MECHANICS OF OPTIMAL STRUCTURAL DESIGN FOR EXTREME LOADS TO PEAK SYSTEM RESPONSES

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ABSTRACT

Over the past decades, with the development of modern manufacturing and information technology, demands of smart and economical structural designs have been increasing considerably. Central to this engineering issue is that a good structural design needs to embrace both necessary capabilities to afford critical load distributions and the best arrangement of materials serving the performance criteria using limited resources. Here, a new analysis technique is proposed to achieve optimal structural designs considering peak system responses as design constraints respective to extreme load distributions. We anticipate that the technique will open a door for designing efficient structural systems which satisfy safety requirements under various sophisticated loadings from the environment.

KEYWORDS: OPTIMAL STRUCTURAL DESIGN, EXTREME LOAD DISTRIBUTION, PEAK SYSTEM RESPONSE, STRUCTURAL MECHANICS

Introduction

With current population on the earth, there is an inevitable pressure from consumption of natural resources. Therefore, we have to gain the financial and environmental benefits by designing smart and efficient structural systems serving their working loads. In this paper, we address both structural mechanics of extreme load distribution identification corresponding to certain peak system response and numerical nonlinear schemes of structural optimization, and then fuse these two techniques to achieve optimal designs considering peak structural behaviors as design constraints corresponding to extreme loads.

Formulations

We may consider the finite element method of structural mechanics [Cook *et al.* (2001)] for implementing structural optimization considering peak system responses as design requirements corresponding to extreme load distributions under the following two main subjects: structural mechanics of extreme load distribution identification to certain peak system response and nonlinear programming in structural optimization involving the peak system behaviors as design constraints. The detailed description of the structural mechanics dealing with identification of extreme loads and the nonlinear programming in structural optimization will be given as follows.

Structural Mechanics of Extreme Load Distribution Identification to Certain Peak System Response

In this paper, extreme loads are adopted as the load distributions which lead a particular extreme quasi-static response of the structure system, and this load pattern is regarded as the most expected loads which result in the special peak system behavior among all the events of Gaussian properties for certain loading duration [Kasperski (1992)]. The maximum structural response of the i^{th} degree of freedom is written as

$$r_i^{\max} = \sum_k a_{ik} \cdot \left(\overline{p}_k + g\ \rho_{r_i p_k} \sigma_{p_k} \right), \tag{1}$$

where $\mathbf{a}$ is the flexibility matrix of the structure, $\overline{\mathbf{p}}$ is the mean dynamic loads, g is the peak factor for a given structural reliability ($g = 3.5$ in the *Eurocode 8* will be adopted in this paper.), $\boldsymbol{\rho}$ is the correlation coefficient matrix between the structural responses $\mathbf{r}$ of the system and the dynamic loads $\mathbf{p}$, and $\boldsymbol{\sigma}$ is the standard deviation of the dynamic loads. Therefore, the equivalent static loads for the maximum response of the i^{th} degree of freedom for the loading duration is

$$p_k^{\max} = \overline{p}_k + g\ \rho_{r_i p_k} \sigma_{p_k}, \text{ (no sum on } k\text{)}. \tag{2}$$

Similarly, the minimum structural response of the i^{th} degree of freedom is

$$r_i^{\min} = \sum_k a_{ik} \cdot \left(\overline{p}_k - g\ \rho_{r_i p_k} \sigma_{p_k} \right), \tag{3}$$

and the equivalent static loads for the minimum response of the i^{th} degree of freedom for the loading duration is

$$p_k^{\min} = \overline{p}_k - g\ \rho_{r_i p_k} \sigma_{p_k}, \text{ (no sum on } k\text{)}. \tag{4}$$

Nonlinear Programming in Structural Optimization involving Peak System Behaviors as Design Constraints

In dealing with structural optimization, the mathematical problem is usually described as

$$\text{minimize } f(\mathbf{x}), \mathbf{x} \in R^n \tag{5}$$

$$\text{subject to } \quad \mathbf{g}(\mathbf{x}) \le 0, \tag{6}$$

$$\mathbf{h}(\mathbf{x}) = 0, \tag{7}$$

where $\mathbf{x}$ is the design variables, $f(\mathbf{x})$ is the objective function, $\mathbf{g}(\mathbf{x})$ and $\mathbf{h}(\mathbf{x})$ denote the inequality and equality constraints respectively. For a truss structure, the objective function is set as the total weight of the structural system; the design variables are the cross-sectional areas of the members $\mathbf{A}$; the inequality constraints are specified by the limited maximum system response of the i^{th} degree of freedom r_i^*; and the lower and upper bounds of the design cross-sectional areas of the truss members are denoted as $\mathbf{A}^{lb}$ and $\mathbf{A}^{ub}$ in this study. Hence, the optimization problem of the truss structure is further written as

$$\text{minimize } f(\mathbf{A}) = \mathrm{A}_i L_i, \mathbf{A} \in R^n \tag{8}$$

