, the 'individual effect' (IE) and the 'collaborative effect' (CE), through which the increase in the external knowledge stock will impact the tolerable coordination costs. In short, the 'net effect' (NE) on the evolvement of $\underline{c}$ relies not only on the authors' absorptive capacity under single and co-authorship, i.e., r and $r+b$, but also on their corresponding coefficients, which are -1 and $\frac{3}{16}\cdot 2^{\frac{2}{\rho}}\cdot K^{2b}$ respectively. The net effect of the expanding external knowledge stock on the tolerable coordination costs is ambiguous since it depends on the contest between IE and CE. Firstly, the coefficients of IE and CE are of opposite signs, suggesting their possibly diverse influences along the expansion of the external knowledge stock. Secondly, as r can be either positive or negative, its value or sign could also generate disparate impacts on $\underline{c}$. Although the net effect is ambiguous, the fact that the coefficient of CE is increasing in K suggests the importance of CE is escalating with the rise of the external knowledge stock.[12]

Proposition 2 summarizes how an author's incentive to engage in co-authorship (that is, the author's tolerable coordination costs, $\underline{c}$) is affected by the volume of the external knowledge stock (K), given the individual and the additional absorptive capacity.

Proposition 2. *With the continuing expansion in the external knowledge stock* (K)*, the tolerable coordination costs* ($\underline{c}$) *of an author will be*

(i) a U-shaped function in K if $r \geq 0$*;*

(ii) a monotonically increasing function in K if $-b \leq r < 0$*;*

[12] Note that $\frac{3}{16}\cdot 2^{\frac{2}{\rho}}\cdot K^{2b} \geq (<) 1$ if $K \geq (<) \underline{K}$.

(iii) an inverted U-shaped function in K before eventually converging to 0 if $r < -b$.
Proof: See Appendix A1.

Figures 2a, 2b, and 2c illustrate these distinct transition patterns of an author's tolerable coordination costs alongside the expansion of the external knowledge stock under three different cases of *r*, as identified in Proposition 2.

<Figure 2a is inserted about here>

<Figure 2b is inserted about here>

<Figure 2c is inserted about here>

Irrespective of the value of an author's absorptive capacity under single- or co-authorship, two common features regarding how the tolerable coordination costs evolve with the external knowledge stock are noteworthy. Firstly, an author's $\underline{c}$ will be negative for very low levels of the external knowledge stock, as can be verified in Equation (23). It suggests an unfavorable position for collaboration, particularly when K is low, so sparse benefit can be derived from co-authorship. Secondly, there exists a threshold value of the external knowledge stock, $\underline{K}$, above which the coefficient of CE in Equation (24) exceeds that of IE in absolute value, implying CE's strengthening impact. It should be noted that $\underline{K}$ also specifies the critical boundary below which an author's tolerable coordination costs remain negative, as stated in Lemma 1.

Intuition derived from these three distinct scenarios in Proposition 2 can be further discussed. Firstly, for authors endowed with a positive individual absorptive ability to assimilate external knowledge under single authorship ($r \geq 0$), as shown in Figure 2a, the increase in K provides a constant benefit for single authorship, and at the same time a negative IE, $(-1) \cdot r$, that weakens their incentive to collaborate with others. On the other hand, because the relative importance of CE, i.e., $\frac{3}{16} \cdot 2^{\frac{2}{\rho}} \cdot K^{2b}$, grows with *K,* CE will be insignificant at extremely low levels of *K*. As a positive but relatively constrained CE will be overshadowed by the negative IE, the ensuing negative NE explains the decreasing $\underline{c}$ in Figure 2a when the external knowledge stock is low. However, as the importance of CE enlarges along the expansion of *K*, the positive CE will grow more significantly and eventually outweigh the impact from the negative IE, resulting in a turnaround in $\underline{c}$ at K_1. Afterward, the authors will find co-authorship more and more lucrative as we see $\underline{c}$ steadily increasing with the expanding K in Figure 2a for $K > K_1$. In words, these authors who benefit from the increasing external knowledge stock while working individually may first find co-authorship less attractive. Nevertheless, as the advantage of collaboration builds up along the accumulating external knowledge stock, co-authorship becomes more and more advantageous and eventually emerges as an inevitable choice of collaboration when engaging in research.

