Frequency-Domain Recursive Hybrid GA to the Identification of a Real Building

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***Abstract* – In the implementation of the recursive hybrid genetic algorithm in the time domain, numerical integration is essential for solving the differential equation. This procedure may result in a huge amount of computational effort since it is required to apply so many times as long as the evolutionary process is proceeded. To accelerate the identification process, a recursive hybrid GA in the frequency domain is developed. The time history of the measurement is divided into a series of time intervals, and then the model of equivalent linear system is employed to identify the modal parameters of the system for each time interval. The differential equation can be transformed into the frequency domain by Fourier transform and the response in the frequency domain can be solved by algebraic equations instead of differentials equations. The process of exploring this new algorithm is similar to that of recursive hybrid genetic algorithm in the time-domain, by using the simulated SDOF system and MDOF system considering noise contamination. Finally, this new strategy is also applied to the identification of areal building.**

1. INTRODUCTION

Although a great deal is known about where earthquakes are likely to occur, there is currently no reliable way to predict the time when an event will occur in any specific location. However, the damages caused by them can be greatly reduced with proper structural design using safer seismic code. In this regard, dynamic behavior of structures under earthquakes should be considered in the process of design. In order to realize the dynamic behavior of structural systems, we can determine the dynamic models and parameters by system identification techniques. However, collecting strong motion data is essential when performing the system identification analysis. Fortunately, the strong motion data recorded by accelerographs, which were installed under the Taiwan Strong-Motion Instrumented Program (TSMIP) since 1993, has accumulated to a remarkable amount. In addition to updating the structural parameters for better response prediction, system identification techniques made possible to monitor the current state or damage state of the structures. The parameters of the structures are identified and referred to the associated baseline states. The current state of a structure condition relative to a baseline state is compared and the degree of damage is determined.

The author [1] applied the real-coded GA to structural identification problems. The validity and the efficiency of the proposed GA strategy were explored for the cases of systems with simulated input/output measurements. Moreover, the strategy was also applied to the real structure. Genetic algorithms (GAs) are global search techniques for optimization. However, GAs are inherently slow, and are not good at hill-climbing. In order to accelerate the convergence to the optimal solutions, a hybrid GA identification strategy that employs Gauss-Newton method as the local search technique was also proposed and verified by the author [2].

The damage of the buildings induced by the Chi-Chi earthquake provides solid evidence that some of the structures have experienced inelastic response. Consequently, the parameters of some real buildings may vary with the change of the amplitude of vibration, so that the identified parameters may not reflect the real state of the structure if we only use a model of a linear system to represent the structure. In this regard, a new system identification method called recursive hybrid genetic algorithm, which can identify the change of parameters, was developed for the purpose of identifying the system parameters of the buildings with accelerographs installed. The time history of the measurement is divided into a series of time intervals, and then the model of equivalent linear system is employed to identify the modal parameters of the system and the initial displacement and velocity for each time interval.

To accelerate the identification process, a recursive hybrid GA in the frequency domain is developed. The time history of the measurement is divided into a series of time intervals, and then the model of equivalent linear system is employed to identify the modal parameters. The differential equation can be transformed into the frequency domain by Fourier transform and the response in the frequency domain can be solved by algebraic equations instead of differentials equations. The process of exploring this new algorithm is similar to that of recursive hybrid genetic algorithm in the time-domain, by using the simulated SDOF system and MDOF system considering noise contamination. Finally, this new strategy is also applied to the identification of t1real building.

2. Hybrid Genetic Algorithm

Genetic algorithm is a stochastic search technique based on natural selection and genetics, developed by Holland [3]. Genetic algorithms model natural processes, such as selection, recombination, mutation, migration, and competition. The algorithms work on populations of individuals instead of a single solution. In this way, the search is performed in a parallel manner. However, better results can be obtained by introducing multiple subpopulations. Every subpopulation evolves over a few generation isolated (like the single population GA) before one or more individuals are exchanged between subpopulation using the mechanisms of migration and competition. The multi-population GA models the evolution of a species in a way more similar to nature than single population

The GA is a parallel and global search technique that searches multiple points and makes no assumption about the search space. However, GAs are inherently slow and are poor at hill-climbing. In order to compensate the computational inefficiency in hill-climbing when the solution yielded by GA approaches the optimal value, a local search operator compatible to GA is merged to the GA strategy. The Gauss-Newton method is the local search operator used in this paper and is performed after completing the evolution process of every 10 generations. Accordingly, a new hybridization of a GA with Gauss-Newton method is formed. Figure 1 shows the structure for such a hybrid GA.