$$\begin{aligned} \text{subject to } \quad \mathbf{g}_1(\mathbf{A}) &= r_i^{\max} - r_i^* \\ &= \sum_k a_{ik}(\mathbf{A}) \cdot \left(\overline{p}_k + g\ \rho_{r_i p_k}(\mathbf{A}) \sigma_{p_k} \right) - r_i^* \le 0, \end{aligned} \tag{9}$$

$$\mathbf{g}_2(\mathbf{A}) = \mathbf{A} - \mathbf{A}^{ub} \le 0, \tag{10}$$

$$\mathbf{g}_3(\mathbf{A}) = \mathbf{A}^{lb} - \mathbf{A} \le 0. \tag{11}$$

For such kind of nonlinear optimization problems, the design variables are suggested to be solved by the sequential quadratic programming method [Byrd *et al.* (1999), Byrd *et al*. (2000) and Waltz *et al.* (2006)].

Practical Application

Consider a truss structure with its supports shown in Fig. 1. The truss is made of the A-36 steel, which has Young’s modulus of 200 GPa, and has a uniform cross-sectional area of 150 mm^2. The structure is subjected to external dynamic loads at all its nodes in both x_1 and x_2 directions, and the means μ and standard deviations σ of these nodal loads are given in Table

1. For the structural design, the vertical displacement of node no.11 of the truss is chosen as the most important response in the system. Fig. 2 shows the structural extreme response of x_2-component deformation at node no.11 of the initial structural design by applying Eq. (1) & (3). In addition, the corresponding extreme load distribution is identified through Eq. (2) & (4) and presented in Fig. 3.

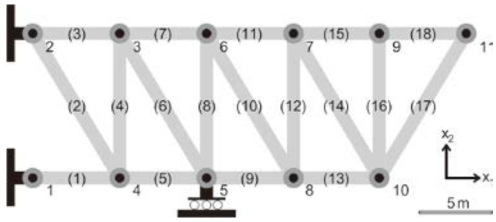

Figure 1: Schematic diagram of the truss structure

Table 1: Information of the external dynamic loading distributions (Unit: kN)

Node-Dir.	3-x_1	3-x_2	4-x_1	4-x_2	5-x_1	6-x_1	6-x_2	7-x_1	7-x_2
μ	0.526	0.496	0.477	0.512	0.524	0.463	0.471	0.481	0.480
σ	1.014	1.033	0.973	1.048	1.004	1.006	0.990	0.992	0.977
Node-Dir.	8-x_1	8-x_2	9-x_1	9-x_2	10-x_1	10-x_2	11-x_1	11-x_2	
μ	0.494	0.475	0.497	0.562	0.530	0.514	0.500	0.511	
σ	0.976	0.997	1.002	1.000	0.962	1.018	0.977	0.981	

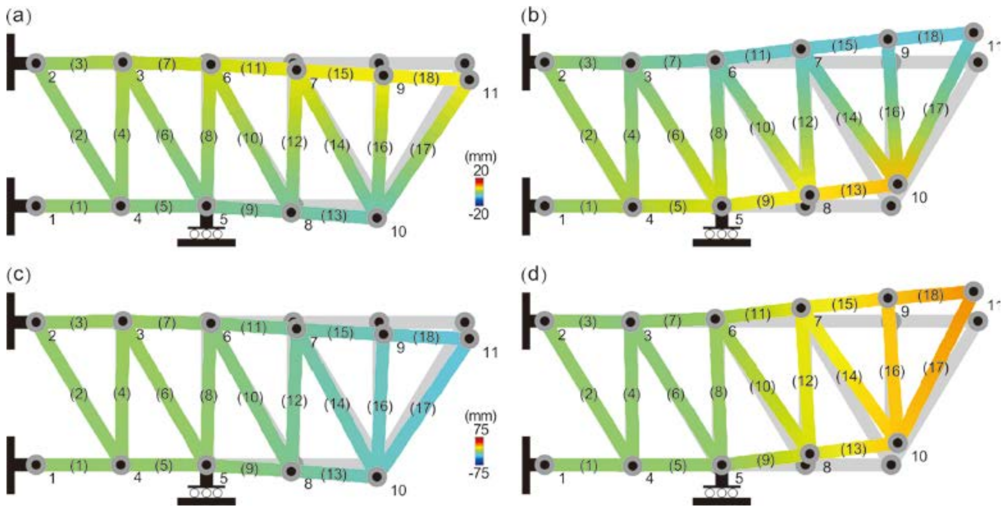

Figure 2: Structural peak response of x_2-component deformation at node no.11: Deformation fields of x_1-component corresponding to the structural (a) minimum and (b) maximum responses, and deformation fields of x_2-component corresponding to the structural (c) minimum and (d) maximum responses under the extreme load distributions.