Alternatively, for authors with lower individual absorptive capacity, $-b \leq r < 0$ (i.e., $r < 0$ and $r + b \geq 0$), Figure 2b illustrates their tolerable coordination costs to increase monotonically in the external knowledge stock despite being negative below $\underline{K}$. On the one hand, $r < 0$ suggests that these authors will face more difficulties to work individually as K increases. This in turn favors the collaboration, i.e., IE effect is positive. On the other hand, as the benefit from co-authorship is big enough to outweigh the potential fishing-out effect (i.e., $r + b \geq 0$), CE always remains positive for these authors. Combining the above together, we can therefore always expect a positive NE, which explains why $\underline{c}$ is always rising with the increase in K. It implies that the incentive to collaborate for the authors with a negative but relatively constrained individual absorptive capacity ($-b \leq r < 0$) will be primarily governed by the strengthening 'comparative advantage' of collaboration, as illustrated in Figure 2b, where $\underline{c}$ is strictly increasing in K.

At last, for authors with the lowest individual absorptive capacity, i.e., $r < -b$, they suffer more from the increasing knowledge stock while working individually. Although the CE becomes negative under a negative combined absorptive capacity, its relative importance is insignificant when the external knowledge stock is low. Hence, the disadvantage of single authorship strengthens their preference toward co-authorship as the external knowledge builds up. Therefore, Figure 2c demonstrates their tolerable coordination costs to be first increasing along the expanding external knowledge stock despite being negative for $K < \underline{K}$. Nevertheless, as K further increases, the impact from the negative CE, i.e., $\frac{3}{16} \cdot 2^{\frac{2}{\rho}} \cdot K^{2b}$, continues to grow and eventually becomes the dominating force for $K \geq K_1$. In other words, K_1 is the threshold value at which NE switches from positive to negative and after which we witness the decreasing tolerable coordination costs. Moreover, it should be addressed that, as the productivity under single authorship has been exhausted early along the external knowledge stock, the worsening productivity of collaboration is now the major drive for the decreasing incentive for co-authorship. Therefore, with the steady expansion in K, $\underline{c}$ converges to zero in the end, not to the negative territory [13] for $r < -b$ and exhibits an inverted U-shaped function in K as shown in Figure 2c.

Although most research fields have witnessed an increasing percentage of co-authored papers as demonstrated by Proposition 2(i) and 2(ii), Wuchty et al. (2007) have documented the decrease of co-authorship in disciplines such as Veterinary Sciences, Marine Engineering, Philosophy, Film, Radio and TV, Dance, Poetry, and American Literature. A similar finding appeared in the work of Henriksen (2016), who recorded a trend of collaboration that is first increasing and then decreasing in Women's Studies, Demography, and Psychoanalysis Psychology. From the perspective of Proposition 2 (iii), the researchers in these fields may have received the most adverse impact from the expanding external knowledge stock due to their negative combined absorptive capacity. As a result, they rely more heavily on working alone, causing a lower percentage of co-authored papers.

In summary, Proposition 2 has signified the complicated role of the absorptive capacities in governing the impact of expanding external knowledge stock on researchers' incentives to collaborate. It is the sum of the individual and the additional absorptive capacities, i.e., the synergy of collaboration, that is the most influential on how eventually the tolerable coordination costs will vary with *K,* instead of either one alone. On the other hand, although co-authorship has been identified by Jones (2009) as a way researchers use to cope with knowledge burden, the possible mechanism is not well-examined yet by previous studies. As portrayed in Proposition 2, when growing knowledge is a burden as when $r < 0$, co-authorship can serve as a shelter to the researchers only if the combined absorptive capacity is positive. Otherwise, when the benefit from collaboration cannot outweigh the fishing-out effect, they will still prefer to work individually.

Moreover, as the evolvement of tolerable coordination costs does not necessarily transform into the variation in co-authorship propensity, the next subsection will further examine how the latter will be affected by the expansion of external knowledge stock.

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[13] In Figure 2a and Figure 2c, K_2 indicates the level of external knowledge stock where $\underline{c}$ switches its concavity. See Appendix A2 for details.