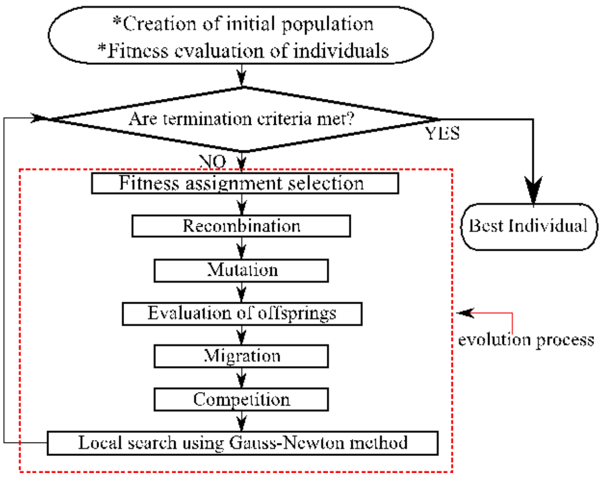

Fig. 1: Structure of hybrid GA

3. Frequency-domain Recursive Hybrid Genetic Algorithm

The new method developed here is called the Frequency-Domain recursive hybrid Genetic Algorithm. In the development of this new algorithm, the time history of the measurement is divided into a series of non-overlapping time intervals, and then the model of equivalent linear system is transformed into frequency domain to identify the modal parameters of the system and the difference of displacement and velocity for each time interval as well. The differential equation can be transformed into the frequency domain by Fourier transform and the response in the frequency domain can be solved by algebraic equations instead of differentials equations. The process of exploring this new algorithm is proceeded by using response of the simulated single degree of freedom (SDOF) system and multiple degree of freedom (MDOF) system considering the effect of noise contamination.

3.1. Identification model for SDOF system

The motion equation of a single degree of freedom linear system when excited by a uni-directional earthquake ground acceleration is

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = -\ddot{u}_g \qquad (1)$$

where ξ = damping ratio, ω = natural frequency, and $\ddot{u}_g$ = ground acceleration in one direction. The motion equation can be transformed into the frequency domain. The finite Fourier transform of relative response of u , $\dot{u}$, and $\ddot{u}$ over a finite record length of T or each non-overlapping time interval can be definded as

$$U^T(\varpi) = \int_0^T u e^{-i\varpi t} dt \tag{2}$$

$$\dot{U}^T(\varpi) = \int_0^T \dot{u} e^{-i\varpi t} dt = U(T) - U(0) + i\varpi U^T(\varpi) \tag{3}$$

$$\begin{aligned} \ddot{U}^T(\varpi) &= \int_0^T \ddot{u} e^{-i\varpi t} dt \\ &= -\dot{U}(T) - \dot{U}(0) + (U(T) - U(0)) i\varpi - \varpi^2 U^T(\varpi) \end{aligned} \tag{4}$$

Transforming equation (1) into frequency domain by using the terms defined in equation (2~4), yields

$$v + (2\xi\omega + i\omega)d + (\omega^2 - \varpi^2 + 2i\xi\omega\varpi)U(\varpi) = -\ddot{U}_g(\varpi) \tag{5}$$

where $U^T(\varpi)$, the Fourier transform of the relative displacement, is denoted as $U(\varpi)$ for simplicity. The parameters v and d are the differences of velocity and displacement between the beginning and the end of the record segment of duration *T*. The measured response in this paper is the relative acceleration and the associated Fourier transformed can be derived as

$$\begin{aligned} \ddot{U}(\varpi) &= -\frac{\varpi^2 \ddot{U}_g(\varpi) + d(\omega^2 i\varpi) + v(\omega^2 + 2\xi\omega i\varpi)}{\omega^2 - \varpi^2 + 2i\xi\omega\varpi} \\ &= H(\varpi)\ddot{U}_g(\varpi) + \frac{-d(\omega^2 i\varpi) - v(\omega^2 + 2\xi\omega i\varpi)}{\omega^2 - \varpi^2 + 2i\xi\omega\varpi} \end{aligned} \tag{6}$$

where $H(\varpi) =$ Transfer function

$$= -\frac{\varpi^2}{\omega^2 - \varpi^2 + 2i\xi\omega\varpi}$$

In order to optimize the system, error function in term of transformed response is defined in such a way that it is quadratic in terms of the parameters and is denoted as the error index, E.I., for the system

$$E.I. = \left[\frac{\sum_{i=1}^{N} \left\{ \left| \mathrm{Re}(Y_i) - \mathrm{Re}(V_i) \right|^2 + \left| \mathrm{Im}(Y_i) - \mathrm{Im}(V_i) \right|^2 \right\}}{\sum_{i=1}^{N} \left\{ \left| \mathrm{Re}(Y_i) \right|^2 + \left| \mathrm{Im}(Y_i) \right|^2 \right\}} \right]^{1/2}$$

h (7)

where Y_i and V_i are the Fourier transforms of the measured response and identified response for the *i*th sampled frequency. The Notations *Re* and *Im* represent the real and imaginary parts of the response.

