Nondestructive Evaluation of Buried Dielectric Cylinders by Asynchronous Particle Swarm Optimization

Wei-Tsong Lee¹, Chi-Hsien Sun², Chien-Ching Chiu¹ and Jyun-Fu Li¹

¹Electrical Engineering Department, Tamkang University
Tamsui, Taiwan, R.O.C.
²Department of Electronic and Computer Engineering, National Taiwan University of Science and Technology, Taipei City, Taiwan, R.O.C.

Abstract

This paper presents the studies of time domain inverse scattering for a two-dimensional inhomogeneous dielectric cylinder buried in a slab medium by the finite difference time domain (FDTD) method and the asynchronous particle swarm optimization (APSO) method. For the forward scattering part, the FDTD method is employed to calculate the scattered E fields. Based on the scattering fields, these inverse scattering problems are transformed into optimization problems. The APSO is applied to reconstruct permittivity of the two-dimensional inhomogeneous dielectric cylinder. In addition, the effects of Gaussian noise on the reconstruction results are investigated. Numerical results show that even when the measured scattered fields are contaminated with Gaussian noise, APSO ten times is able to yield good reconstructed quality.

Index Terms – FDTD, asynchronous particle swarm optimization
I. INTRODUCTION

The objective of the inverse problem is to determine the electromagnetic properties of the buried scatterer from the scattered field measured outside. There are many applications such as geophysical prospecting, medical imaging, non-destructive evaluated, determination of underground tunnels and pipelines, etc [1]-[10]. However, it is well known that one major difficulty of inverse scattering is its nonlinearity because it involves the product of two unknowns: the electrical properties of object, and the electric field within the object [11].

In general, the nonlinearity of the problem is coped with by applying iterative optimization techniques, concretely two techniques are employed. The first is traditional deterministic methods, such as Born iterative method (BIM) [12] and the distorted Born iterative method (DBIM) [13]. Furthermore, for a gradient-type method, it is well known that the convergence of the iteration depends highly on the initial guess. If a good initial guess is given, the speed of the convergence can be very fast. On the other hand, if the initial guess is far away from the exact one, searching tends to get fail [14].

The second is stochastic methods. These algorithms based on strategies offer advantages relative to local inversion algorithms including strong search ability, simplicity, robustness and insensitivity to nonlinearity. In contrast to traditional deterministic methods, evolutionary searching schemes provide a more robust and efficient approach for solving inverse
scattering problems [15]-[17]. Thus, some stochastic methods, such as genetic algorithm (GA) [18]-[21], particle swarm optimization (PSO) [22]-[26], differential evolution/dynamic differential evolution (DE/DDE) [17], [26]-[34] are proposed to search the global extreme of the inverse problems to overcome the drawback of the deterministic methods. However, these papers only focus on homogeneous dielectric cylinder or perfectly conducting cylinder cases.

Recently, there are a few reports on this subject of 2-D object about inhomogeneous dielectric cylinder under time domain [32]-[34]. The reference [33] focuses on two-dimensional inhomogeneous dielectric cylinder buried in a half-space by using dynamic differential evolution (DDE) and non-uniform steady state genetic algorithm (NU-SSGA). To the best of our knowledge, there is still no investigation on using the APSO to reconstruct the electromagnetic imaging of inhomogeneous dielectric cylinders with arbitrary cross section in a three-layered slab medium under time domain.

This paper presents a time domain computational scheme for the microwave imaging of a 2D inhomogeneous dielectric cylinder. The forward problem is solved by the FDTD method. The inverse problem is formulated into an optimization problem, and then the global searching scheme APSO is used to search the parameter space. In Section II, the theoretical formulation for the electromagnetic imaging is presented. The general principle of the APSO scheme and the way we applied it to the imaging problem are described. In section III, some numerical results are presented. Finally, in section IV some conclusions are drawn.
II. THEORETICAL FORMULATION

Consider a 2-D inhomogeneous dielectric cylinder buried in a slab medium material medium as shown in Figure 1. The cylinder is parallel to z axis buried between the planar interfaces separating three homogeneous spaces: the region 1 \( (\varepsilon_1, \mu_1) \), the region 2 \( (\varepsilon_2, \mu_2) \) and region 3 \( (\varepsilon_1, \mu_1) \). The permittivity of the buried dielectric object is denoted by \( \varepsilon_3(x, y) \). The dielectric object is illuminated by a Gaussian pulse line source located at the points denoted by \( T_x \) in the first layer and scattered waves are recorded at those points denoted by \( R_x \) in the same layer. The computational domain is surrounded by the perfect matching layers (PML) absorber [35] to reduce the reflection from the environment-PML interface.