3.2 The Impacts of the Expanding External Knowledge Stock on the Co-authorship Propensity

In this sub-section, we go on to discuss the potential impacts of the expanding external knowledge stock on the likelihood of co-authorship formation, that is, the *co-authorship propensity (COP)*. We assume that the actual coordination cost of each author, c_i, is independently and identically drawn from the same distribution $F(c)$ with $c \in [0, \underline{d}]$, where $\underline{d} > 0$. Consequently, the establishment of a co-authorship will also be a stochastic process. According to Equation (18), the *COP* in equilibrium can be expressed by:

$$COP = Pr\left(c_i \leq \underline{c}, \forall i \in N\right) = \prod_{i \in N} Pr(c_i \leq \underline{c}) = \left[F(\underline{c})\right]^2. \tag{25}$$

The impact of the expanding external knowledge stock on *COP* can be assessed by taking the derivative of *COP* with respect to *K*:

$$\frac{dCOP}{dK} = 2F(\underline{c}) \cdot f(\underline{c}) \cdot \frac{d\underline{c}}{dK}, \tag{26}$$

where $f(\underline{c})$ is the density function of the coordination costs evaluated at $\underline{c}$.

Since probability functions must be non-negative, we have $F(\underline{c}) \geq 0$ and $f(\underline{c}) \geq 0$. Accordingly, Equation (26) suggests a close connection between $dCOP/dK$ and $d\underline{c}/dK$, such that the way the external knowledge stock influences *COP* will be primarily driven by the way in which it impacts on the author's tolerable coordination costs:

$$sign\left(\frac{dCOP}{dK}\right) = \begin{cases} sign\left(\frac{d\underline{c}}{dK}\right), & \underline{c} \geq 0 \\ 0, & \underline{c} < 0 \end{cases}. \tag{27}$$

Therefore, we can further verify how the changing external knowledge stock will impact the *COP* based upon our earlier findings regarding $d\underline{c}/dK$ in Proposition 2. The results are summarized below:

Proposition 3. *The co-authorship propensity* (*COP*) *between the two authors will be* 0 *when* $K < \underline{K}$ *where* $\underline{K} = \left(\frac{16}{3} \cdot 2^{\frac{-2}{\rho}}\right)^{\frac{1}{2b}}$ *and it becomes:*

(*i*) *an increasing function in K when* $K \geq \underline{K}$, *if* $r + b \geq 0$ *for both of the authors;*

(*ii*) *an inverted U-shaped function in K before converging to* 0 *as K goes to infinity when* $K \geq \underline{K}$, *if* $r + b < 0$ *for both authors.*

Proof: See Appendix A2.

Proposition 3 first points out the critical role played by $\underline{K}$, below which COP will be zero, regardless of the level of individual and additional absorptive capacity. It restates that some external knowledge stock is an essential requirement for boosting the co-authorship formation. As aforementioned, this threshold depends on how complementary the research efforts are among the authors. A lower $\underline{K}$ and thus a more favorable environment to foster collaboration will be expected for researchers with more complementary efforts as in Physics. However, regardless of the value of ρ, given the positive actual coordination costs ($c_i \geq 0\ \forall i \in N$) for the authors, neither of them will have an incentive to engage in collaboration, and thus, $COP = 0$ whenever $K < \underline{K}$, as depicted in both Figure 3a and Figure 3b.

After the external knowledge stock surpasses $\underline{K}$, the authors' tolerable coordination costs become positive, and so does COP. Although this may be based on our assumption of $b > 0$ so that increasing external knowledge favors collaboration, nevertheless, how COP will evolve with K will completely pivot on the sign of the combined absorptive capacity, which will be further discussed below:

For authors with $r + b \geq 0$, i.e. $r \geq -b$, whether they have a positive or negative individual absorptive capacity, Proposition 2 finds their tolerable coordination costs to be both increasing in the external knowledge stock when it is beyond $\underline{K}$, causing COP to be monotonically increasing in K as shown in Figure 3a. Eventually, COP will approach 1 even though both authors' $\underline{c}$ continues to rise with the expansion in K. This finding is consistent with most observations in practice witnessing a positive relationship between the percentage of co-authored papers and external knowledge stock (McDowell and Melvin, 1983; Barnett, Ault, and Kaserman, 1988; Wuchty et al., 2007). It may also provide an explanation for the increasing COP in both Physics and Economics fields as shown in Figure 1.