In the new method, the excitation measurement and the response measurement are automatically divided into non-overlapping time intervals or time windows by setting the number of time instants or sampling points, *na*, in the beginning. In order to account for the effect of the difference of velocity and displacement between the beginning and the end of the each interval, *v* and *d* are also implemented as the parameters to be identified in addition to the system parameters $A_1 = 2\xi\omega$ and $A_3 = \omega^2$. Then the model of equivalent linear system in the frequency domain and the recursive hybrid GA is employed to identify the parameters of the system and the differences of displacement and velocity for each time interval. The response in frequency domain can be computed using equation (6).The best five individuals or system parameters are replicated and used as the individuals of the initial population of next interval. This process will be continued until the measurements of the rest of the time interval are implemented. Figure 2 shows the procedure of the proposed frequency-domainrecursive hybrid genetic algorithm where the red dashed box represented the evolution process of hybrid genetic algorithm illustrated in Figure 1.

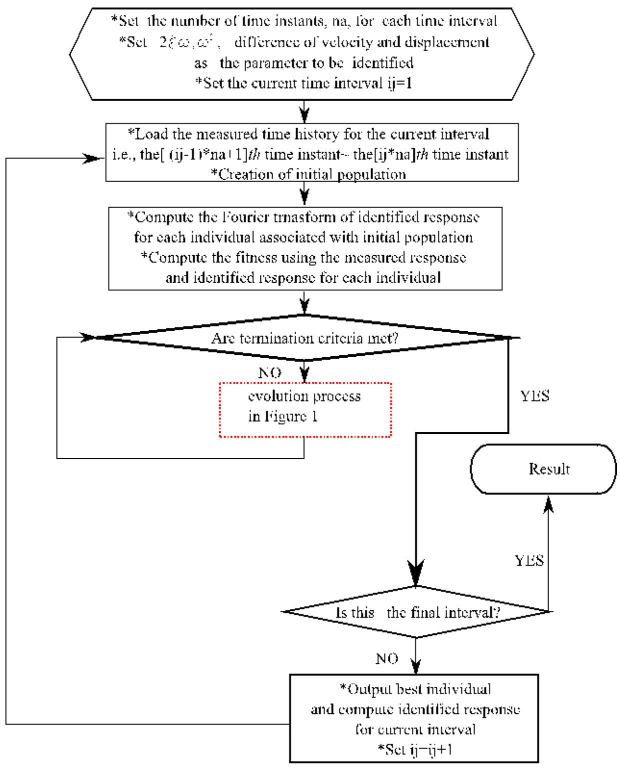

Fig. 2: Structure of frequency-domain recursive hybrid GA in the for SDOF system

3.2. Identification of a SDOF linear system

In order to verify the fast convergence of the proposed recursive hybrid GA strategy in frequency domain to system identification problem, we apply the new strategy to the SDOF linear systems with $2\xi\omega = 0.232$、 $\omega_1^2 = 44.256$. In this case, we did not divide the time history into several intervals. In other word, we set the number of time instants, *na*, as the total sampling number of the time history. As expected, the identified values are the same as the true values and the error index, 1.4584×10^{-8}, is extremely small. Figures 3~4 illustrate the comparison of the true acceleration with the identified one in the frequency domain and the time domain.

For realistic simulation, the time histories of the applied excitation as well as the acceleration response of the mass were noise contaminated. Consequently, the identification strategy should preferably be not too sensitive to the noise of the input and output measurements. In this section, the proposed frequency-domain recursive hybrid strategy is also explored for the case of noise existing. Figures 5 illustrates the comparison of the true acceleration with the identified one in the the time domain. Again, the strategy is demonstrated to be able to identify the system parameters even though the signals are contaminated.

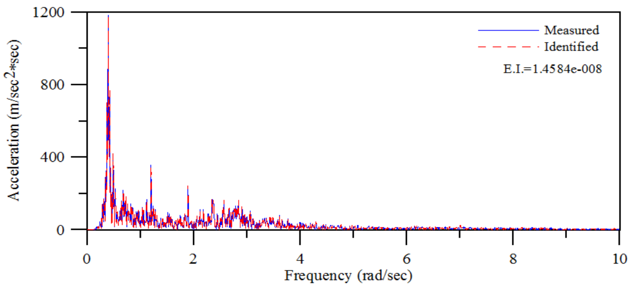

Fig. 3: Comparison of measured response with identified one of SDOF system in the frequency domain.

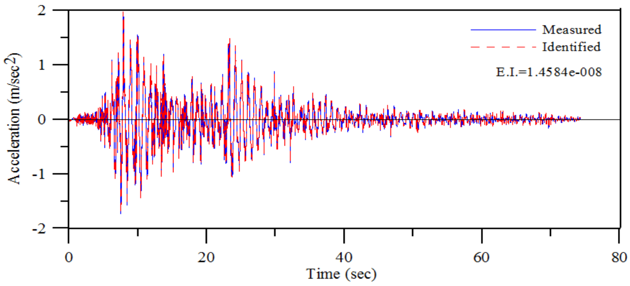

Fig. 4: Comparison of measured response with identified one of SDOF system in the time domain.