A. Forward Problem

The direct scattering problem is to calculate the scattered electric fields for a given permittivity \( \varepsilon_3 \). The scatterer region is subdivided by a rectangular grid with R and U subdivisions along the x and y axis, respectively. Then, the relative permittivity is expressed as

\[
\varepsilon_3(x, y) = \sum_{r=1}^{R} \sum_{u=1}^{U} \alpha_{ru} I_{ru}(x, y)
\]  

(1)
where

\[ II_m(x,y) = \begin{cases} 1 & \text{if } (x,y) \in [x_r,x_{r+1}] \times [y_u,y_{u+1}] \\ 0 & \text{elsewhere} \end{cases} \]  

(2)

are the pulse functions.

B. Inverse Problem

For the inverse scattering problem, the permittivity of the dielectric cylinder is reconstructed through the given scattered electric fields obtained at the receivers. This problem is formulated as an optimization problem, for which the global searching scheme APSO is employed to minimize the following objective function (OF):

\[
OF = \frac{\sum_{n=1}^{N_i} \sum_{m=1}^{M} \sum_{k=0}^{K} \left| E_z^{\exp}(n, m, k\Delta t) - E_z^{cal}(n, m, k\Delta t) \right|}{\sum_{n=1}^{N_i} \sum_{m=1}^{M} \sum_{k=0}^{K} \left| E_z^{exp}(n, m, k\Delta t) \right|}

(3)

where \( E_z^{exp} \) and \( E_z^{cal} \) are the experimental electric fields and calculated electric fields, respectively. The \( N_i \) and \( M \) are the total number of the transmitters and receivers, respectively. \( K \) is the total time step number of the recorded electric fields.

C. Asynchronous Particle Swarm Optimization (APSO)

Particle swarm global optimization is a class of derivative-free, population-based and self-adaptive search optimization technique which was introduced by Kennedy and Eberhart [36]. PSO has proven to be a useful method of optimization for difficult and discontinuous multidimensional engineering problems [37]-[39] due to its efficiency of exploring the entire search space.

Particles (potential solutions) are distributed throughout the searching
space and their positions and velocities are modified based on social behavior. The social behaviors in PSO exhibit a population of particles moving toward the most promising region of the search space. Clerc [40] proposed the constriction factor to adjust the velocity of the particle for obtaining the better convergence; the algorithm was named as constriction factor method (CFM). PSO starts with an initial population of potential solutions that is randomly generated and composed of $N_p$ individuals (also called particles), of which each represents the permittivity distribution of the cylinder.

After the initialization step, each particle of population is associated with a randomized velocity and position. Thus, each particle has a position and velocity vector, and can move through the problem space. In each generation, the particle changes its velocity according to its best experience, called $x_{pbest}$, and the best particle in the swarm, called $x_{gbest}$.

Assume there are $N_p$ particles in the swarm that is in a search space of $D$ dimensions, the position and velocity of the $i$-th particle is determined according to the constriction factor method as follows:

$$v_{id}^k = \chi \cdot (v_{id}^{k-1} + c_1 \cdot \phi_1 \cdot (x_{pbest, id} - x_{id}^{k-1}) + c_2 \cdot \phi_2 \cdot (x_{gbest, id} - x_{id}^{k-1}))$$  \hspace{1cm} (4)

$$x_{id}^k = x_{id}^{k-1} + v_{id}^k$$  \hspace{1cm} (5)

where $\chi = \frac{2}{2 - \phi - \sqrt{\phi^2 - 4\phi}}$, $\phi = c_1 + c_2 \geq 4$. $c_1$ and $c_2$ are the learning coefficients used to control the impact of the local and global component in velocity equation (4). $v_{id}^k$ and $x_{id}^k$ are the velocity and position of the
\( i \)-th particle with \( d \)-th dimension at the \( k \)-th generation, \( \varphi_1 \) and \( \varphi_2 \) are both random numbers between 0 and 1.

