Abstract of thesis entitled

"SOME ASPECTS OF FINITE DIMENSIONAL CONES"

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This thesis consists of four chapters. They deal with several aspects of general cones and matrix cones in euclidean spaces.

In chapter 1, we consider the polarity operator $\wp_K: \mathcal{F}(K) \to \mathcal{F}(K^*)$ of a full cone K. The injectivity and the surjectivity of $\wp_K$ are examined in details. It appears that in the literature the polarity operator is studied only for a polyhedral cone but not for a general cone.

In chapter 2, we first prove that a cone is polyhedral if and only if it has a finite number of maximal faces. From this we deduce several properties of polyhedral cones, among which is a theorem of Weyl. Then we review the notion 'decomposable cone'. Its corresponding notion for general convex sets is also given. After this, we introduce the new notion 'subcone'. It will be shown that an n-dimensional indecomposable cone with more than 2n-2 extreme rays must contain a proper indecomposable subcone of the same dimension. Finally we give an equivalent condition for a self-dual cone to be decomposable.

Chapter 3 is devoted to a study of $\pi(K)$, the cone of all real matrices which leave a full cone K invariant. We characterize its dual cone, give some results related to its polarity operator, and

study those faces of $\pi(K)$ of the form $\pi_{F,G} = \{A \in \pi(K) : AF \subset G\}$ $(F, G \lhd K)$, in particular its maximal faces. In terms of $\pi(K)$ we also give some equivalent conditions for K to be polyhedral, simplicial, self-dual or simplicial self-dual.

In chapter 4, we consider matrices cross-positive on a full cone K. The main result obtained is some new characterizations of matrices cross-positive on K in terms of the partial ordering induced by K. We shall give a counter-example which answers two open questions posed by Schneider and Vidyasagar in 1969. The chapter concludes with an example of a non-polyhedral cone K for which $\pi_1(K) = \Sigma(K)$.