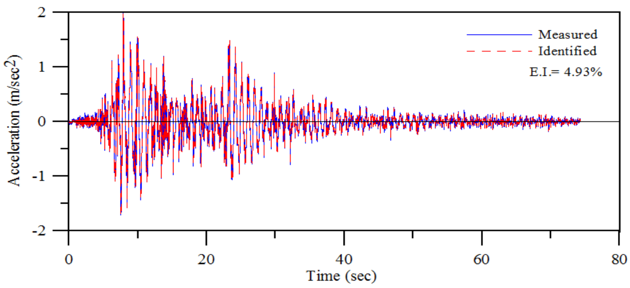

Fig. 5: Comparison of measured response with identified one of SDOF system in the time domain (5% noise level)

3.3. Identification of a SDOF linear system with *na* =400

In order to verify the accuracy of the proposed recursive strategy to system identification problem with divided intervals, we apply the new strategy to the same SDOF linear system shown in the previous section. Set the number of time instants for each interval to be 400. Identified parameters associated with ω^2 are shown in Figure 6. The identified values are close to the true value. Figure 7 illustrates the comparison of the true acceleration with the identified one. From the error index in the figure, the same conclusion can be drawn, too.

The proposed recursive hybrid strategy is also explored for noise level of 5%. Figure 8 shows true parameter versus identified parameter for each interval. Since the identified parameters fluctuate, the average value of identified parameter ω^2 is also computed as 43.8776 and plotted in the same figure for comparison. The average identified parameter is close to the true one. In order to alleviate the effect of variation of noise level for each time interval on the identified parameters, we will present the smoothed parameters for the rest of the paper.

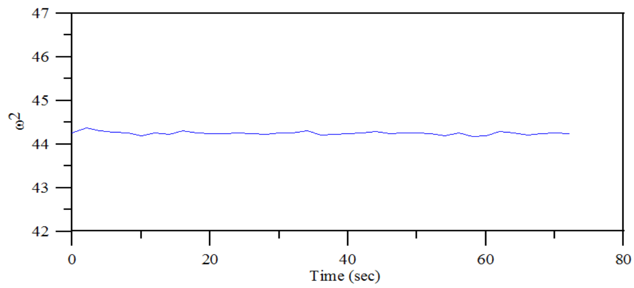

Fig. 6: Identified parameters for SDOF system (*na* =400)

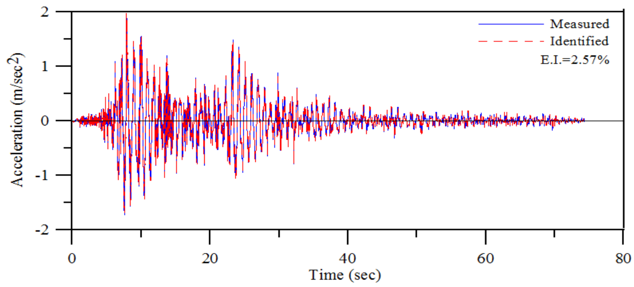

Fig. 7: Identified response for SDOF system (*na* =400)

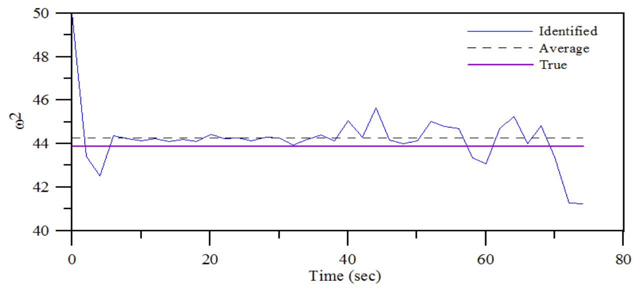

Fig. 8: Identified parameters for SDOF system with 5% noise level (*na* =400)

3.4. Identification of a SDOF nonlinear system

In order to demonstrate the accuracy of the proposed strategy to nonlinear SDOF system problem, the excitation is divided into four segments and the response for each segment is computed using different parameter values.

For this case, the number of time instants for each interval is set to be 400. The identified parameters associated with the frequency are shown in Figure 9. Figure 10 illustrates the comparison of true acceleration measurement with the identified response for this case. The error index computed for this case is extremely small and the identified parameters are exactly the values we set. Consequently, the strategy is demonstrated to be able to capture the system parameters even though the system is nonlinear.