On the other hand, for those authors with $r + b < 0$, i.e., $r < -b$, their $\underline{c}$ is an inverted U-shaped function in K for $K \geq \underline{K}$ and converging to 0 in the end as stated in Proposition 2(iii). Accordingly, COP will be also an inverted U-shaped function in K for these authors. As shown in Figure 3b, COP will be initially increasing in K for $K < K_1$, and then becomes decreasing in K afterward before converging to 0. This finding may explain why Henriksen (2016) has documented a trend of collaboration that is first increasing and then decreasing in areas such as Women's Studies, Demography, and Psychoanalysis Psychology amid the accumulating external knowledge stock, while other areas have experienced a tendency of increasing collaboration. Although the positive additional absorptive capacity may help authors under co-authorship to assimilate the external knowledge as it grows, that benefit eventually will be overshadowed by their negative individual absorptive capacity. Co-authorship thus will be dampened when collaboration cannot overcome the knowledge burden in these fields. Amid the escalating challenging brought by the accumulating external knowledge stock, our contribution thus is to identify that it is the combined absorptive capacity, instead of individual or additional absorptive capacity alone, to cause a decreasing percentage of co-authored papers in these fields.

<Figure 3a is inserted about here>

<Figure 3b is inserted about here>

In summary, Proposition 3 describes how the co-authorship propensity may evolve along the expanding external knowledge stock. In addition to identifying a threshold level of the external knowledge stock necessary to foster co-authorship, it demonstrates *COP*'s possible diverse patterns. The results not only echo Jones' (2009) claim that teamwork would be more common in areas facing a more significant knowledge burden but also demonstrate another possibility that is seldom addressed. As the external knowledge stock accumulates, a diminishing percentage of co-authored papers will likely appear if the authors' potential benefit from the collaboration is overwhelmed by the knowledge burden.

Specifically, Proposition 3 has signified the critical role of the authors' combined absorptive capacities, $r + b$, for these distinct transition patterns, instead of either one alone. As it suggests, *COP* eventually will decrease in the external knowledge stock if the authors' individual and the additional absorptive capacities sum up to be negative. This finding helps explain why some existing literature has documented a decreasing percentage of co-authored papers or an inverted U-shaped pattern in selected fields as the external knowledge stock expands (Wuchty et al., 2007; Nowell and Grijalva, 2011; Henriksen, 2016).

4 Conclusions

The present study is motivated by the general observation of the coexistence of single and co-authorships and the persistence of the former amid the increasing prevalence of the latter in many academic fields. Although the rising co-authorship is primarily attributed to the “stand on the shoulders of giants” effect, the continuing expansion in the external knowledge stock is often accompanied by the “knowledge burden” (Jones, 2009) that is seldom addressed. We consider both positive and negative absorptive capacities that individual researchers may have in assimilating the external knowledge stock under a two-person game to explore their roles in the collaboration choices of the authors amid the expanding external knowledge stock.

Our study contributes to the literature as follows:

Firstly, our model considers the constant elasticity of substitution (CES) production function in representing the authors’ joint research output instead of a linear form commonly used in the related literature. Because many research collaborations are made of heterogeneous members whose efforts may be either complementary or substitutive, the model with CES production function would be more capable of addressing the difference in their productivities and incentives to collaborate.

Secondly, our framework helps explain the coexistence of single and co-authorship, which is rarely addressed in the existing literature. We attribute the scenario partly to the consequence of the coordination game the authors play when choosing whether to collaborate with their colleagues. As they may both choose to collaborate or both choose not to collaborate in equilibrium, the discrepancy in co-authorship tendency across academic fields may be related to the researchers’ focal point choices based on the distinctive features and history.

Thirdly, based on the CES joint production function, our framework confirms that the authors with more complementary efforts are more likely to form partnerships. This intuitional connection between the authors’ co-authorship tendency and their research effort complementarity offers another justification for the empirical observation that the co-authorship percentage in natural sciences like Physics is usually higher than that in social sciences like Economics.

