Linearly Constrained RLS Algorithm with Variable Forgetting Factor for DS-CDMA System

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Abstract—**In this paper we present a novel linearly constrained (LC) RLS algorithm with variable forgetting factor (VFF), and apply to the DS-CDMA system for suddenly joined narrowband interference suppression. Under Rayleigh fading channel environment, the VFF could be employed to improve the tracking capability compared with the conventional LC-RLS algorithm with fixed value of forgetting factor (FF). From computer simulation results, we verify that the proposed scheme outperforms the conventional optimal FF LC-RLS algorithm in terms of rapid tracking capability and strong narrow-band suppression.**

Keywords—linearly constrained RLS; variable forgetting factor; DS-CDMA; narrowband interference suppression; SINR

I. INTRODUCTION

The direct-sequence code-division multiple-access (DS-CDMA) is one of the primary modulation techniques which have been widely studied in the literatures [1]-[3]. It has emerged as the predominant multiple access technique for third-generation (3G) cellular systems because it increases capacity and facilitates network planning in a cellular network. In the downlink, these systems rely on the orthogonality of the spreading codes to separate the different user signals. By exploiting the spatial diversity of multiple transmit-antenna and receive-antenna (MIMO), channel capacity of the DS-CDMA systems can be increased effectively [4][5]. Moreover, MIMO systems with the space-time coding (STC) [4][6]-[8] can be used to counteract the effect due to channel fading and multipath propagation. A space-time (ST) encoding is bandwidth efficient schemes that improve the reliability of data transmission in MIMO systems. By doing so, more reliable detection can be achieved in the receiver. As described in [7] to reduce the computation complexity at receiver end, in the two-branch filter bank receiver design of ST-BC MIMO-CDMA systems, the weight-vector of one branch can be updated simply using the one already obtained with the proposed LC-RLS algorithm, with simple pre-calculated transform. In this paper, we simply focus on the single-input single-output (SISO) CDMA systems, since this idea can be extended easily to the ST-BC MIMO-CDMA systems [7]. In the DS-CDMA system, due to the inherent structure interference, named as the multiple access interference (MAI), system performance might degrade, significantly. To combat the effect of MAI, many efficient techniques such as the de-correlating, minimum mean-square error (MMSE), and minimum output energy (MOE) detectors were proposed [12]-[17]. In which, the adaptive multiuser detectors addressed in [16][17] are based on the modified MMSE structure with the linear constraint. With the modified MMSE structure described in [16] many adaptive filtering techniques were suggested to compensate the effect due to channel amplitude and phase variation, and hence improving the system performance. Also, in [17] the channel estimation scheme is accomplished at the output of the adaptive filter, in which the desired signal-to-interference-plus-noise-ratio (SINR) should be substantially higher than the input of adaptive filter, and has good capability for wide band interference suppression. In wireless communication systems, the realistic channel or environment is generally time-varying. For instances, the suddenly appearance of narrow-band interference (NBI), due to other systems. This may degrade the performance greatly in the DS-CDMA systems [18][19].

To deal with this problem many adaptive filtering techniques, such as the adaptive least mean squares (LMS) and recursive least squares (RLS) algorithms [9], have been extensively used to effectively extract the desired signals from the noisy received signal with interference. Among the existing adaptation algorithms the RLS algorithm generally offers better convergence rate, steady-state mean square-error (MSE), and tracking capability over the LMS-based algorithm [9]. Recently, the linearly constrained (LC) adaptive filtering approach with the RLS algorithm has been intensively used in the wireless communication systems to further suppressing various interference signals [7][8][19]. Depending on the stationarity of system environment, the value of forgetting factor (FF) (λ) in the RLS algorithm, which is used to control the stability and tracking capability, should be chosen properly. In the conventional exponentially weighted (EW) least square (LS) approach, the error is accumulated from the initial time up to the present with fixed value of FF. In fact, with fixed value of FF the RLS algorithm may have problem that it might not reflect properly to the changing statistics under time varying environment. As indicated in [19], under such circumstance the LC-RLS algorithm with sliding window (SW) could be used to combat this effect. With the SW-RLS algorithm, it discards old data to reduce the influence of the past estimate, thus enhancing the tracking capability in time-varying environment, with the penalty for requiring much more computation complexity.