Figure 11 illustrates the comparison of true acceleration measurement with the identified response for the same case but with 5% noise. The error index computed for this case is 6.39% which is consistent with the noise level. In summary, it can be concluded that the proposed recursive hybrid GA strategy is not sensitive to the noise involved in the input and output measurements for a SDOF nonlinear system, too.

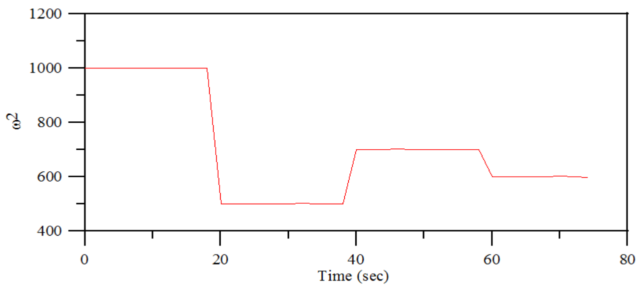

Fig. 9: Identified parameter for SDOF nonlinear system (na =400)

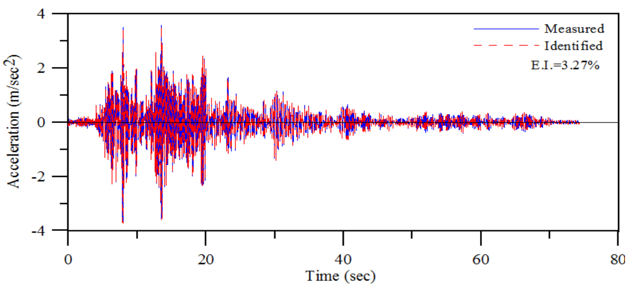

Fig. 10: Identified response for SDOF nonlinear system (na =400)

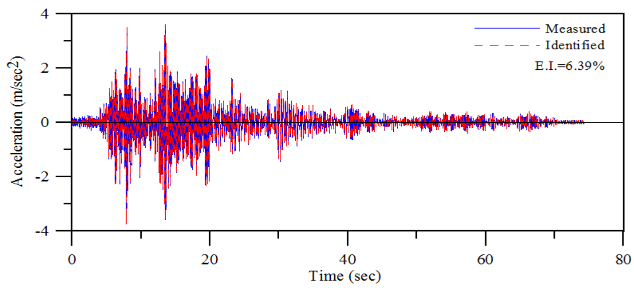

Fig. 11: Identified response for SDOF nonlinear system with 5% noise level (na =400)

3.5. Identification of a MDOF system

In this section, we will apply the proposed strategy to the identification of a multiple degree of freedom system with multiple outputs or measurements. The motion equation of a MDOF system subjected to a single excitation can be transformed into modal equation through mode superposition as

$$\ddot{u}_{sm} + 2\xi_m \omega_m \dot{u}_{sm} + \omega_m^{2} u_{sm} = -P_{sm}\ddot{u}_g \text{, s=1,2,..,n} \quad (8)$$

where u_{sm} is the modal displacement in mode m at the sth DOF, and P_{sm} the effective participation factor in mode m at the sth DOF associated with the ground motion $\ddot{u}_g$

$$P_{sm} = \frac{\phi_{sm}\{\phi_m\}^T[M]\{l\}}{\{\phi_m\}^T[M]\{\phi_m\}} \quad (9)$$

In which $[M]$ is the mass matrix, $\{\phi_m\}$ the mode shape in mode m, and $\{l\}$ the ground influence coefficient matrix with elements 0 and 1. Again, employing the finite Fourier transform, the following forms can be yielded.

$$U_{sm}^{\ T}(\varpi) = \int_0^T u_{sm} e^{-i\varpi t} dt \quad (10)$$

$$\dot{U}_{sm}^{\ T}(\varpi) = \int_0^T \dot{u}_{sm} e^{-i\bar{\omega} t} dt \quad (11)$$

$$= U_{sm}(T) - U_{sm}(0) + i\varpi U_{sm}^{\ T}(\varpi)$$

$$\ddot{U}_{sm}^{\ T}(\varpi) = \int_0^T \ddot{u}_{sm} e^{-i\bar{\omega} t} dt$$

$$= \dot{U}_{sm}(T) - \dot{U}_{sm}(0) + (U_{sm}(T) - U_{sm}(0))i\varpi - \varpi^2 U_{sm}^{\ T}(\varpi)$$

(12)

Transforming the mth modal equation (8) produces, for the sampling frequencies,

$$\ddot{U}_{sm}(\varpi) = -\frac{\varpi^2 P_{sm}\ddot{U}_g(\varpi) + d_{sm}(\omega_m{}^2 i\varpi) + v_{sm}(\omega_m{}^2 + 2\xi_m\omega_m i\varpi)}{\omega_m{}^2 + 2i\xi_m\omega_m\varpi - \varpi^2} \quad s=1,2,..,n \quad (13)$$