At last, we focus on how the increase in the external knowledge stock will influence the authors’ tendencies to collaborate by decomposing its impacts into individual and collaborative effects. As proposed by McDowell and Melvin (1983) and Barnett, Ault, and Kaserman (1988), collaborations are more likely to be formed if the productivity of the authors is enhanced by the co-authorships, which is valid under the “on the shoulders of giants” effect. However, because the growing external knowledge could also bring a knowledge burden or the fishing-out effect (Jones, 2009) to the researchers, our contribution is to confirm that even under such circumstances, it may still promote collaboration when the authors have a positive combined absorptive capacity under co-authorship. Alternatively, the expanding external knowledge may discourage co-authorship formation when the fishing-out effect dominates the potential benefit from collaboration, i.e., under a negative combined absorptive capacity, causing the coexistence of single and co- authorships in academia or the persistence of the former. In summary, either the individual absorptive capacity or the additional absorptive capacity under co-authorship alone cannot independently shape the evolvement of co-authorship as the external knowledge stock expands. It is the combined absorptive capacity, the sum of them, that eventually determines how the expanding external knowledge stock will impact the co-authorship propensity.

Our analyses in the present study may be further extended in the following ways. The first is to verify our findings empirically to better integrate theory with practice. The second would be extending the current model to a multiple-player game, which may enable us to draw more general policy implications. As most government-supported projects aim to promote R&D collaboration among many parties, a more general framework would be appropriate for examining these public projects' efficacy. The third is to consider the practical scenario where the authors can choose to write single- or co-authored papers simultaneously. Last but not least would be to relax our assumption of identical researchers to help explain the gap in co-authorship propensity across disparate academic fields and, more importantly, how they evolve along the expanding external knowledge stock.

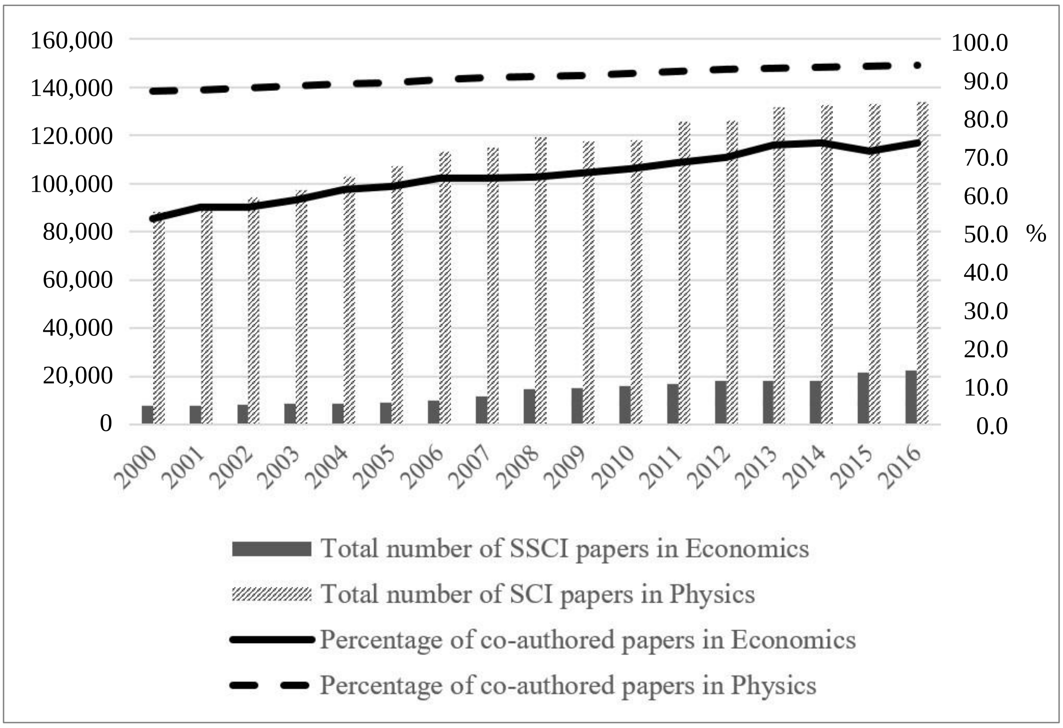

Figure 1: Total number of SSCI/SCI papers and the percentage of co-authored papers in Economics and Physics, 2000-2016