In [10][11], it has been proved that the use of the RLS algorithm with the variable forgetting factor (VFF) could achieve better tracking property over the conventional RLS algorithm. This method is adapted to a non-stationary signal by an extended prediction error criterion which accounts for the non-stationarity of the signal. It has good adaptability in the nonstationary situation and low variance in the stationary situation. In this paper, we derive a novel LC-RLS algorithm with VFF. The performance improvement, in terms of SINR, with our proposed algorithm will be verified via computer simulations. With the same receiver model of [17], the proposed algorithm is devised and compared with the conventional exponential windowed (EW) linearly constrained (LC)-RLS and the orthogonal decomposition-based LMS algorithms.

II. SYSTEM MODEL DESCRIPTION

Let us consider a synchronous DS-CDMA system with K users and strong narrow band interference over Rayleigh fading channels. The received signal is represented by

$$r(t) = \sum_{k=1}^{K} A_k \alpha_k(t) e^{j\varphi_k(t)} \sum_{j=0}^{N-1} b_k(t) s_k(t-jT_c) + v(t) + \sigma u(t) \quad (1)$$

where $b_k(t)$ is the information bearing waveform of user *k*, A_k is the signal amplitude, $s_k(t)$ is the corresponding signature code sequence of user *k* with the length of the PN codes to be *N,* and T_c is the chip interval. Also, $\alpha_k(t)e^{j\varphi_k(t)}$ denotes the complex attenuation due to the Rayleigh fading parameter $\alpha_k(t)$ and Doppler frequency shift $\varphi_k(t)$ of user *k*. Moreover, *v*(*t*) and *u*(*t*) are the NBI signal and the additive white Gaussian noise (AWGN) with zero-mean and variance σ^2, respectively. For simplicity, we assume that the desired signal is the first user.

The VFF of the LC-RLS algorithm is derived, based on the adaptive constrained MMSE receiver [17] as illustrated in Fig. 1. In which, the channel parameter is estimated after the adaptive filter to degrade the effect due to noise. The modified constrained MMSE criterion is defined as [17]:

$$E\left[|e(n)|^2\right] = E\left[|\hat{c}_1(n)d_1(n) - \mathbf{w}^H(n)\mathbf{r}(n)|^2\right] \quad (2)$$

, subject to $\mathbf{w}^H(n)\mathbf{s}_1 = 1$. In (2) the reference signal is $d_1(n)$ multiplied by the channel estimate $\hat{c}_1(n)$, $\mathbf{w}(n)$ and $\mathbf{s}_1$ are the tap weight vector and the spreading code vector of the desired user, respectively. Also, $(\bullet)^H$ denotes the Hermitian operation. The complex-valued channel can be represented by $c_1(n) = \alpha_1(n)e^{j\varphi_1(n)}$, where $\alpha_1(n)$ and $\varphi_1(n)$ are amplitude and phase of fading channel, respectively. Similar to (2), the modified LC-LS criterion with the cost function is defined as

$$\xi(n) = \sum_{i=1}^{n} \lambda^{n-i} |e(n)|^2 = \sum_{i=1}^{n} \lambda^{n-i} |\hat{c}_1(n)d_1(n) - \mathbf{w}^H(n)\mathbf{r}(n)|^2 \quad (3)$$

, subject to $\mathbf{w}^H(n)\mathbf{s}_1 = 1$. In next section, the VFF-LC-RLS algorithm will be addressed.