Combining the modal contributions gives the following expressions for the transformed relative acceleration

$$\ddot{U}_S = \sum_{m=1}^{M}\ddot{U}_{sm}(\varpi) = \sum_{m=1}^{M}\frac{v_{sm}(\omega_m{}^2 + 2\xi_m\omega_m i\varpi) + d_{sm}(\omega_m{}^2 i\varpi) + \varpi^2 P_{sm}\ddot{U}_g(\varpi)}{(\omega_m{}^2 - \varpi^2) + 2\xi_m\omega_m i\varpi} \quad (14)$$

The parameters v_{sm} and d_{sm} are the differences in the mth modal velocity and displacement at the sth DOF between the beginning and the end of the record segment of duration T. The error index in this case is expressed as the average of the error indices of the measurements.

If there are three accelerographs installed, the measurements expressed in Equation (14) can be implemented simultaneously with DOF s representing the stations where accelerographs mounted. The measured response associated with the MDOF system can again be simulated by assuming that the system behavior be represented by a three-mode model. The values of system parameters are summarized in Table 1 as the true parameters. The error index identified in this case is 4.61%. This implies that the identified response and the measured one are almost overlapped. Figure 12 illustrates the true acceleration measurement versus the identified one at the 3th DOF. As expected, the identified response coincides with the measured one and thus the strategy is proved to be quite superior in this case.

The identified parameters associated with the first modal frequency are also shown in Figure 13 for the same case with 5% noise level. Figure 14 illustrates the comparison of true acceleration measurement with the identified response at the 3th DOF for this case. The error index computed associated is 6.76% which is consistent with the noise level. In summary, it can be concluded that the proposed recursive hybrid GA strategy is not sensitive to the noise involved in the input and output measurements for a MDOF system, too.

Table1: True parameters of a MDOF linear system with 3 outputs based on three-mode analysis

	Mode m	$2\xi\omega$	ω^2	P_{1m}	P_{2m}	P_{3m}
True parameters	1	0.232	44.256	0.5001	1.0061	1.2680
	2	0.065	413.99	0.3666	0.1874	-0.3049
	3	0.068	1004.7	0.1333	-0.1935	0.1333

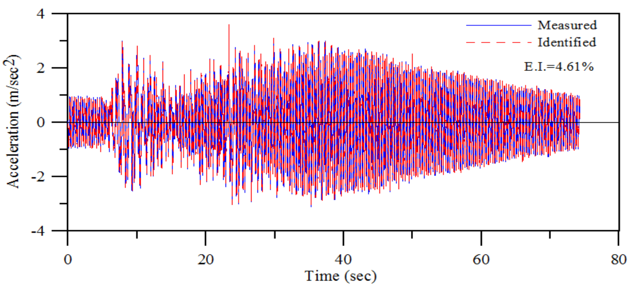

Fig. 12: Identified response for MDOF system (na =400)

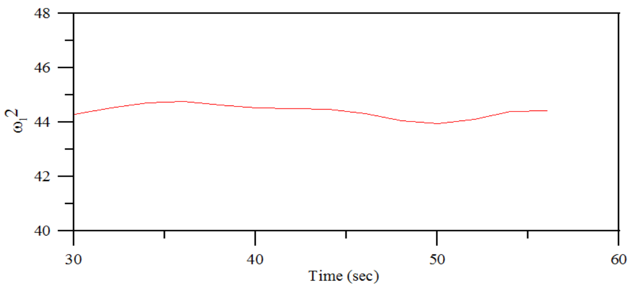

Fig. 13: Identified 1st modal frequency for MDOF system with 5% noise level (na =400)

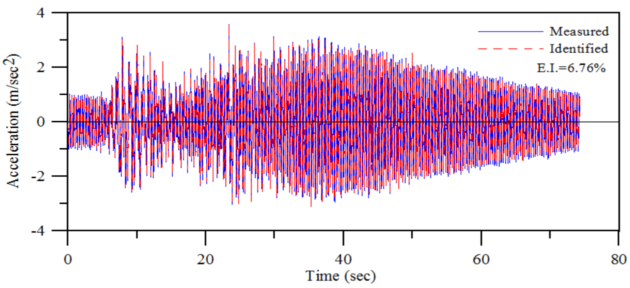

Fig. 14: Identified response for MDOF system with 5% noise level (na =400)

4. Identification of a real building

The real building identified here is the Taiwan Electricity Main Building located in Taipei. This building is a 27-story steel frame structure. This building seems to experience no visible damages under the attacks of the earthquakes. However, the dynamic parameters may be altered even though the damage of the structure is slight or invisible. Three-component seismographs are installed on the floor of the basement, as well as on the nineteenth floor and the top floor of the building. Accelerograms collected during earthquakes can be utilized to monitor the change of the structural parameters. Through spectral analysis, estimation of frequencies is made and rational ranges of these parameters can be obtained when performing the frequency–domain recursive hybrid GA identification procedure. Modal damping ratio, modal frequencies, and participations factors can be obtained on the basis of either longitudinal (L or X) or transverse (T or Y) measurements.