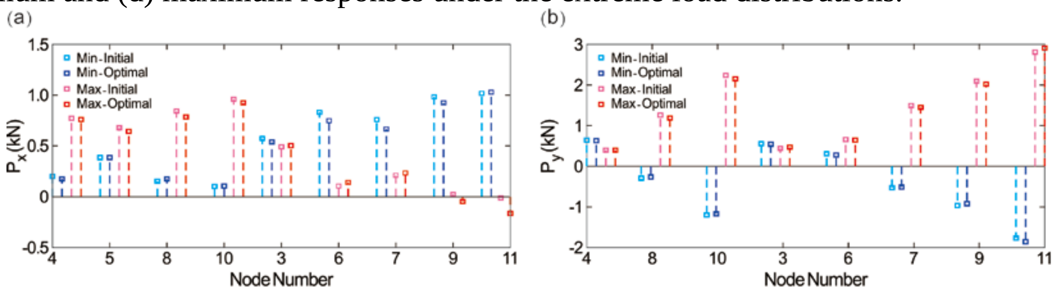

Figure 3: Load distributions to the peak responses of x_2-component deformation at node no.11 of the initial and optimal structural designs. (a) The extreme load distributions of x_1-component and (b) the extreme load distributions of x_2-component.

Afterward, the peak vertical displacement of node no.11 is also selected as the design constraint for structural optimization of gaining the optimal system with the lightest weight which still satisfies the design requirement. Fig. 4 presents the optimal cross-sectional areas of the structural members for the design constraint specified as 75 mm. The system peak response of x_2-component deformation at node no.11 of the optimal design is shown in Fig. 5.

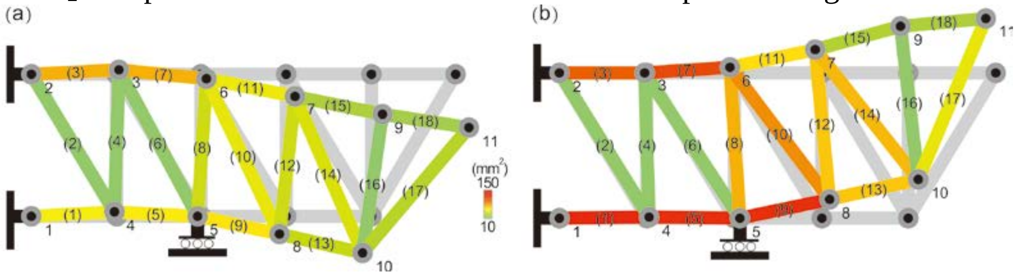

Figure 4: Optimal cross-sectional member areas to (a) minimum and (b) maximum responses

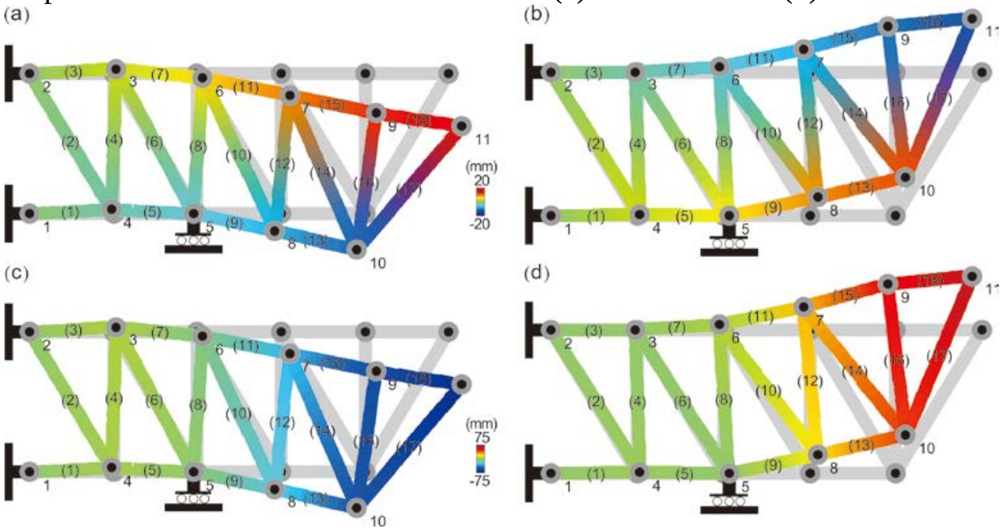

Figure 5: Structural peak response of x_2-component deformation at node no.11 of the optimal design: Deformation fields of x_1-component corresponding to the structural (a) minimum and (b) maximum responses, and deformation fields of x_2-component corresponding to the structural (c) minimum and (d) maximum responses under the extreme load distributions.

Conclusions

We proposed a novel analysis technique integrating the extreme load distribution identification to certain peak system response and the nonlinear programming in optimization for realizing optimal structural designs considering a specific system response as design constraints corresponding to the extreme load distribution. As the computational results presented above, the technique provides a unique way of dealing optimization design of structures for their peak mechanical behaviors under temporal loadings.

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