III. THE LC-RLS ALGORITHM WITH NOVEL VARIABLE FORGETTING FACTOR

With (3) and using the similar approach as [8][19], the updating equation of tap-weight vector with LC (EW)-RLS algorithm is given by

$$\mathbf{w}(n) = \mathbf{w}(n-1) + [\mathbf{I} - \mathbf{g}(n)\phi^{-1}(n)\mathbf{s}_1^T]\mathbf{k}(n)e^*(n\,|\,n-1) \quad (4)$$

where $\mathbf{g}(n) = \mathbf{R}^{-1}(n)\mathbf{s}_1$ and $\phi(n) = \mathbf{s}_1^T\mathbf{R}^{-1}(n)\mathbf{s}_1$ are an $N\times1$ vector and a scalar, respectively. $\mathbf{k}(n)$ denotes as *Kalman gain* vector that is given by

$$\mathbf{k}(n) = \mathbf{R}^{-1}(n)\mathbf{r}(n) \quad (5)$$

where $\mathbf{R}(n)$ is defined as the input signal autocorrelation matrix, and the inverse of $\mathbf{R}(n)$ can be gained via $\mathbf{R}^{-1}(n-1)$ as follows

$$\mathbf{R}^{-1}(n) = \tfrac{1}{\lambda}[\mathbf{R}^{-1}(n-1) - \mathbf{k}(n)\mathbf{r}^H(n)\mathbf{R}^{-1}(n-1)] \quad (6)$$

To derive the variable FF of LC-RLS algorithm, the cost function is defined as [19]:

$$J'(n) = \tfrac{1}{2}E\left[|e(n\,|\,n-1)|^2\right] \quad (7)$$

where $e(n|n-1)$ denotes the a priori estimation error and is given by

$$e(n\,|\,n-1) = \tilde{d}(n) - \mathbf{w}^H(n-1)\mathbf{r}(n) \quad (8)$$

and in (8) $\tilde{d}(n) = \hat{c}_1(n)d_1(n)$ [19]. The objective is to find the optimal value of λ, that minimizes the cost function of (7). By applying the steepest descent method, a recursive form of $\lambda(n)$ is obtained to iteratively minimize (7) with respect to λ [9]:

$$\lambda(n) = \lambda(n-1) - \alpha\nabla_\lambda(n) \quad (9)$$

In (9) α is a positive and small value, it is related to the learning-rate, and the gradient of $J'(n)$, with respect to λ, is denoted as

$$\nabla_\lambda(n) = \frac{\partial J'(n)}{\partial\lambda} = -\mathrm{Re}[\boldsymbol{\psi}^H(n-1)\mathbf{r}(n)e^*(n\,|\,n-1)] \quad (10)$$

In (10) vector $\boldsymbol{\psi}(n)$ is defined as $\boldsymbol{\psi}(n) = \partial\mathbf{w}(n)/\partial\lambda$. From (4), we learn that weight-vector $\mathbf{w}(n)$ is simply a function of $\mathbf{R}^{-1}(n)$. For further discussion, we define matrix $\mathbf{M}(n) = \partial\mathbf{R}^{-1}(n)/\partial\lambda$, it can be represented as

$$\begin{aligned}\mathbf{M}(n) = \lambda^{-1}[\mathbf{I}-\mathbf{k}(n)\mathbf{r}^{H}(n)]\mathbf{M}(n-1)[\mathbf{I}-\mathbf{r}(n)\mathbf{k}^{H}(n)] \\ +\lambda^{-1}\mathbf{k}(n)\mathbf{k}^{H}(n)-\lambda^{-1}\mathbf{R}^{-1}(n)\end{aligned} \tag{11}$$