Source: Web of Science

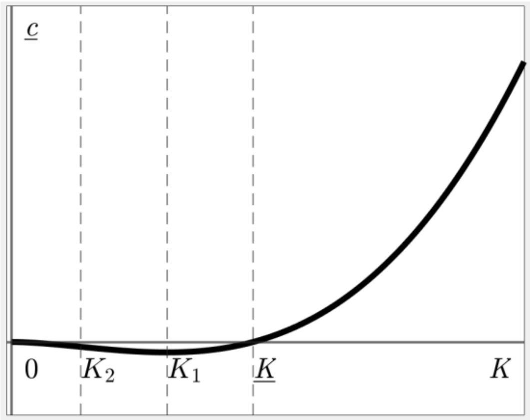

Fig.2a: $\underline{c}$ is a U-shaped function in *K* for $r \geq 0$

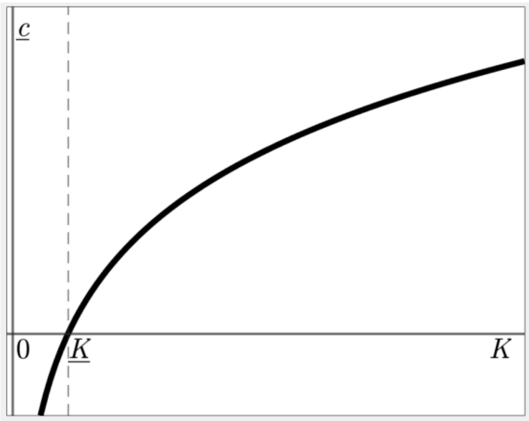

Fig.2b: $\underline{c}$ is an increasing function *K* for $-b \leq r < 0$

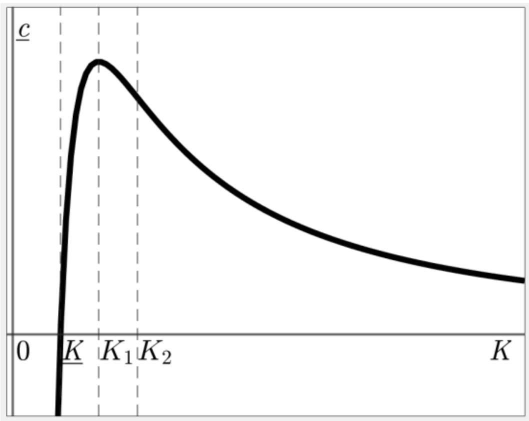

Fig.2c: $\underline{c}$ is an inverted U-shaped function in *K* for $r < -b$

Figure 2: Impacts of increasing external knowledge stock (*K*) on an author's tolerable coordination costs ($\underline{c}$) under the three scenarios of the authors' absorptive capacity

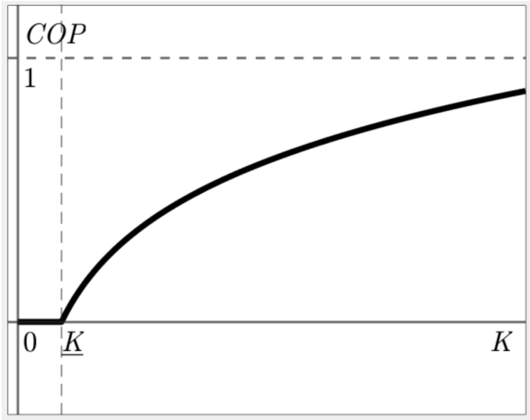

Fig.3a: *COP* is an increasing function in *K* for $r + b \geq 0$

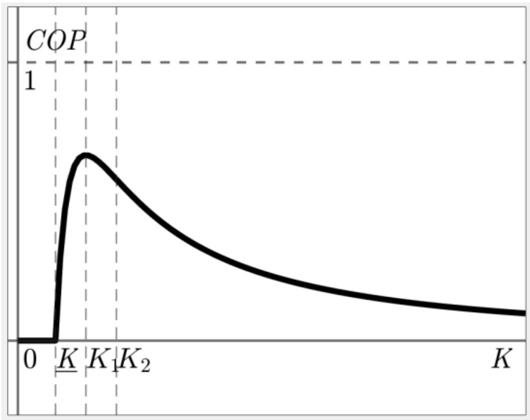

Fig.3b: *COP* is an inverted U-shaped function in *K* for $r + b < 0$

Figure 3: Impacts of the increasing external knowledge stock (*K*) on the co-authorship propensity (*COP*) given different authors' individual absorptive capacity (*r*) and additional absorptive capacity (*b*)