Twelve sets of strong motion records collected during earthquakes occurred on December 1 of 1995 (with magnitude of 5.7), January 22 of 1996 (with magnitude of 5.1), March 5 of 1996 (with magnitude of 6.4), July 29 of 1996 (with magnitude of 6.1), May 7 of 1999 (with magnitude of 5.4), September 21 of 1999 (Chi-Chi earthquake with magnitude of 7.3), one aftershock of Chi-Chi earthquake (with magnitude of 6.4), June 14 of 2001 (with magnitude of 6.3), March 31 of 2002 (with magnitude of 6.8), May 15 of 2002 (with magnitude of 6.2), November 8 of 2004 (with magnitude of 6.6), and November 11 of 2004 (with magnitude of 6.1), are analyzed. To get knowledge of the state of the structure, single-input-multiple-output model is utilized to perform the identification of modal parameters of the structure.

The data recorded on the basement can be employed as the input and those on the nineteenth floor and the top floor can be employed as output. Through spectral analysis of the record collected on December 1 of 1995, estimation of frequencies is made and rational boundaries of these parameters can be obtained before performing the new hybrid GA identification procedure. The first three modes are identified using the longitudinal excitation and response. Fig. 15 shows the comparison between the measured response and the identified one of the nineteenth floor based on the three-mode analysis. The error index is 22.38%. In this case, we conclude that the first three modes can provide sufficient accurate results for the top floor response of the specific building, since the measured response is in good agreement with the predicted one.

The procedure of recursive hybrid GA can also be applied to the record of the rest of the earthquakes. We can study the variation of the identified parameters by plotting them in the same figure. Figure 16~19 shows the change of the modal parameters for the motions in the longitudinal direction during these earthquakes. The values of the peak ground acceleration (PGA) for all the earthquakes are also plotted in the same figure for comparison. From the plot of the fundamental frequency, we can see that the modal frequency ranged from 2.33 $rad/\sec$ to 2.64 $rad/\sec$. Although this change is slight, it seems to follow the usual trend of increasing frequency with decreasing excitation amplitude. During the struck of the Chi-Chi earthquake, the fundamental frequency dropped to 2.235 $rad/\sec$. One and a half years later, the fundamental frequency during the earthquake on March 31, 2002 decreased to 2.224 $rad/\sec$. Finally, the fundamental frequency increased to 2.3622 $rad/\sec$ which is lower than the value yielded during the first earthquake. Similar changes were also indicated by the frequency of the second mode. The frequency of the second mode decreased to 6.6 $rad/\sec$ which is lower than the value yielded during the first earthquake. For the trend of modal damping, it seems to follow the usual trend of increasing modal damping with increasing excitation amplitude. The fundamental modal damping identified during the first earthquake was about 1.5%. This damping increased to 2.58% during the earthquake occurred on March 31, 2002. Figure 20~21 show the change of the modal frequencies for the motions recorded on the nineteenth floor and top floor in the transverse direction during these earthquakes. The fundamental modal frequency decreased from 2.29 $rad/\sec$ down to 2.248 $rad/\sec$ under the struck of the earthquake on March 31, 2002. Finally, the fundamental frequency increased to 2.33 $rad/\sec$. The modal frequency of the second mode started from 7.046 $rad/\sec$ and decreased to 6.823 $rad/\sec$.

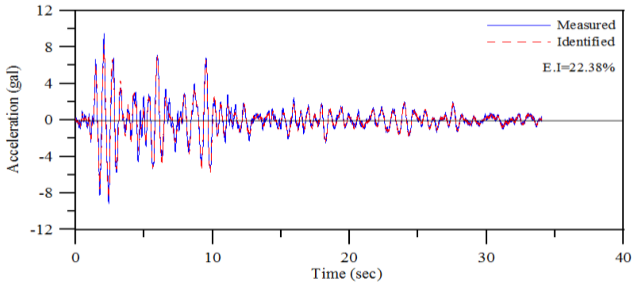

Fig. 15: Identified response (na =1000)

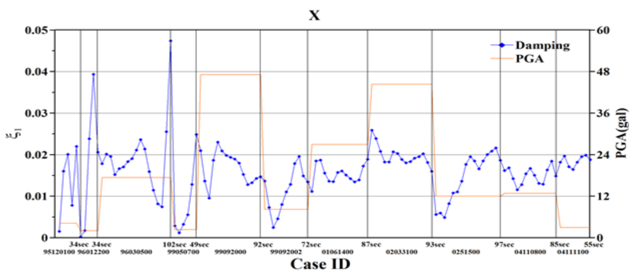

Fig. 16: Variation of first modal damping ratio in the longitudinal direction (na =1000)