Asynchronous PSO use the following rule: after each particle position updates if the new best position is better than the current best position, the new best position will replace the current best position and be used in the next particles swarm immediately. In this way, the swarm reacts more quickly to speedup the convergence because the updating occurs immediately after cost function evaluation for each particle.

The “damping boundary condition” was proposed by Huang and Mohan [41] to ensure the particles move within the legal search space. In many practical optimization problems, the dimensionality and the location of the global optimum is usually difficult to know \textit{a priori}. It is therefore desirable to have a single boundary condition that can offer a robust and consistent performance for the PSO technique regardless of the problem dimensionality and the location of the global optimum. When the particle exceed a certain dimension of the solution search space, the position of particle will be re-located in the search boundary and its velocity component in the reverse of this dimension and multiplied by a random number (between 0-1).

Mutation scheme is introduced in this algorithm to speed up the convergence when particles are around global optimum. The mutation scheme can also avoid premature convergences in the searching procedure and help the \( x_{gbest} \) escape from the local optimal position. It can increase the robustness of the algorithm with full search capabilities to avoid falling into the regional value. More details about the APSO
algorithm can be found in [41]-[42].

The flowchart of the modified APSO is shown in Figure 2. Asynchronous PSO goes through seven procedures as follows:

I. Initialize a starting population: randomly generate a swarm of particles.

II. Calculate the $E$ fields by a home-made FDTD code.

III. Evaluate the population using cost function: the asynchronous PSO algorithm evaluates the cost function (3) for each individual in the population.

IV. Find $x_{pbest}$ and $x_{gbest}$.

V. Mutation scheme: the particle swarm optimization algorithm has been shown to converge rapidly during the initial stages of a global search, but when around the global optimum, the search can become very slow. For the reason, mutation scheme is leaded into asynchronous PSO. As shown in Figure 2, there is an additional competition between the $x_{gbest}$ and $x_{gbest_{mu}}$. The current $x_{gbest}$ will be replaced by the $x_{gbest_{mu}}$ if the $x_{gbest_{mu}}$ is better than the current $x_{gbest}$. The $x_{gbest_{mu}}$ is generated by following way:

$$x_{gbest_{mu}} = \begin{cases} x_{gbest} - \phi_3 \cdot \left[ c_3 - (c_3 - c_4) \cdot \frac{k}{k_{max}} \right] \cdot (x_{max} - x_{min}), & \text{if } \phi_{mu} < 0.5 \\ x_{gbest} + \phi_3 \cdot \left[ c_3 - (c_3 - c_4) \cdot \frac{k}{k_{max}} \right] \cdot (x_{max} - x_{min}), & \text{if } \phi_{mu} \geq 0.5 \end{cases}$$

(6)

where $c_3$ and $c_4$ are the scaling parameter. $\phi_3$ and $\phi_{mu}$ are both the random numbers between 0 and 1. $k$ is the current iteration.
number. $k_{\text{max}}$ is the maximum iteration number. $x_{\text{max}}$ and $x_{\text{min}}$ are the upper limit and lower limit of the search space, respectively.