Appendix

A1. Proof of Proposition 2:

Based on $\frac{d\underline{c}}{dK} = K^{2r-1}\left[-r + \left(\frac{3}{16}\cdot 2^{\frac{2}{\rho}}\cdot K^{2b}\right)(r+b)\right]$ in Equation (24), one can further derive the following:

$$\frac{d^2\underline{c}}{dK^2} = 2K^{2r-2}\left[-r\left(r-\frac{1}{2}\right) + \left(\frac{3}{16}\cdot 2^{\frac{2}{\rho}}\cdot K^{2b}\right)(r+b)\left(r+b-\frac{1}{2}\right)\right]. \quad \text{(A1)}$$

Combining these derivatives, we discuss below how $\underline{c}$ would evolve with K for r in the following cases separately.

- Case 1: $r \geq 0$.

  In this case, we have $r + b \geq r \geq 0$. According to Equation (24), we obtain that

$$\frac{d\underline{c}}{dK}\begin{cases} \geq 0, & if\ K \geq K_1 \\ < 0, & otherwise \end{cases}, \quad \text{(A2)}$$

  where $\underline{K} = \left(\frac{16}{3}\cdot 2^{\frac{-2}{\rho}}\right)^{\frac{1}{2b}}$, $K_1 = \left[\left(\frac{r}{r+b}\right)\right]^{\frac{1}{2b}}\underline{K}$, and $\underline{K} \geq K_1$.

  Consequently, for $r \geq 0$, $\underline{c}$ is initially decreasing in K when $K < K_1$ and will turn increasing in K for $K \geq K_1$. In other words, K_1 is the level of external knowledge stock where $\underline{c}$ reaches its minimum.

  On the other hand, according to Equation (A1), $\frac{d^2\underline{c}}{dK^2}$ depends on $(r - 1/2)$ and $(r + b - 1/2)$. Therefore, the concavity of $\underline{c}$ depends on the level of K and which of the following subcases $r + b$ belongs to:

  (i) when $r \geq 1/2$ so that $r + b \geq r \geq 1/2$, we have $\frac{d^2\underline{c}}{dK^2} \geq 0$ if $K \geq K_2$, and $\frac{d^2\underline{c}}{dK^2} < 0$, otherwise, where $K_2 = \left[\left(\frac{r}{r+b}\right)\left(\frac{r-\frac{1}{2}}{r+b-\frac{1}{2}}\right)\right]^{\frac{1}{2b}}\underline{K}$ and $K_2 < K_1 < \underline{K}$.

  (ii) when $r < 1/2$ and $r + b \geq 1/2$, we have $\frac{d^2\underline{c}}{dK^2} \geq 0$ for all possible K.

  (iii) when $r < 1/2$ and $r + b < 1/2$, we have $\frac{d^2\underline{c}}{dK^2} \geq 0$ if $K \leq K_2$, and $\frac{d^2\underline{c}}{dK^2} < 0$, otherwise.

- Case 2: $-b \le r < 0$.

  In this case, the facts that $r < 0$ and $r + b \ge 0$ collectively ensure $\frac{d\underline{c}}{dK} \ge 0$ for all K by Equation (24) so that $\underline{c}$ is monotonically increasing in K. Moreover, from Equation (A1), if $r + b \le 1/2$ we have $\frac{d^2\underline{c}}{dK^2} \le 0$ for all K so that $\underline{c}$ is a concave function through all range of K. In contrast, if $r + b > 1/2$, we have $\frac{d^2\underline{c}}{dK^2} \ge 0$ only for $K \ge K_2$ and $\frac{d^2\underline{c}}{dK^2} < 0$ otherwise. In words, $\underline{c}$ is initially concave in K, but turns into convex for $K > K_2$.

- Case 3: $r < -b$.