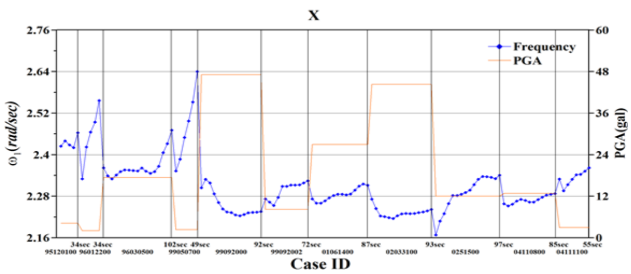

Fig. 17: Variation of first modal frequency in the longitudinal direction (na =1000)

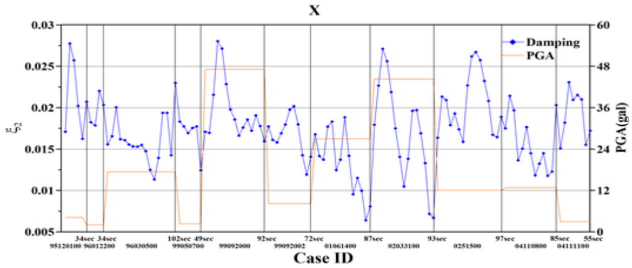

Fig. 18: Variation of second modal damping ratio in the longitudinal direction (na =1000)

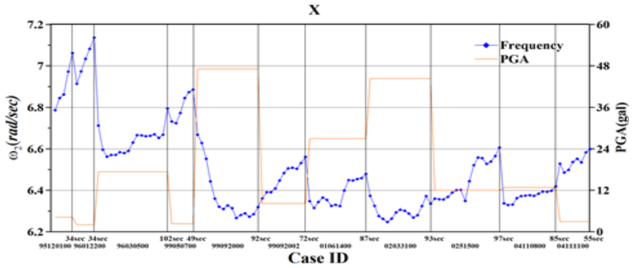

Fig. 19: Variation of second modal frequency in the longitudinal direction (na =1000)

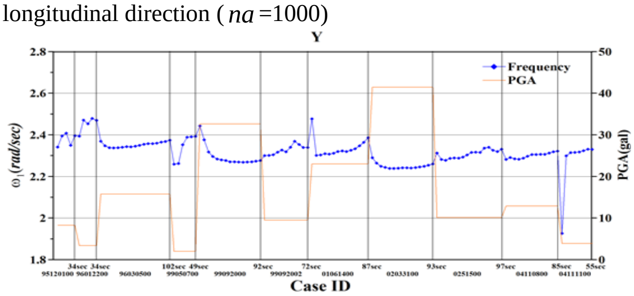

Fig. 20: Variation of first modal frequency in the transverse direction (na =1000)

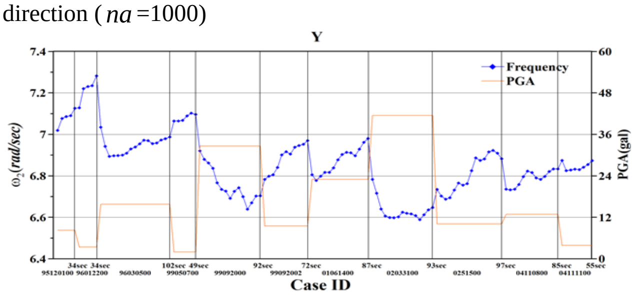

Fig. 21: Variation of second modal frequency in the transverse direction (na =1000)

5. CONCLUSION

This paper developed a new identification strategy called the frequency-domain recursive hybrid genetic algorithm for nonlinear system. The time history of the measurement is divided into a series of time intervals, and then the model of equivalent linear system in the frequency domain is employed to identify the modal parameters of the system and the differences of velocity and displacement between the beginning and the end of the record for each interval. The numerical accuracy can be improved a lot especially for the case of nonlinear behavior and the following conclusions can be made:

1. The new recursive hybrid GA identification strategy has been applied to the simulated input/output measurements of SDOF linear and nonlinear dynamic systems as well as a MDOF dynamic system. The identified parameters are very close to the true one and the error index is extremely small in each case. Also, the identified response and the measured response are almost overlapped in all the cases. Consequently, the applicability of the propose strategy to structural dynamic parameter identification is proved. Moreover, the strategy is also shown to be not sensitive to the noise contamination. This assures the feasibility of application to the measurements of real systems.

2. From the identification results of a real high–rise steel building under the struck of twelve earthquakes with slight to moderate intensity, the first three modes can provide sufficient compatible results for the response of the specific building, since the measured response coincides quite well with the predicted one in time domain. According to the results of system identification, the modal frequency decreased about 9% of the original frequency after the earthquake on March 31 of 2002.

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