VI. Update the velocity and position.

VII. Stop the process and print the best individual if the termination criterion is satisfied; else, go to step 2.

III. NUMERICAL RESULTS

As shown in Figure 1, the problem space is divided in $68 \times 68$ grids with the grid size $\Delta x = \Delta y = 5.95\text{mm}$. The inhomogeneous dielectric cylinder is buried in a lossless a slab medium ($\sigma_1 = \sigma_2 = \sigma_3 = 0$). The transmitters and receivers are placed in free space above the inhomogeneous dielectric slab. The permittivities in region 1, region 2 and region 3 are characterized by $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = 8\varepsilon_0$ and $\varepsilon_3 = \varepsilon_0$, respectively, while the permeability $\mu_0$ is assumed for each region, i.e., only non-magnetic media are concerned here. The cylindrical object is illuminated by a transmitter at two different positions, $N_t=2$, which are located at $(154.7\text{mm}, 440.3\text{mm})$ and $(440.3\text{mm}, 440.3\text{mm})$, respectively. The scattered E fields for each illumination are collected at the five receivers, $M=5$, which are equally separated by $23.8\text{mm}$ in the x-axis and the first receiver is located at $(119\text{mm}, 416.5\text{mm})$. For the representation of the scatterer properties, seven subdivisions are considered along both x and y axis ($R = U = 7$) and axis resulting in $49$ equal square cells. There are $49$ unknown parameters to retrieve, which include the scatterers permittivities distribution. The search range of unknowns for permittivity
is chosen from 1.0 to 6.0, determined by the prior knowledge of the object. The related coefficients of the APSO are set below. The learning coefficients \( c_1 \) and \( c_2 \) are set to 2.8 and 1.3\(^{[42]-[43]}\), respectively. The mutation probability is 0.4. Moreover, the relative mean square error of the reconstructed data profile is defined as

\[
err = \left[ \frac{\sum_{i=1}^{N} \left| \Delta \tau_i \right|^2}{\sum_{i=1}^{N} \left| \tau_{i}^{\text{exact}} \right|^2} \right]^{\frac{1}{2}} \times 100\% \quad (7)
\]

where \( \tau_{i}^{\text{exact}} \) and \( \Delta \tau_i \), respectively, denote the exact value of the relative permittivity of the \( i \)th cell and the difference between the reconstructed and exact values. Reconstruction is carried out on an Intel PC (2.83 GHz/2GB memory/500 GB HD). The software is developed on FORTRAN VISION 6.0 in WINDOWS XP system environment. In our simulation, the typical CPU time for 1000 generations is about 4 hours on a home-made Fortran program that runs on an Intel PC (3.4 GHz/4G memory/500G).

In the first example, we consider a practical case to detect water buried in the wall, as shown in Fig. 3(a). The whole problem space is considered. The reconstructed error is 27.02 %. From Fig. 3(b), we conclude that the water in the wall is detectable by APSO.

In the second example, let us consider a crisscross-like object, as shown in Fig. 4(a). The relative permittivity of the crisscross axis is 4.5 with four leaves of 2.5. The reconstructed error is 3.89 %. Fig. 4(b) shows that it is clear that the reconstructed result is good by APSO.

In the third example, we consider an X-like object, as shown in Fig. 5(a). The permittivity of the crisscross axis is 4.5 with four leaves of 2.5.
The reconstructed error is 3.09%. Again, Fig. 5(b) shows it is seen that good reconstruction is achieved by APSO.

In the fourth example, we report on multi-inhomogeneous dielectric objects buried in a lossy slab medium \( (\sigma = 2 \times 10^{-3} \text{ s/m}) \), as shown in Fig. 6(a). The reconstructed error is 9.68%, as shown in Fig. 6(b). It is clear that the reconstruction error is less well for the inhomogeneous dielectric objects with high permittivity. However, we can see that reconstruction is better for the illuminated side than that obtained for the backside.

In the last example, to test the problem of reference [33], we have conducted the following studies: a two dimensional inhomogeneous dielectric cylinder buried in half space and slab medium, respectively. Bulges object of \( \varepsilon = 3.5 \), surrounding area of the object is 2.5, as shown in Fig. 7(a). The reconstructed result is shown in Fig. 7(b) and Fig. 7(c), respectively. The reconstructed error is 2.48% and 2.61% in slab medium and half space, respectively. It is found that APSO is suitable for inverse scattering problem whether inhomogeneous dielectric cylinders buried in half or slab medium.