  Based on $r < r + b < 0$, the impacts of K on $\underline{c}$ in this case can be summarized as follows:

$$\frac{d\underline{c}}{dK}\begin{cases}\ge 0, & if\ K \le K_1 \\ < 0, & otherwise\end{cases}, \tag{A3}$$

$$\frac{d^2\underline{c}}{dK^2}\begin{cases}\ge 0, & if\ K \ge K_2 \\ < 0, & otherwise\end{cases}, \tag{A4}$$

  where $\underline{K} < K_1 < K_2$. Therefore, in this case, $\underline{c}$ is initially increasing in K before it reaches its maximum at K_1 and becomes decreasing afterward.

Moreover, given $\underline{c} \equiv \frac{K^{2r}}{2}(\frac{3}{16} \cdot 2^{\frac{2}{\rho}} \cdot K^{2b} - 1)$ from Equation (23) it can be shown that $\lim_{K\to\infty} \underline{c} = 0$ when $r < 0$ and $r + b < 0$.

In conclusion, $\underline{c}$ is initially increasing in K, reaching its peak at K_1, and then becoming decreasing before approaching 0 as K goes to infinity. In addition, it is first concave in K before $K_2 > K_1$, and turns into convex afterward. ■

A2. Proof of Proposition 3:

According to Equation (23), an author's equilibrium tolerable coordination cost, $\underline{c}$, will be positive only for $K \geq \underline{K} = \left(\frac{16}{3} \cdot 2^{\frac{-2}{\rho}}\right)^{\frac{1}{2b}}$ regardless of the value of the individual absorptive capacity. Since an author's actual coordination cost is positive ($c_i \geq 0 \ \forall i \in N$), it is clear that $Pr(c_i \leq \underline{c}) = 0$ for any external knowledge stock below $\underline{K}$. Combining the above fact with Equation (24), we have the following:

$$COP = Pr\left(c_i \leq \underline{c}, \forall i \in N\right) = \prod_{i \in N} Pr(c_i \leq \underline{c}) = 0. \quad \text{(A5)}$$

In words, no co-authorship will be successfully established when the external knowledge stock is relatively low.

As for $K \geq \underline{K}$ where the tolerable coordination cost of the author becomes positive, the way *COP* evolves along the external knowledge stock depends on how $\underline{c}$ changes with $\underline{K}$ as shown in Equation (26). Based on the ways $\underline{c}$ evolves amid increasing *K* in Proposition 2*,* we examine below the transition of *COP* for $K \geq \underline{K}$ under two possible scenarios: $r \geq -b$ and $r < -b$.

- Scenario 1: $r \geq -b$ (or $r + b \geq 0$ after rearranging terms)

  According to Proposition 2(i) and (ii), $\underline{c}$ is positive and increasing in *K* for $K \geq \underline{K}$. Thus, *COP* is increasing in *K* for $K \geq \underline{K}$.

  In summary, *COP* will initially be 0 for $K < \underline{K}$ and becomes increasing in *K* for $K \geq \underline{K}$ before eventually converging to the value of 1, as illustrated in Figure 3a.

- Scenario 2: $r < -b$ (or $r + b < 0$ after rearranging terms)

  According to Proposition 2(iii), $\underline{c}$ is initially increasing in $K \geq \underline{K}$ before reaching its maximum at $K = K_1$ and then becomes decreasing in *K* until it eventually converges to 0. Consequently, as *K* expands beyond $\underline{K}$, *COP* is initially increasing in K for $K < K_1$, and then becomes decreasing in *K* afterward before converging to 0.

  In summary, *COP* is initially 0 for $K < \underline{K}$ and becomes an inverted U-shaped function in *K*. It eventually converges to the value of 0, as depicted in Figure 3b. ■

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外部知識存量對合著形成之影響

李文基
淡江大學外交與國際關係學系

翁明宏
國立成功大學經濟學系

鄭秀玲
國立台灣大學經濟學系

宮崎浩一
廣島大學經濟學院

本文以賽局模型分析在外部知識存量膨脹過程中影響合著形成之因素。除了研究者間較好的協調外，彼此間較高之互補程度，亦將導致較多之合著結果。此外，隨著外部知識存量累積，唯有對其聯合吸收能力為正向之研究人員，才會提高其合著之傾向。

關鍵詞: 可容忍合作成本, 合著傾向, 吸收能力, 外部知識存量

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