Consequently, the earlier defined vector $\boldsymbol{\psi}(n) = \partial\mathbf{w}(n)/\partial\lambda$, can be obtained:

$$\begin{aligned}\boldsymbol{\psi}(n) = [\mathbf{I}-\mathbf{k}(n)\mathbf{r}^{H}(n)+\mathbf{R}^{-1}(n)\mathbf{s}_1\phi^{-1}\mathbf{s}_1^{T}\mathbf{k}(n)\mathbf{r}^{H}(n)]\boldsymbol{\psi}(n-1) \\ +\{\mathbf{M}(n)\mathbf{r}(n)-\mathbf{M}(n)\mathbf{s}_1\phi^{-1}\mathbf{s}_1^{T}\mathbf{k}(n) \\ +\mathbf{R}^{-1}(n)\mathbf{s}_1\phi^{-1}\mathbf{s}_1^{T}\mathbf{M}(n)[\mathbf{s}_1\phi^{-1}\mathbf{s}_1^{T}\mathbf{k}(n)-\mathbf{r}(n)]\}e^{*}(n\,|\,n-1)\end{aligned} \tag{12}$$

Finally, with (9) and its related terms defined in (10)-(12), the variable FF of the LC-RLS algorithm could be updated, iteratively.

IV. COMPUTER SIMULATIONS

In this section, to document the merits of the proposed algorithm, computer simulation results are given. The uplink channel is considered, and assumed that all active users are experiencing different Rayleigh fading channels. In the computer simulations, the non-orthogonal Gold codes with length N=31 are employed and the number of active users K=5 is assumed. Also, synchronous pilot symbol-aided BPSK DS-CDMA system is adopted. The channel bandwidth is 3.968 MHz, the carrier frequency is 2.0 GHz (consistent with that given in [17]), and the mobile speed is set as 50 km/h. E_b/N_0 of the desired user is set to be 20dB, and the powers of all active users are set to be equal, where E_b and N_b denote the signal and noise power, respectively. With the adaptive receiver described in Fig.1, the channel coefficients are estimated with pilot signals, that is

$$\hat{c}_1(i) = \frac{1}{N_p}\sum_{j=1}^{N_p} b_{1p}(i-jM)\mathbf{w}^{H}(i-jM)\mathbf{r}(i-jM) \tag{13}$$

In (13) each pilot symbol is periodically inserted into data symbol streams at every eight data bits (i.e. M=8), and the number of pilot symbols used for channel estimation is $N_p = 3$ [17]. The tap weight vector for the adaptive filter is initialized as $\mathbf{s}_1$.

To demonstrate the superiority of the proposed algorithm, a specific case is investigated. That is, a strong NBI is appeared and joined at 800 number of bit iteration, while the output of the adaptive receiver is in steady state. The NBI signal $v(n)$ that is generated by the AR(2) model, that is, $v(n)+a_1v(n-1)+a_2v(n-2)=\beta(n)$, where $\beta(n)$ is a white Gaussian noise, $a_1 = -1.98$, and $a_2 = 0.9801$. The forgetting factor of conventional LC-RLS algorithm is chosen to be λ=1 and λ=0.98, respectively, while the step size $\mu = 1/\text{maximal tap weight power}$ for LMS algorithm is selected. In all simulations, the results are the averaging of 100 independent runs, and the initial value of the proposed forgetting factor of VFF-LC-RLS algorithm is set as $\lambda(0)$=1.Two cases are considered to investigate the performance. The results of learning curves of output SINR are shown in Fig.2 and Fig.3, respectively.

In these two cases, the values of jammer-to-signal ratio (JSR) for the joined strong narrow band interference are selected as JSR=20dB and JSR=30dB, respectively. In which the step-size α of the proposed VFF-LC-RLS algorithm is given as 10^{-5} and 10^{-6}, respectively. As observed from Fig.2, the output SINR with the VFF-LC-RLS algorithm performs the best. Also, the conventional EW-LC-RLS algorithm performs superior than the one with the orthogonal decomposition-based LMS algorithm. However, other two algorithms could not track well, when the NBI jointed suddenly, as the proposed algorithm. Particularly, when the JSR=30 dB is considered, the tracking ability of orthogonal decomposition-based LMS algorithm becomes worse. Again, we observed that with lower value of forgetting factor, it achieves faster convergence rate as our proposed VFF-LC-RLS algorithm after NBI being joined at 800th iteration. But, it performs inferior in the steady state. Finally, we may conclude that the proposed scheme could work successfully in practical wireless communication systems, in which the MAI and NBI, introduced by other communication systems, could be suppressed, simultaneously.