From last four examples, we conclude that, in the case of noisy free data, the relative reconstruction errors for all parameters are lower than 10%. In order to investigate the sensitivity of the imaging algorithm against random noise, the additive white Gaussian noise of zero mean with standard deviation \( \sigma_g \) is added into the recorded scattered electric fields to mimic the measurement errors for examples 2 to 4. The signal to noise ratio (SNR) is defined as:
\[ SNR = 10 \log_{10} \frac{\sum_{n=1}^{N_i} \sum_{m=1}^{M_i} \sum_{b=0}^{B} |E_z^{\exp}(n, m, b\Delta t)|^2}{\sigma_g^2(N_i)(M_i)(B)} \]  

Figure 8 shows the reconstructed results for the cylinder under the condition that the recorded scattered fields are contaminated by noise, of which the SNR includes 40dB, 30dB, 20dB, 10dB and 3dB. It is observed that good reconstruction can be obtained when the SNR is above 10dB.

IV. CONCLUSION

In this paper, the time domain inverse scattering of inhomogeneous dielectric cylinders of arbitrary cross section in three layer space is investigated. By combining the FDTD method and the APSO scheme, good reconstructed results are obtained. Through the APSO scheme, dielectric constant of the object can be successfully reconstructed even when the dielectric constant is relatively large even with lossy or lossless surrounding media. The effects of noise on microwave imaging are examined, and good reconstruction has been obtained even in the presence of white Gaussian noise in experimental data.
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Caption

Fig. 1 Geometry for the inverse scattering of a dielectric cylinder buried in a slab medium.

Fig. 2 Flowchart for the asynchronous particle swarm optimization

Fig. 3(a) Original permittivity distribution

Fig. 3(b) Reconstructed permittivity distribution with a 27.02% error

Fig. 4(a) Original permittivity distribution

Fig. 4(b) Reconstructed permittivity distribution with a 3.89% error

Fig. 5(a) Original permittivity distribution

Fig. 5(b) Reconstructed permittivity distribution with a 3.09% error

Fig. 6(a) Original permittivity distribution

Fig. 6(b) Reconstructed permittivity distribution with a 9.68% error

Fig. 7(a) Original permittivity distribution

Fig. 7(b) Reconstructed permittivity distribution buried in slab medium

Fig. 7(c) Reconstructed permittivity distribution buried in half space

Fig. 8 Reconstruction error as function of SNR by APSO
Fig. 1 Geometry for the inverse scattering of a dielectric cylinder buried in a slab medium
Generate initial population $x^0$ randomly and uniformly in the search range, $k=0$

$k > k_{\text{max}}$ Yes → Stop: giving $x_{\text{gbest}}$, optimal solution.

No → $id = 1$

Calculate $E$ field by FDTD codes

Evaluate cost function value for the id-th particle’s position in the population, $F(x_{id}^k)$

Find $x_{pbest}$

Find $x_{gbest}$

Yes → mutation

$OF(x_{\text{gbest}}_{\text{mu}}) < OF(x_{\text{gbest}})$

$\begin{align*}
    x_{\text{gbest}}_{\text{mu}} &= x_{\text{gbest}} \\
    x_{\text{gbest}} &= x_{\text{gbest}}
\end{align*}$

No →

Update the velocity and position of id-th particle in the population

$k = k + 1$ Yes → $id = \text{population size}$

No → $id = id + 1$

Fig. 2 Flowchart for the asynchronous particle swarm optimization
Fig. 3(a) Original dielectric constant

Fig. 3(b) Reconstructed permittivity distribution with a 27.02% error
Fig. 4(a) Original dielectric constant

Fig. 4(b) Reconstructed permittivity distribution with a 3.89% error
Fig. 5(a) Original dielectric constant

Fig. 5(b) Reconstructed permittivity distribution with a 3.09% error
Fig. 6(a) Original dielectric constant

Fig. 6(b) Reconstructed permittivity distribution with a 9.68% error
Fig. 7(a) Original dielectric constant

Fig. 7(b) Reconstructed permittivity distribution buried in slab medium
Fig. 7 (c) Reconstructed permittivity distribution buried in half space

Fig. 8. Reconstruction error as function of SNR by APSO