V. CONCLUSIONS

In this paper, the variable forgetting factor (VFF) of the LC-RLS algorithm has been developed and applied to the Rayleigh fading channel in the DS-CDMA system for MAI cancellation, under the consideration of suddenly joined narrow-band interference due to other systems. From simulation results, it verified that the proposed algorithm outperformed the orthogonal decomposition-based LMS and the conventional EW-LC-RLS algorithm with fixed forgetting factor, in terms of SINR. It has the advantage of faster convergence and better tracking capability compared with the conventional EW-LC-RLS algorithm and the orthogonal decomposition-based LMS algorithm [14], when the NBI joined suddenly to the DS-CDMA system. Besides, we also proved that the proposed algorithm could maintain the performance when system is in stationary situation while the lower values of forgetting factor are g*iven, it may decrease the output performance.*

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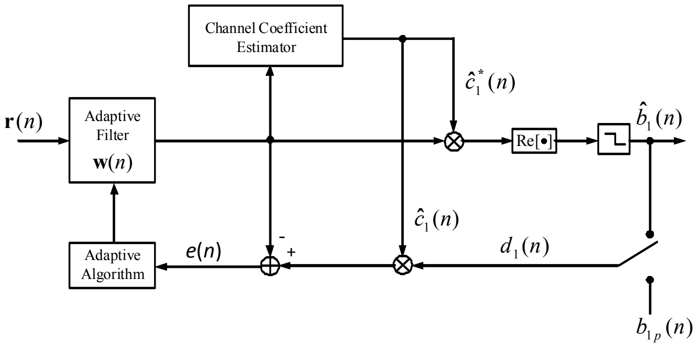

Fig.1. Block diagram of an adaptive receiver with the phase and amplitude compensation.

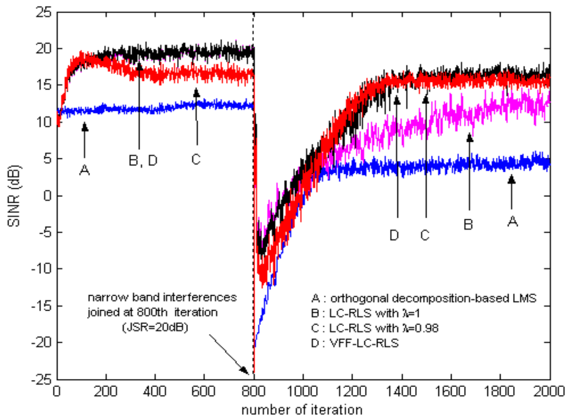

Fig.2. Learning curves comparison with different techniques under fading channel with a suddenly joined narrow-band interference for JSR=20db.

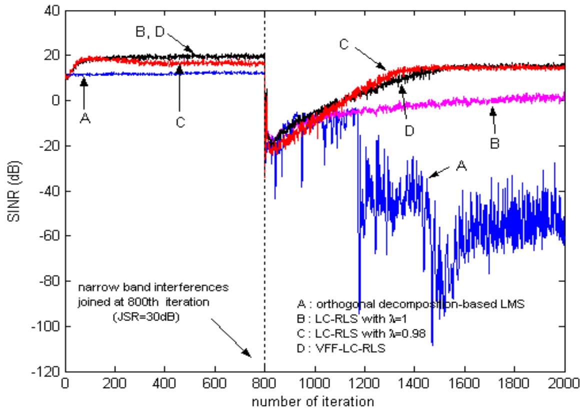

Fig.3.Learning curves comparison with different techniques under fading channel with a suddenly joined narrow-band interference for JSR=30db.